Longitudinal radiation excitation in an electron storage ring

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In an electron storage ring, even if electrons lose the same amount of energy by photoemissions in one turn, their path lengths vary according to where the photoemissions take place. Usually this fluctuation of the path length hides behind the well-known radiation excitation of energy spread. In a quasi-isochronous ring, however, the fluctuation of path length dominates over the bunch stretch owing to the radiation excitation of energy spread. We give an analytical expression for the equilibrium state within linear optics including the fluctuation of the path length. This formula gives an intrinsic lower limit of the equilibrium bunch length in a quasi-isochronous storage ring. [S1063-651X(96)51511-4]

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A quasi-isochronous ring is expected to be a solution for producing an ultrashort bunch in electron or positron storage rings. The ultrashort bunch can offer short-pulsed brilliant light in a synchrotron light source or high luminosity in lepton colliders. The idea of utilizing a quasi-isochronous ring to shorten the bunch length comes from a well-known expression that the equilibrium bunch length is proportional to the square root of the linear momentum compaction factor $\sqrt{\alpha_1}$ [1]. The shortening of the equilibrium bunch length by means of reducing α_1 has been demonstrated at several synchrotron light source rings, such as NSLS [2], UVSOR [3], ALS [4], Super-ACO [5], and ESRF [6] and is planned at LNLS [7], SPEAR [8], and the new ring at Himeji Institute of Technology [9]. In their experimental and theoretical works [10–13] it is shown that the nonlinear part of the momentum compaction factor (mainly α_2) or instability by the high beam density restricts the bunch to shorten. However, in the recent designs of a quasi-isochronous ring, α_2 can be adjusted to zero by using sextupole families [3,7,12]. Therefore, in these new rings with low beam current the limit to the bunch shortening is not clarified. In this paper we introduce the idea of *longitudinal quantum radiation excitation*, which gives a limit to the shortening of equilibrium bunch length in a quasi-isochronous ring. From now on we will refer to it simply as *longitudinal excitation*, while the conventional radiation excitation of energy spread is *energy excitation*.

The idea of longitudinal excitation is that a stochastic fluctuation of where the photoemission takes place produces a fluctuation of path length of one revolution. If the photoemission takes place at the first bending dipole, the electron goes around most of the ring with lower energy compared with the starting one. On the other hand, if the photoemission happens at the last bending dipole, the electron goes around most of the ring with starting energy and finishes the revolution with the same energy as in the previous case. When α_1 is positive, the revolution time of the former case is shorter than that of the latter. This variation of the path length is statistical because it comes from the stochastic variation of where the photoemission takes place.

In usual storage rings the longitudinal excitation is buried in the energy spread excitation and can hardly be observed. On the other hand, in a quasi-isochronous storage ring the longitudinal excitation becomes conspicuous, since one can reduce α_1 to zero but cannot do the longitudinal excitation. Another characteristic of a quasi-isochronous storage ring is that a synchrotron oscillation period can approach a radiation damping time, where the energy and the time displacements couple weakly with each other. In that case the effect of the longitudinal excitation on the equilibrium bunch length is dominant because the radiation damping of the energy spread scarcely works in the longitudinal axis. Our calculation gives a limit to shortening the equilibrium bunch length by reducing α_1 in electron (or positron) storage rings. In the following, ignoring the nonlinear effect, we refer to the linear momentum compaction factor α_1 simply as α.

To derive analytical expressions for the longitudinal excitation, we assume the following: (a) circulating electrons or positrons are ultrarelativistic; (b) the energy spread is very small compared with the reference energy of circulating particles; (c) the amounts of the radiation damping and the radiation excitations per revolution are very small compared with the equilibrium energy spread and the bunch length; (d) the ring has only one rf gap; (e) all events of photoemission are purely stochastic and independent of each other; (f) the absolute value of curvature radius in bending dipoles is constant. These assumptions permit one to average statistic quantities over ensembles independently.

We assume that the rf gap is located at $s=0$ or $s=L_0$, where s is the azimuthal coordinate and L_0 is the circumference of a ring. At the lowest order of perturbation, the change of path length in one turn caused by N photoemissions is expressed by

$$\Delta L=\sum_j^N \int_{s_j}^{L_0}\frac{1}{\rho(s)}\left[\eta(s)\frac{u_j}{E_0}+x_j\left(s,\frac{u_j}{E_0}\right)\right]ds, \qquad (1)$$

where $\eta(s)$, $\rho(s)$, and E_0 are, respectively, the horizontal dispersion function, the radius of curvature, and the synchronous energy. Symbols u_j and x_j are the energy loss and the betatron displacement produced by the jth photoemission at the azimuthal location s_j. In this paper Δ stands for the change by the photoemissions in one turn. Both the loss of energy and the change of betatron oscillation produce the change of the path length of one turn. However, the betatron oscillation caused by a photoemission can be treated separately from the synchrotron oscillation because it makes no time displacement after being averaged over revolutions. Then, we omit the contribution of the excited betatron oscillation from Eq. (1) as follows:

$$\Delta L=\sum_{j}^{N}\int_{s_j}^{L_0}\frac{\eta(s)}{\rho(s)}\frac{u_j}{E_0}ds. \tag{2}$$

