Convective roll patterns in vertically vibrated beds of granules

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This paper describes dynamic transitions of convection motions of granules under vertical vibrations. The granule motion of one convection mode, which has been studied by many authors, is downward along the vertical side walls and upward in the middle forming a heap. The present study focuses on the granule motion of the other convection mode that occurs in rectangular containers: granules move upward along the vertical side walls, and downwards in the middle forming a valley. This latter convection mode is stable enough to yield multiple pairs of convection rolls. The number of convection rolls is strongly dependent on the vibrational acceleration, granular size, and to a lesser extent on frequency, given the bed height and width. The critical vibrational acceleration, where a transition occurs between the two convection modes, is dependent on the bed height but not on the bed width in the range investigated (100–200 mm). In the case of large granules (mean diameter $\overline{d} \geqslant 0.78$ mm) a chaotic state (where convective motion of granules ceases to exist) is found to appear in between the two convection modes mentioned above. [S1063-651X(96)05307-X]

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I. INTRODUCTION

Noncohesive granular materials are held together mainly by gravity. The discreteness allows the granules to fluidize, exhibiting fluidlike behavior under certain conditions: for example, granules flow through apertures and avalanches in piles of sands. As a means of fluidizing granular materials, vibrated beds are widely used in solid processing industries. In vibrated beds, granular materials behave like both continuous and discrete media, yielding many interesting phenomena [1]. Heap formation in vibrated beds is a phenomenon that has recently attracted increasing attention among physicists. Attempts to explain the mechanism of this phenomenon have been done both experimentally [2] and theoretically using computer simulations [3]. However, consensus on the origin of the heap has not been reached yet. Nearly all studies have been concerned with the convection wherein the granule motion is downward along the vertical side walls and upward in the middle, forming a heap. The above granule motion does not always give rise to heap formation: the surface can be nearly flat. Hereafter this mode of convection is referred to as downward mode convection, referring to the motion near to the vertical sidewalls.

Another mode of convection, wherein granules move upward along the vertical walls and downward in the middle forming a valley, hereafter referred to as upward mode, was observed earlier in our laboratory [4], and a brief account of the upward mode was also given in [5]. However, systematic studies on the conditions when the upward mode convection occurs have not been done yet. It should be noted here that we are concerned with the convection in rectangular containers having vertical side walls; the upward mode appears to be the norm for conical containers or when the side walls are slanted outwardly [6].

The upward mode has also been observed by computer simulations [7]. It has been known that irrespective of the models (whether it is a distinct element method [8] or hard-core models), the direction of particle movement changes depending on the relative magnitude between the granule-wall and granule-granule shear friction coefficients [9]. These simulation studies indicate that when the aspect ratio W/H, where W is the bed width and H the height, is much greater than unity convection rolls appear only near to the walls [10]. In between the two rolls no clear-cut pattern formation is observed. No multiple pairs of convection rolls have been reported to date by particle dynamics computer simulations.

This paper presents dynamic phase transitions in the roll pattern during granular convection. We have investigated the conditions when the transition occurs between the upward and downward modes of convections; furthermore, we have discovered that under certain conditions there appear multiple pairs of convection rolls, equivalent to the roll patterns of Rayleigh-Bénard convection in fluids. In studying the granular convection special attention should be given to the role of the upward mode convection, because the multiple pairs of convection rolls appear only with the upward mode in rectangular containers with vertical side walls. We have found that the number of multiple pairs of convection rolls is dependent strongly on the vibrational acceleration, granular size, and to a lesser extent on frequency (It should also be mentioned that the convection rolls appear only in a limited range of frequency), given the bed height and width. Systematic experiments have been carried out to investigate the effect of operational parameters—such as vibrational acceleration, frequency, particle size, and the aspect ratio of the container—on the pattern of granular convection. A full understanding of the dynamics of granule motion must await theoretical progress, for which the present study presents valuable data. Experimental setup and procedures are explained in Sec. II. Section III describes results consisting of six subsections, the first section giving a general description of experimental observations. The following subsections examine influences of experimental parameters on the dynamic transitions. Section IV contains discussion and concluding remarks.

II. EXPERIMENTAL SETUP

We used parallelepipedic glass vessels fixed on an electromagnetic vibrator (available frequency $f=$ 5–3000 Hz, maximum amplitude 5 mm, and maximum vibrational acceleration $30g$, where g is the gravitational acceleration) which created motions of vertical sine waves with deformation less than 0.5%. The electromagnetic vibrator was placed on a vibration-proof foundation. Experimental findings with the use of three types of glass containers I, II, and III having the same thickness (30 mm) and height (200 mm) but different widths 100, 150, and 200 mm, respectively, are reported in this paper (preliminary experiments were carried out using acrylic containers with thickness 15, 30, 40, and 100 mm, the results of which will be briefly explained in Sec. IV). After applying an electrostatic inhibitor and drying, we filled the glass container with granules to varying heights to investigate the effects of the bed height. Dyed granules of the same size range were placed on the top of the bed (thickness 2 mm) for visualization when photographs were taken. To examine the size effect nine sizes of glass beads whose mean diameter $\overline{d}=0.10$, 0.23, 0.34, 0.46, 0.61, 0.78, 0.93, 1.09, and 1.29 mm were used. In addition, some experiments were carried out with two sizes of polystyrene, millet seeds, and cylindrical plastic pellets. After filling the container with glass beads to a set height it was subjected to vertical sinusoidal vibrations. Occurrence of the upward mode convection was observed at frequencies 20 Hz $<f<$ 200 Hz, although the convection rolls deformed considerably near the upper and lower frequency limits. The heap convection (downward mode) occurred for a wider frequency range. The vibrational acceleration $\Gamma=A(2\pi f)^2$, where A is the amplitude of the vibration was gradually increased or decreased in the range of $0<\Gamma<10.5g$: it was possible to measure the precision of hundredth of g by a vibration pickup attached to the vessel. However, the study on reproducibility indicated that there existed experimental deviation of $\pm 0.5g$. This was not due to the specification of the experimental apparatus, but rather due to the intrinsic nature of granular materials where instability played a crucial role. Nevertheless a good reproducibility of the overall trend of these data has been confirmed. The glass vessel was coated with electrostatic inhibitor and dried once every acquisition of five data points, since granules subjected to vibration for a long time will be charged with static electricity, altering their performance.

III. RESULTS

A. General description of phenomenon

When granular beds are subjected to vertical vibrations, they start to fluidize at a certain threshold value of Γ, slightly greater than gravity. With the increase of Γ value a convective motion in the downward mode appears. The convection in the downward mode is not very stable, hence the symmetry of the heap easily breaks resulting in an inclined slope. If the magnitude of Γ is increased further a transition occurs, yielding the upward mode: the granules move upward near to the wall and downward in the middle forming a valley. This transition, hereafter referred to as DU transition for brevity, has a much more significant meaning than just a change in the direction of granular motion, because compared to the downward mode the upward mode yields a very stable roll pattern. Thus once the valley is formed its position does not vary even though the convection speed increases considerably after the transition. This indicates that the mechanism of the upward mode convection is different from that of oscillating granular waves in shallow beds [11,12] in that the free surface profile does not depend on the vibration phase. Considering the speed of upward convection, which is an order of magnitude faster than that of the downward mode after the transition, it can be said that the vibrational energy is transmitted to the kinetic energy (as collective motions of granules) more effectively in the upward mode than in the downward mode. The upward convection is stable enough to form multiple pairs of convection rolls. Doubling the container width leads to a twofold increase in the number of convection rolls. The dynamic process that shows the formation of four rolls is presented in Fig. 1. The last three photographs show that the dyed beads have moved approximately half a circle in nine seconds. The experiment with a container of half the width (100 mm), with other condition fixed, led to a pair of convection rolls (not shown).

Multiple pairs of convection rolls can be obtained not only by changing the aspect ratio of the bed, but also by increasing the value of Γ. Figures 2 and 3 show the dynamics of convection roll formation under different values of Γ, with other condition fixed. The initial state in the experiments of Fig. 3 is identical to that shown in Fig. 2(a), thus excluded for brevity. It is clearly seen that the numbers of rolls multiply at higher values of Γ.

As a representative case, the phase diagrams of $\overline{d}=0.78$ mm, measured in a glass container of width 100 mm at 50 Hz, are shown in Fig. 4. The values of Γ_c when the numbers of rolls (represented by symbols) appear with increasing Γ are plotted in Fig. 4(a) while the values of Γ_c when the numbers of rolls disappear with decreasing Γ are plotted in Fig. 4(b). The downward mode prevails in the region denoted by D, and the upward mode in the region denoted by U, which is followed by the number of rolls. From Fig. 4 we can derive such information that at a given bed height $H=16$ mm the convection mode changes from the downward to the upward at $\Gamma=4.6g$. Further increases in the value of Γ result in formations of four convection rolls at $\Gamma=8.0g$, six rolls at $\Gamma=8.4g$, and eight rolls at $\Gamma=9.4g$. The pattern of eight convection rolls persists until Γ reaches $10.5g$. When the value of Γ is decreased from $10.5g$, the number of rolls changes from eight to six at $\Gamma=6.4g$. Further decreases in the value of Γ lead to reductions of the numbers of rolls, from six to four at $\Gamma=5.2g$, from four to two at $\Gamma=4.6g$, and finally resulting in the transition from the upward to the downward mode at $\Gamma=4.4g$. For beds of $H<16$ mm, the phase of four rolls in the upward mode directly changes into the phase of downward mode with decreasing Γ. The transition involving the change in the direction of granule motion (transition from region D to region U) occurs around $4.5g<\Gamma<5.1g$ whether Γ is increased or decreased and only small hysteresis exists when $H<45$ mm. A sudden jump in the value of Γ_c occurs when Γ is increased for beds of $H\geqslant 45$ mm, giving rise to a large hysteresis. This has an implication with an indirect transition, the details of which are explained in Sec. III D. Focusing attention on the