The displacement in relative energy by N photoemissions and that of the arriving time from the reference are

$$\frac{\Delta E}{E_0}=\sum_{j}^{N}\frac{u_j}{E_0}, \tag{3}$$

$$\Delta\tau=\frac{T_0}{L_0}\sum_{j}^{N}\int_{s_j}^{L_0}\frac{\eta(s)}{\rho(s)}\frac{u_j}{E_0}ds, \tag{4}$$

where T_0 is a revolution period. Introducing the partial momentum compaction factor $\tilde{\alpha}(s_j)$ defined by

$$\tilde{\alpha}(s_j)=\frac{1}{L_0}\int_{s_j}^{L_0}\frac{\eta(s)}{\rho(s)}ds, \tag{5}$$

Eq. (4) is simply rewritten by

$$\Delta\tau=T_0\sum_{j}^{N}\frac{u_j}{E_0}\tilde{\alpha}(s_j). \tag{6}$$

For the sake of convenience we write the variance of $\tilde{\alpha}$ as I_α, which is calculated only from the geometrical parameters of a ring:

$$I_\alpha=\frac{1}{L_0^2}\left\langle\left[\int_{s_j}^{L_0}\frac{\eta(s)}{\rho(s)}ds-\langle\tilde{\alpha}\rangle L_0\right]^2\right\rangle. \tag{7}$$

It should be emphasized that I_α approaches a nonzero constant value as α diminishes to zero and substantially gives the lower limit of the bunch length in a quasi-isochronous ring. By the assumption that the statistics of N, u_j, and s_j can be treated independently, the variances of $\Delta E/E_0$ and $\Delta\tau$ are calculated as

$$\left\langle\left\langle\left\langle\left(\frac{\Delta E}{E_0}-\left\langle\left\langle\left\langle\frac{\Delta E}{E_0}\right\rangle\right\rangle\right\rangle\right)^2\right\rangle\right\rangle\right\rangle=\langle N\rangle\left\langle\frac{u^2}{E_0^2}\right\rangle, \tag{8}$$

$$\langle\langle\langle(\Delta\tau-\langle\langle\langle\Delta\tau\rangle\rangle\rangle)^2\rangle\rangle\rangle=T_0^2I_\alpha\langle N\rangle\left\langle\frac{u^2}{E_0^2}\right\rangle, \tag{9}$$

where the brackets $\langle\ \rangle$ represent an average over N, over photon energy or over s_j only at the bending dipoles. Here we have used the fact that N obeys the Poisson distribution.

To calculate the equilibrium bunch length, we start with a matrix equation of the linearized synchrotron oscillation in a storage ring. The method is the same as that used to calculate the effect of the rf noise [14]. A one turn transfer of a synchrotron oscillation is described by

$$X_n=AX_{n-1}+D_n. \tag{10}$$

Here the vectors X_n and D_n denote, respectively, the state of a particle in the synchrotron oscillation phase space and the radiation excitation in the nth turn and A is a transfer matrix including radiation damping. The state vector X_n is defined at just before the rf gap ($s=L_0$) by

$$X_n=\begin{pmatrix}\left.\frac{\delta E}{E_0}\right|_n\\ \left.\delta\tau\right|_n\end{pmatrix}, \tag{11}$$

where $(\delta E/E_0)|_n$ and $\delta\tau|_n$ are the relative energy displacement and the arriving time displacement from the barycenter at just before the rf gap in the nth turn. In this paper δ stands for a displacement from the barycenter. A matrix notation for the motion in the rf gap is

$$\begin{pmatrix}1 & \frac{eV'}{E_0}\\ 0 & 1\end{pmatrix} \tag{12}$$

with V' the gradient of the rf acceleration voltage. A transfer through a circumference except for the rf gap including the radiation damping is described by a matrix

$$\begin{pmatrix}1-2\kappa_sT_0 & 0\\ -\alpha T_0 & 1\end{pmatrix}, \tag{13}$$

where κ_s denotes the radiation damping coefficient. Hence the one turn transfer matrix A is written as

$$A=\begin{pmatrix}1-2\kappa_sT_0 & \frac{eV'}{E_0}(1-2\kappa_sT_0)\\ -\alpha T_0 & 1-\alpha T_0\frac{eV'}{E_0}\end{pmatrix}. \tag{14}$$

The vector D_n is described by

$$D_n=\begin{pmatrix}\left.\frac{\Delta E}{E_0}\right|_n-\left\langle\left\langle\left\langle\frac{\Delta E}{E_0}\right\rangle\right\rangle\right\rangle\\ \Delta\tau|_n-\langle\langle\langle\Delta\tau\rangle\rangle\rangle\end{pmatrix}. \tag{15}$$

The suffix n in the averaging brackets is omitted because the averaged values are independent of the turn number n.