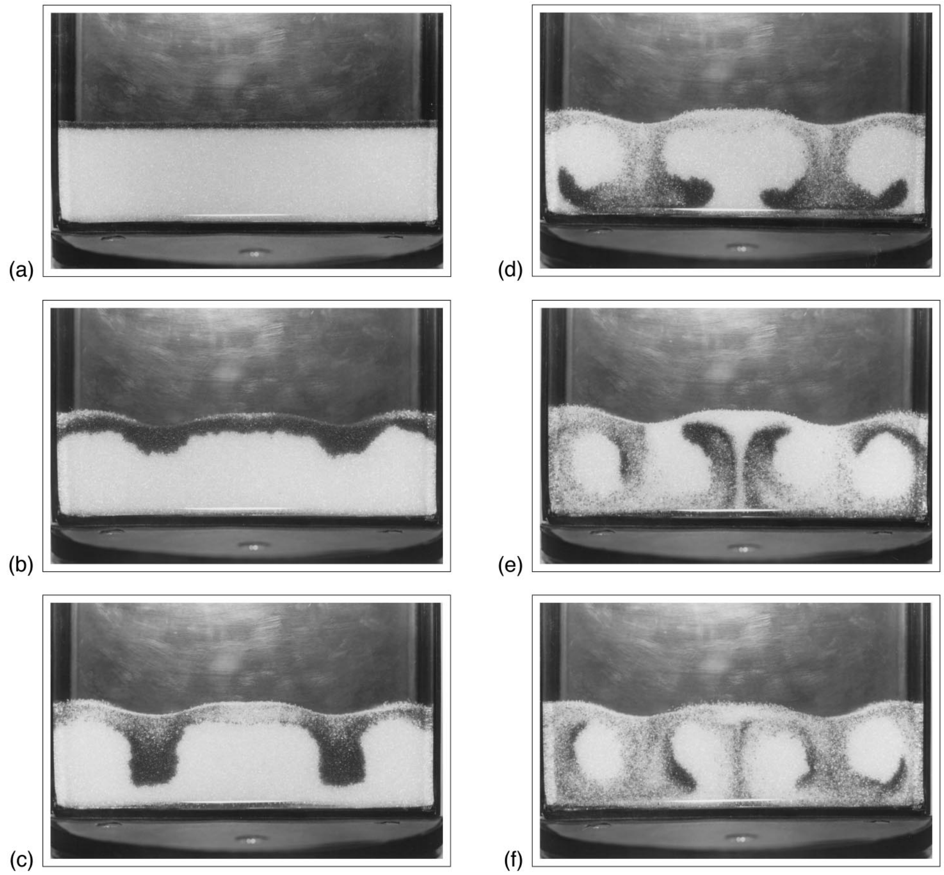

FIG. 1. Formation of two pairs of convection rolls of glass beads of $\bar{d}=0.61$ mm in a glass container of 200 mm width, 30 mm thickness, and 200 mm height: vibrational acceleration $6.4g$, frequency 50 Hz, and initial bed height $H=50$ mm at (a) 0 sec, (b) 5 sec, (c) 8 sec, (d) 14 sec, (e) 23 sec, and (f) 32 sec.

case of four rolls, one can see the hysteresis to decrease with increasing bed height, and at $H=42$ mm the phase of four rolls ceases to exist, never appearing again in the phase diagram for lager values of H. A similar tendency can be seen for the cases of larger roll numbers ($N\geqslant 6$), but due to the experimental limitation for safety, $\Gamma<10.5g$, it is not clear whether the phases of larger roll numbers disappear altogether. The values of Γ_c depend on H rather than on the aspect ratio W/H of the bed as will be seen in the next subsection. The number of rolls N, however, is dependent on W/H. It can also be seen from Fig. 4 that multiple convection rolls emerge with both smaller and larger aspect ratios than $W/H=N$ where the rolls can be cylinders. This implies that multiple pairs of convection rolls can be elongated in vertical or horizontal direction depending on the bed height. Comparison of Fig. 4(a) and 4(b) reveals that the hysteresis involving the transition between the upward and the downward modes is usually small; in contrast, the hysteresis involving the change in the number of convection rolls is usually quite large. A similar phase diagram was obtained for the case of $\bar{d}=0.61$ mm although slightly larger Γ values were necessary for the transitions to occur.

The phase diagrams reported in this paper were obtained by slowly increasing or decreasing Γ: the bed was at the steady state at each Γ. If we rapidly increase Γ from 0 to $10g$ at $H=16$ mm for the conditions given in Fig. 4, eight rolls immediately appear: four rolls will not appear prior to the appearance of eight rolls. Similar transitions occur when Γ is decreased rapidly.

B. Bed height dependence and granular size dependence

To investigate whether the critical vibrational acceleration Γ_c is simply a function of bed height alone or is dependent on the aspect ratio W/H, experiments were carried out using three containers I, II, and III, the widths of which were 100, 150, and 200 mm, respectively. Representative results when Γ was increased at $f=50$ Hz are shown in Figs. 5 to 7, each representing the results for beds of $\bar{d}=0.34$, 0.46, and 0.61 mm, respectively. In the case of small glass beads, $\bar{d}=0.34$ mm, only the DU transition occurs, thus only one branch of the data exists (Fig. 5). In contrast, for cases of larger glass beads ($\bar{d}=0.46$ and 0.61 mm) the second transition (change in the roll numbers in upward mode) occurs also, resulting in additional branches of data in Figs. 6 and 7. In all cases Γ_c increases with increasing bed height, and the following relationship holds:

$$\Gamma_c^{2\mathrm{nd}}/\Gamma_c^{1\mathrm{st}}\simeq 1.7.$$

It appears from Figs. 5–7 that Γ_c can be correlated (especially for the DU transition) by the bed height alone. The number of rolls N , however, is dependent on W/H, although general relation between N and W/H remains to be explored.

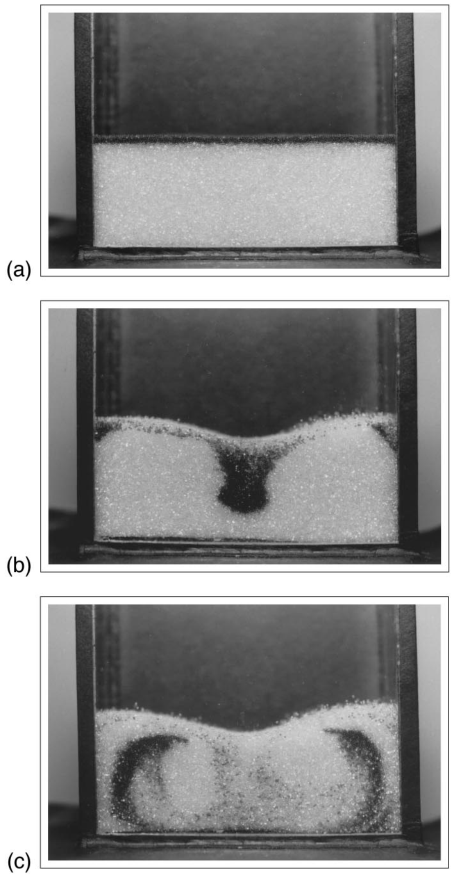

FIG. 2. Formation of a pair of convection rolls of glass beads of $d=0.61$ mm in a glass container of 100 mm width, 30 mm thickness, and 200 mm height: vibrational acceleration $5.9g$, frequency 50 Hz, and initial bed height $H=37$ mm at (a) 0 sec, (b) 9 sec, and (c) 19 sec.

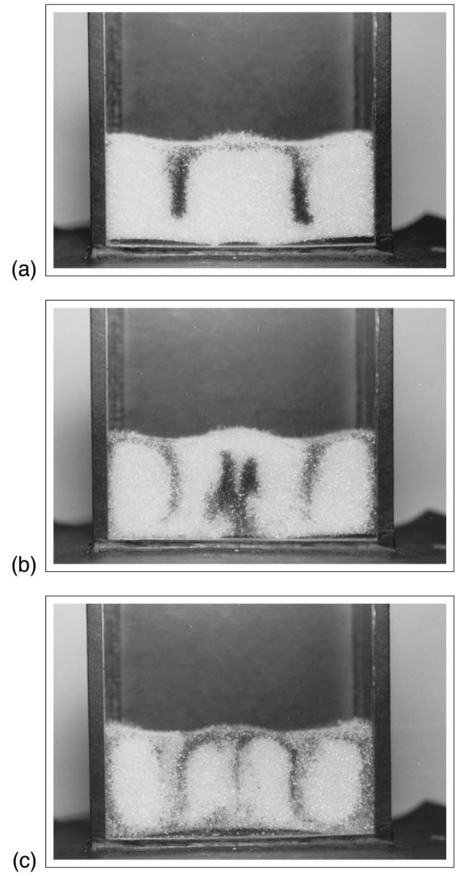

FIG. 3. Formation of two pairs of convection rolls of glass beads of $d=0.61$ mm in a glass container of 100 mm width, 30 mm thickness, and 200 mm height: vibrational acceleration $8.5g$, frequency 50 Hz, and initial bed height $H=37$ mm at (a) 9 sec, (b) 19 sec, and (c) 29 sec.

It also should be noted that Γ_c cannot be simply scaled by H/d. The upward convection was not observed for beds of $d=0.10$ and 0.23 mm within the acceleration range investigated ($\Gamma<10.5g$). Extrapolation of Γ_c values suggests that the DU transition may occur at $\Gamma_c \simeq 12g$ for the bed of $d=0.23$ mm with $H=$ 20 mm.

C. Frequency dependence

We have thus far dealt with the case of $f=50$ Hz. Frequency dependence of Γ_c is explored in this section. Figures 8 and 9 show Γ_c vs frequency f for beds of $d=0.46$ and 0.61 mm, respectively, with the initial bed height kept at 20 mm in container I. Critical vibrational acceleration Γ_c of the DU transition in beds of $d=0.46$ mm (Fig. 8) is seen to decrease with increasing frequency. This tendency is less noticeable for beds of $d=0.61$ mm (Fig. 9), and for beds of $d \geqslant 0.78$ mm the dependence of Γ_c on f diminishes. Overall, the Γ_c is dependent less on frequency than on the bed height, i.e., the number of rolls do not depend on frequency except for a narrow region close to Γ_c. This is in contrast to the case of oscillating granular layers giving rise to stripe and square patterns [12], since the wavelength of oscillating surface pattern is easily tuned by frequency. It should be noted, however, that the frequency range in which the upward mode appears is limited. The second transition (change in the number of rolls) can only be observed at higher frequencies ($f>80$ Hz) for beds of $d=0.46$ mm (Fig. 8), which is presumably because of the experimental limitation $\Gamma<10.5g$. When the frequency is large, say $f=150$ Hz, the maximum amplitude becomes small (0.12 mm) because of the limitation $\Gamma<10.5g$. This may explain why we fail to observe stable roll convection for $f>150$ Hz. The frequency range where stable convection rolls appear with the upward mode is discussed in Sec. III F.

D. Indirect transition

We have thus far discussed the direct transition from the downward to the upward mode convections and the transition within the upward mode when the roll number changes. In this subsection, we discuss an indirect transition which appears for beds of larger glass beads ($d \geqslant 0.78$ mm). When Γ is increased gradually under a given set of conditions a chaotic state, where the convection motion of the downward mode is replaced by a chaotic one with jets of beads erupting from the bed surface, appears before the transition to the upward mode takes place. This chaotic state can easily be distinguished from the convective ones since the sound emitted from the vibrated bed is significantly larger in the chaotic state than in the other states, suggesting that a good portion

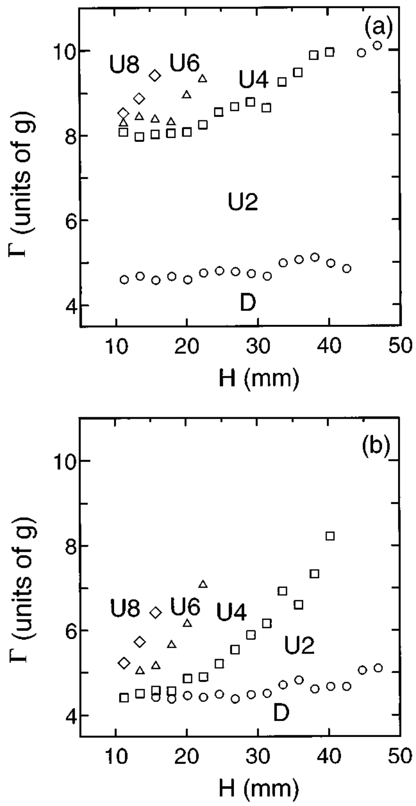

FIG. 4. Critical vibrational acceleration Γ_c for beds of glass beads $\bar{d}=0.78$ mm vs initial bed height H for the (a) appearance of rolls when Γ is increased and (b) disappearance of rolls when Γ is decreased at $f=50$ Hz: two rolls (○), four rolls (□), six rolls (△), and eight rolls (◇). D and U refer to the regions where granules move downward and upward, respectively, near to the walls. Number of rolls are written after U.

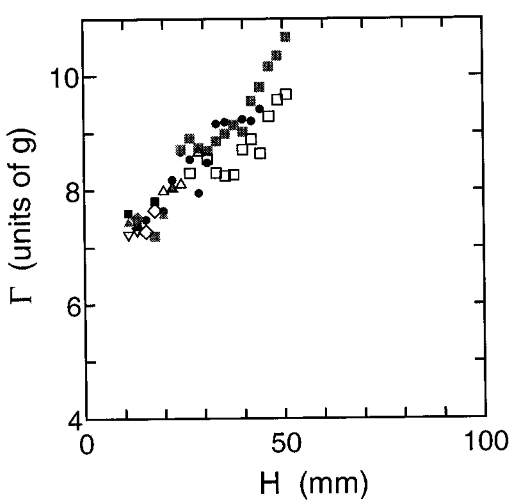

FIG. 5. Critical vibrational acceleration Γ_c for the appearance of upward mode convection when Γ is increased vs initial bed height H for beds of glass beads $\bar{d}=$ 0.34 mm at $f=50$ Hz. Symbols ○, □, △, ◇, ▽ refer to two, four, six , eight, and ten rolls, respectively. Values for different glass containers I (100 mm width), II (150 mm width), and III (200 mm width) are identified by closed, half-tone, and open symbols, respectively.