From Eq. (10), we can rewrite the state vector X_n in terms of the initial state X_0 as

$$X_n=A^nX_0+\sum_{m=0}^{n-1}A^mD_{n-m}. \tag{16}$$

Both absolute values of two eigenvalues of A are less than unity when $\alpha eV'>0$ whether the eigen values are complex or real. This condition certifies the existence of an equilibrium state, so that

$$X_\infty=\sum_{m=0}^{\infty} A^m D_m . \tag{17}$$

Here, using the statistic independence of the radiation excitation, we have changed the suffix of D_{n-m} as D_m.

Although the energy spread is constant over the circumference, the equilibrium bunch length varies along s [15]. The state vector after infinite turns at any place s in the ring $X_\infty(s)$ is given by a transformation from $X_\infty=X_\infty(L_0)$ as follows

$$\begin{pmatrix} 1 & 0 \\ -\tilde{\alpha}(s)T_0 & 1 \end{pmatrix} X_\infty(s)=X_\infty . \tag{18}$$

The equilibrium energy spread is given by a variance of the energy displacement of $X_\infty(s)$, i.e.,

$$\sigma_E^2=\frac{1+(\Omega^* T_0)^2 I_\alpha/\alpha^2}{1-(\frac{1}{2}\,\Omega^* T_0)^2}\,\sigma_{EN}^2, \tag{19}$$

with

$$(\Omega^* T_0)^2=\frac{\alpha eV'}{E_0}\,T_0 . \tag{20}$$

Here σ_{EN}^2 is the conventional natural energy spread given by

$$\sigma_{EN}^2=\frac{1}{4\kappa_s T_0}\langle N\rangle\left\langle\frac{u^2}{E_0^2}\right\rangle. \tag{21}$$

The second term of the numerator in Eq. (19) comes from the longitudinal excitation. In many storage rings other than quasi-isochronous ones, $\Omega^* T_0$ is a synchrotron tune times 2π and I_α/α^2 is almost 1/12. Therefore the energy excitation is dominant against the longitudinal one.

The variance of the equilibrium bunch length $\sigma_\tau^2(s)$ is a quadratic function to $\tilde{\alpha}(s)$ and takes a minimum

$$\sigma_{\tau,\,\mathrm{min}}^2=\left[1+(\Omega^* T_0)^2\frac{I_\alpha}{\alpha^2}+\frac{4(\kappa_s T_0)^2 I_\alpha}{\alpha^2+(\Omega^* T_0)^2 I_\alpha}\right]\left(\frac{\alpha}{\Omega^*}\right)^2\sigma_{EN}^2 \tag{22}$$

at

$$\tilde{\alpha}(s)=\frac{\alpha}{2}\left[1-\frac{4\kappa_s T_0 I_\alpha}{\alpha^2+(\Omega^* T_0)^2 I_\alpha}\right]. \tag{23}$$

In Eq. (22), the second and the third terms in the square brackets represent the contribution from the longitudinal excitation. In the case of a sufficiently small α, Eq. (22) is approximated as

$$\sigma_{\tau,\,\mathrm{min}}^2\approx T_0^2\left[I_\alpha+4\left(\frac{\kappa_s E_0}{eV'}\right)^2\right]\sigma_{EN}^2. \tag{24}$$

One can easily confirm that I_α approaches a nonzero constant as $\alpha\to 0$ and that $\sigma_{\tau,\,\mathrm{min}}$ has a lower limit. Because $4[(\kappa_s E_0)/(eV')]^2$ is always positive, $\sigma_{\tau,\mathrm{min}}$ cannot be smaller than

$$\sigma_{\tau,0}^2=T_0^2 I_\alpha \sigma_{EN}^2. \tag{25}$$

This is an intrinsic limit of the bunch length determined by the ring geometrical parameters I_α, T_0, and σ_{EN}^2. In this case the bunch length takes the minimum value at

$$\tilde{\alpha}(s)\approx-\frac{2\kappa_s E_0}{eV'}. \tag{26}$$

Although in a quasi-isochronous ring $\tilde{\alpha}(s)$ sometimes becomes negative, it does not always satisfy Eq. (26), i.e., the bunch length does not necessarily take the minimum value throughout the circumference.

Now we qualitatively discuss what would happen in an ideal isochronous ring. In the time axis the bunch length is not frozen but continues spreading because of the longitudinal excitation. This is one of the most significant differences from an electron linac, which is isochronous. In the energy axis, when there is a voltage gradient, a spread in the time axis produces a difference of the energy gain per revolution. The energy barycenter at any $\delta\tau$ would approach the energy at which the energy gain and the radiation loss are balanced. Therefore in the $\delta E/E_0$-$\delta\tau$ phase space the beam would continue spreading along a tilting line, so that the beam cannot be stored in the isochronous ring.

The effect of the path-length fluctuation by stochastic photoemission in an electron storage ring is discussed. In usual (far from isochronous) storage rings the longitudinal excitation can be ignored. However, in the isochronous case ($\alpha\approx 0$) the longitudinal excitation is dominant against the energy excitation. In other words, the bunch length of a quasi-isochronous ring is limited by the longitudinal excitation.

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