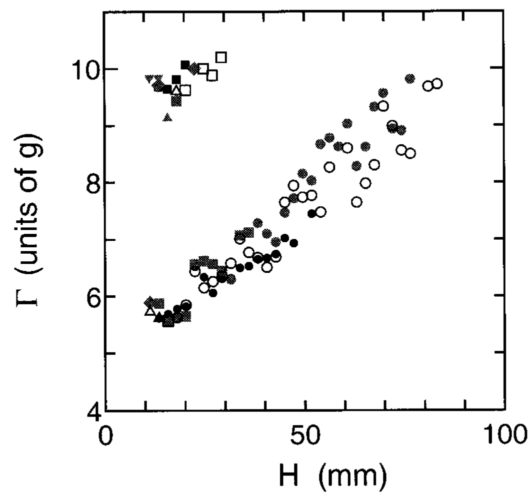

FIG. 6. Critical vibrational acceleration Γ_c for appearance of upward mode convection when Γ is increased vs initial bed height H for beds of glass beads $\bar{d}=0.46$ mm at $f=50$ Hz. Symbols refer to identical roll numbers and containers as those in Fig. 5.

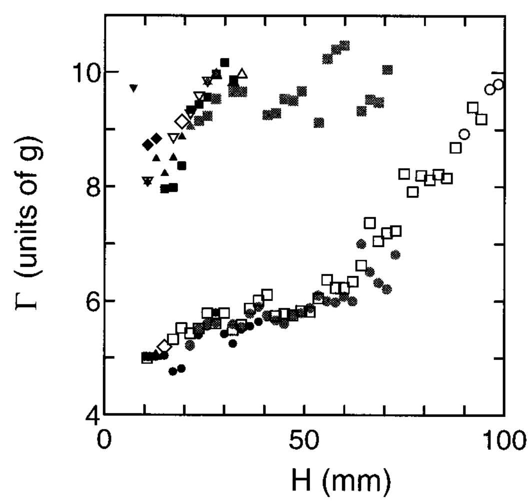

FIG. 7. Critical vibrational acceleration Γ_c for the appearance of upward mode convection when Γ is increased vs initial bed height H for beds of glass beads $\bar{d}=0.61$ mm at $f=50$ Hz. Symbols refer to identical roll numbers and containers as those in Fig. 5.

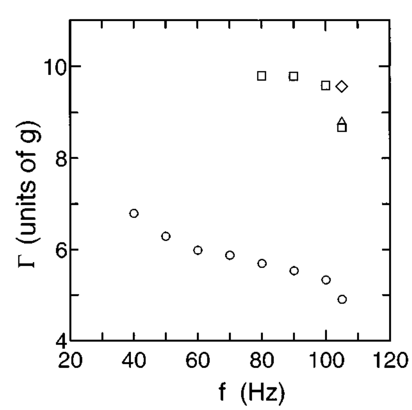

FIG. 8. Frequency dependence of critical vibrational accelerations Γ_c's when Γ is increased for beds of glass beads $\bar{d}=$ 0.46 mm with initial bed height $H=$ 20 mm in glass container I (100 mm width). Symbols refer to identical roll numbers as those in Fig. 4.

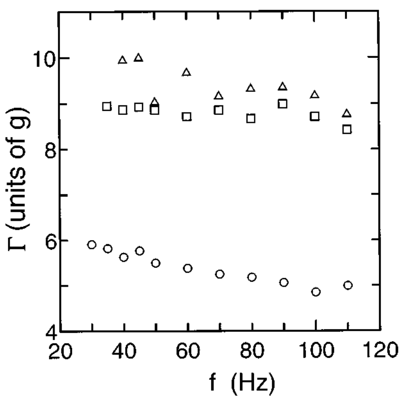

FIG. 9. Frequency dependence of critical vibrational accelerations Γ_c when Γ is increased for beds of glass beads $\overline{d}=$ 0.61 mm with initial bed height $H=$ 20 mm in glass container I (100 mm width). Symbols refer to identical roll numbers as those in Fig. 4.

of the vibrational energy is dissipated through random motions of particles associated with the noises. The chaotic state appears only when Γ is increased: no chaotic state occurs when the transition from the upward to the downward modes takes place. As a result we have a large hysteresis when the chaotic state occurs as shown in Figs. 10 and 11.

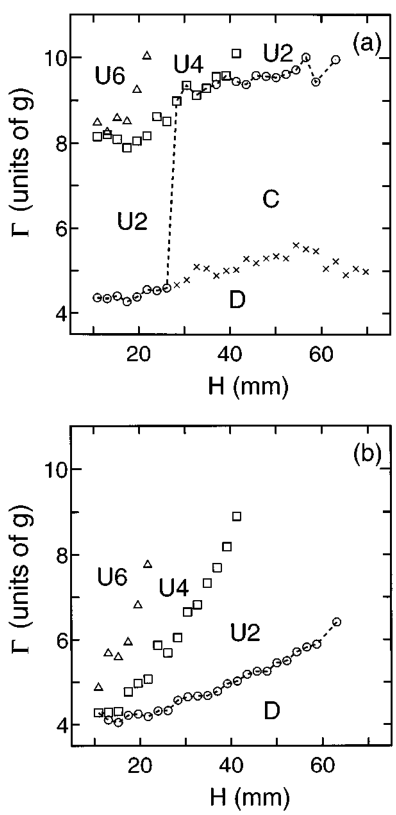

FIG. 10. Critical vibrational acceleration Γ_c for beds of glass beads $\overline{d}=$ 0.93 mm vs initial bed height H for (a) appearance of state when Γ is increased and (b) disappearance of state when Γ is decreased at $f=50$ Hz in container I (100 mm width): two rolls (○), four rolls (□), six rolls (△), and chaotic state (×). D and U refer to the regions where granules are in downward and upward convective modes, respectively, and C refer to chaotic intermediate state. Number of rolls are written after U.

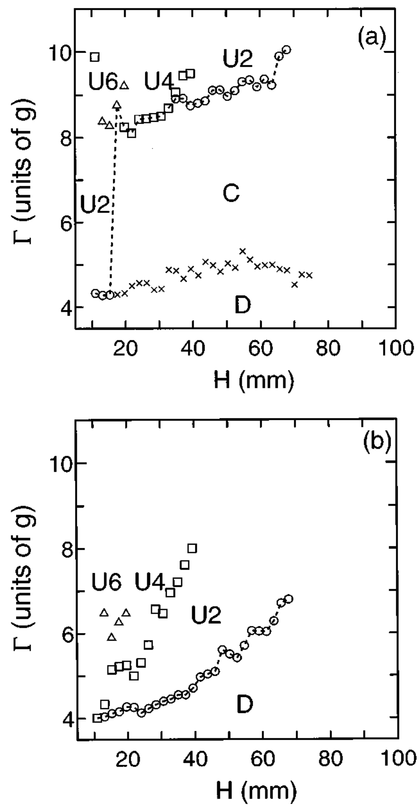

FIG. 11. Critical vibrational acceleration Γ_c for beds of glass beads $\overline{d}=1.09$ mm vs initial bed height H for the (a) appearance of state when Γ is increased and (b) disappearance of state when Γ is decreased at $f=50$ Hz in container I. Symbols refer to identical roll numbers as those in Fig. 10.

Figures 10 and 11 show Γ_c vs H data with increasing Γ (a) and decreasing Γ (b) at $f=50$ Hz, for beds of $\overline{d}=0.93$ and 1.09 mm, respectively.

The dashed line is drawn to mark the Γ_c for the appearance of the upward convection mode, which in effect shows clearly the region of the chaotic state. As seen in Figs. 10(a) and 11(a) there arises, at a critical height above which the chaotic state sets in, a sharp increase in the value of Γ_c that represents the appearance of the upward mode. It should be noted, however, that in spite of the existence of the chaotic state, the second branch of Γ_c (data points in the upper region) in Figs. 10(a) and 11(a) exhibits almost a smooth line: there is no discontinuity in the second branch of Γ_c at the critical bed height. Comparison of Figs. 10(b) and 11(b) with Fig. 4(b) reveals that the effect of the bed height on Γ_c (for decreasing Γ) is stronger when the transition from the chaotic state (to the upward mode) occurs compared to when the DU transition occurs with increasing Γ.

Even for beds of large beads ($\overline{d} \geqslant 0.78$ mm), which leads to the chaotic state under given conditions, Γ_c is better correlated by H rather than by W/H. Figure 12 shows the Γ_c vs H data with increasing Γ for beds of $\overline{d}=0.93$ mm at $f=50$ Hz. In container III we observe the chaotic state but not the upward mode convection presumably because of the limitation $\Gamma < 10.5g$. (We did not continue to measure Γ_c of chaotic state for $H \geqslant 65$ mm, since no upward convection appears thereafter). The minimum bed height H_c that leads to

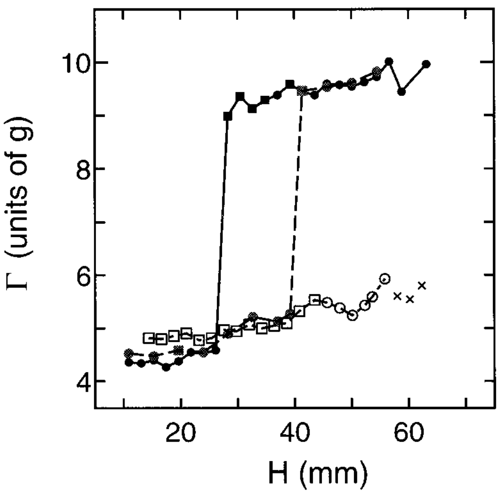

FIG. 12. Critical vibrational acceleration Γ_c for appearance of upward mode convection when Γ is increased vs initial bed height H for beds of glass beads $\bar{d}=0.93$ mm at $f=50$ Hz. Values for containers I (100 mm width), II (150 mm width), and III (200 mm width) are identified by closed (solid), halftone (broken), and open symbols (dot-dash lines), respectively.

the chaotic state depends on the container width W. Results of the investigation on how the container width affects the value of H_c for beds of $\bar{d}=0.78$, 0.93, and 1.09 mm are summarized in Table I. For beds of $\bar{d}=0.93$ mm, H_c is nearly propotional to W/H. There is a tendency that the values of H_c decrease with increasing $\bar{d}$ for containers of $W \leqslant 150$ mm.

E. Downward mode with different materials (polystyrene)

As granules other than glass beads, two sizes of polystyrene beads $\bar{d}=0.93$ and 1.09 mm were used to investigate the dynamic transitions of convective motions. Figure 13 shows results for beds of $\bar{d}=1.09$ mm at $f=50$ Hz. It is of interest to compare Fig. 13 to the phase diagram of glass beads of the same size range $\bar{d}=1.09$ mm (Fig. 11). The overall trend between Figs. 11 and 13 is similar excepting the case of chaotic state, when the values of Γ_c are slightly smaller in Fig. 11(a) than in Fig. 13(a): Fig. 11(a) appears to be a stretched state of Fig. 13(a) in the abscissa. The same tendency is observed for the phase diagram of polystyrene beads of $\bar{d}=0.93$ mm. The density of glass beads is 2.5 times greater than that of the polystyrene beads, thus it is natural that appearance of the chaotic state or disappearance of the downward mode begins at a smaller H in beds of glass beads.

TABLE I. The minimum bed height H_c (mm) where indirect transition appears for $f=50$ Hz.

	Container width W (mm)		
$\bar{d}$ (mm)	100	150	200
0.78	45	53	49
0.93	28	41	58
1.09	18	36	50
1.29	16	27	54

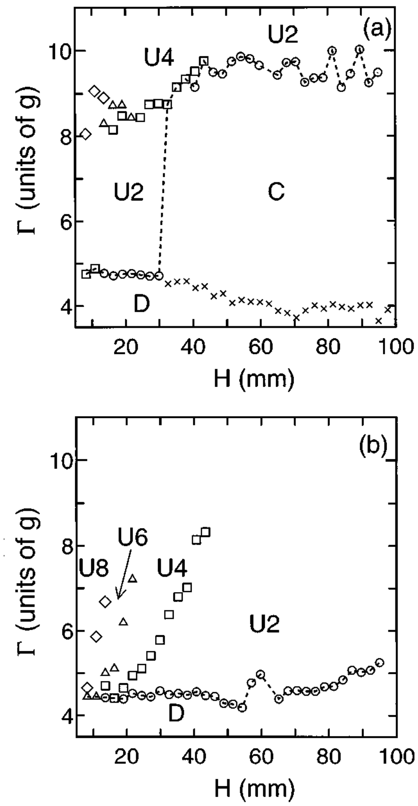

FIG. 13. Critical vibrational acceleration Γ_c for beds of polystyrene beads $\bar{d}=1.09$ mm vs initial bed height H for the (a) appearance of state when Γ is increased and (b) disappearance of state when Γ is decreased at $f=50$ Hz in container I (100 mm width). Symbols refer to identical states as those in Fig. 10.

When Γ is decreased a significant difference appears for Γ_c of the upward-downward ($U2$ to D) transition [Figs. 11(b) and 13(b)]. For beds of glass beads [Fig. 11(b)] Γ_c of the $U2$-D transition increases with H. In contrast Γ_c in beds of polystyrene beads is nearly independent of H.

It should be mentioned that the upward convection was

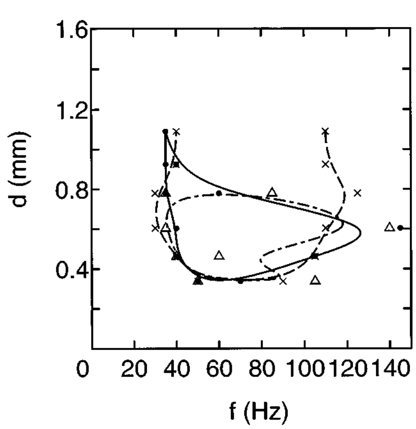

FIG. 14. Stability diagram for roll patterns in vibrated beds of glass beads in the space of vibrational frequency f vs mean diameter of granules $\bar{d}$ in container I (100 mm width): Values for bed height $H=20$ mm, 30 mm, and 40 mm are identified by (×, broken line), (●, solid line), and (△, dot-dash line), respectively.

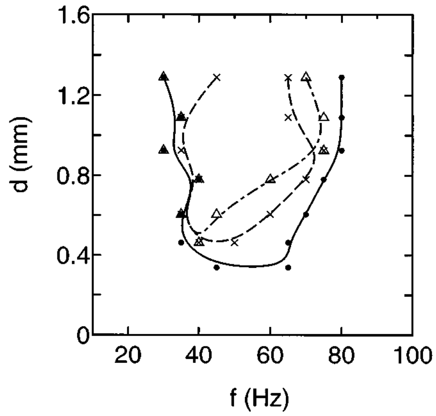

FIG. 15. Stability diagram for roll patterns in vibrated beds of glass beads in the space of vibrational frequency f vs mean diameter of granules $\bar{d}$ in container III (200 mm width). Symbols refer to the identical height as those in Fig. 14.

observed in beds of millet seeds ($\bar{d} \simeq 2$ mm) and of cylindrial plastic pellets (diameter$\simeq 2-3$ mm, height$\simeq 3$ mm) as well.

F. Stability diagram

We have reported in Sec. III C that the upward convection and its roll patterns appear only in a limited frequency range. To get an overall picture where the upward convection appears in vibrated beds of glass beads, we plot stability diagrams in the space of frequency vs characteristic length $\bar{d}$ of the granules. Figures 14 and 15 show the stability diagrams of glass beads, for containers I and III, respectively. The inner region surrounded by the symbols and lines are the one where stable roll patterns appear. In Fig. 14, the regions for the stable roll convection are expressed by closed loops for bed heights $H \geqslant 30$ mm. There is an upper limit of $\bar{d}$ for $H = 20$ mm: roll patterns in the upward mode are not stable at any frequency for glass beads of $\bar{d} =$ 1.29 mm. However, we do not know precisely where the upper limit is (because we could not get glass beads of the size range 1.09 $< \bar{d} <$ 1.29 mm), preventing us from carrying out further experiments. Figure 15 shows that the frequency range where stable roll convection appears is smaller compared to that in Fig. 14: note the difference in the scale of f for Figs. 14 and 15. However, the size range where stable roll convection can be observed is wider in Fig. 15. In container III, the lower limits are $\bar{d} = 0.43$ mm for $H = 20$ and 40 mm and $\bar{d} = 0.34$ mm for $H = 30$ mm. The upper limit of $\bar{d}$ is not known in Fig. 15. Beds of $H =$ 30 mm yield the widest stable region in Fig. 15 and the lower frequency limit merges onto the same line for $H =$ 30 and 40 mm.

IV. DISCUSSION AND CONCLUDING REMARKS

We have investigated effects of various parameters on the dynamic transitions of the convective motion of granules in vibrated beds. The critical value Γ_c is dependent on both the bed height H and the granular size $\bar{d}$. However, it cannot be simply scaled by $H/\bar{d}$ since the bed height for a given value of Γ_c is not linearly dependent on $\bar{d}$. It is rather dependent linearly on $\bar{d}^3$ (not completely satisfactory though). This, combined with the phase diagrams of similar size granules of different densities [glass beads vs polystyrene beads in Figs. 11 and 13($\bar{d} = 1.09$ mm) along with phase diagrams of $\bar{d} = 0.93$ mm (not shown)], suggests that it may be possible to use a scaled bed height $H(\bar{d},\rho)$ (combining the three parameters) in describing the transitions in vibrated beds. To examine the above possibility in detail, however, we need similar size granules of a wide range of density (which is in practice difficult to obtain).

One of the most intriguing features of the dynamic transitions we treated in this paper is the hysteresis observed with increasing and decreasing Γ. In the case of the first transition (direct transition) between the downward and the upward modes, there is only a slight difference in the values of Γ_c with increasing and decreasing Γ: with good reproducibility we observe that the Γ_c values are slightly larger for the DU transition (with increasing Γ). The hysteresis for the second transition (including indirect transition), which is larger than that of the first, has a tendency to become smaller with increasing bed height. In some cases the hysteresis nearly extinguishes at a certain bed height. As can be seen from Figs. 4, 10, 11, and 13, the appearance of four rolls (U4) diminish toward a point where $H \simeq 41$ mm in container I for beds of $\bar{d} \geqslant 0.78$ mm (a slightly higher value $H \simeq 44$ mm for polystyrene). The reason why we cannot observe the point where U4 disappears in all other containers and for $\bar{d} \leqslant 0.61$ mm in container I is presumably due to the experimental limitation $\Gamma < 10.5g$ (refer to Fig. 4). The disappearance of $U4$ suggests that the hysteresis and the existence of the rolls cease at a certain anisotropy in the shape of rolls $W/HN \simeq 0.6$: rolls which are more vertically elongated than $W/HN = 0.6$ have not been observed [13]. The upper limit of the anisotropy of the roll shape W/HN is less limited and takes a value up to $W/HN \simeq 5$. However, this does not necessarily mean that the rolls can be much more anisotropical in shape in the horizontal direction. For beds with large values of W/HN, there exist fairly large dead zones, (where convective flows are not observable) near to the wall or in the middle of pair rolls. The existence of these large dead zones along with convective roll patterns is a significant feature of the granular convection different from that of thermal convection in fluids, suggesting the need for a different theoretical approach (at some points) between granules and fluids.

In the case of granular convection the direction of the shear force, which the side walls apply to the granular bed, is opposite to that of the bed when the bed lifts off and when the bed makes contact with the container bottom. The granules are more densely packed when not suspended thus the total shear force is larger when the bed lifts off. Consequently there exists net downward flux of granules near the side walls, resulting in the downward mode of convection. The above mechanism, which Lee [3] puts forward with some computations, is valid for relatively small Γ. With the increase of Γ values, the time span when the bed is in flight increases, sometimes larger than half a cycle time of the container [14]. Under such conditions the container in the upward phase may exert upward shear force to the granules adjacent to the side walls, resulting in the DU transition.

Continuum models appear inappropriate to describe the vibrated granular bed at high vibrational intensities, when the trajectory of the bed in flight can be very complex with large fluctuations of local density. Under such conditions the body force cannot be expressed by a simple periodic function of time. Furthermore, it may be difficult to explain the drastic change that takes place in the rate of granular circulation at the DU transition.

Preliminary experiments indicated that the effects of thickness (in the 15 to 40 mm range) on the roll patterns was insignificant. In the case of a 100 mm square bed, stable convection rolls were also observed. However, a slight alteration of the initial surface configuration of the bed (before subjected to vibrations) caused the axis of convection rolls to change by a right angle, which indicates the horizontal balance of the vibrating bed being well kept in the experiment. The transition corresponding to the DU transition in rectangular containers was observed to occur even for three-dimensional configurations such as cylindrical beds. Convection rolls with a heap (the downward mode in our work) have been observed in annular cells with narrow gaps [15] indicating that convection rolls appear without side walls (however, the flow pattern of granules confined in a curved geometry is different from that in a planer geometry). Furthermore, multiple pairs of convection rolls were observed in two dimensional beds (30 mm thick) whether the side walls are 5° slanted inwardly or outwardly. Therefore, it appears that there exist some kinds of intrinsic mechanisms, notwithstanding the importance of the shear force at the side walls, that give rise to the DU transition. The core of this study is the finding of stable multiconvection rolls in the granular bed under vibrations.

The Navier-Stokes equation has recently been applied to simulate the behavior of granules under vibrations [16,17]. A case of two pairs of convection rolls (four rolls) with the downward mode has been shown in [16]. However, in the present study multiple pairs of convection rolls have not been observered with the downward mode in the granular convection except in a container with walls slanted inwardly (the rolls adjacent to the slanted walls were considerably deformed with the presence of a dead zone at each of the bottom corners). Moreover, considering the fact that the upward mode is the norm for conical containers [6] the multiple pairs of convection rolls are unlikely to occur with the downward mode when containers with outwardly slanted walls are used.

Simulation studies indicate that the relative magnitude between the particle-particle and particle-side wall shear friction coefficients determines the direction of convection [9]. In our experiments glass beads and glass containers are used, suggesting that microscopic physical properties of the granule alone cannot be claimed for the transition to occur. The convection mechanism suggested by the simulation models is not sufficient to explain the transition caused by the change in the magnitude of vibrational acceleration [18]. Taguchi performed computer simulations on the granule flow in vibrated beds of large aspect ratios, with side walls [9] and also with a periodic boundary condition [10]. On the basis of simulation results he concludes that the reason why a clear array of convection rolls, like that of Rayleigh-Bérnard convection, does not appear in the granular bed is due to an instability inherent in the granule flow. The finding of multiconvection rolls calls for a more refined simulation model that would better explain the difference or for that matter similarity between granules and fluid flows.

We have described dynamic transitions involving roll type convection in vibrated beds of granules. Set conditions which lead to transitions, involving the change in the direction of granule motion and increase in the number of rolls, have been investigated systematically. The downward convection, which occurs at low Γ, is not very stable, and thus symmetry of the heap easily breaks resulting in an inclined slope. By contrast, the upward mode convection, where granules move upward near to the wall, is quite stable and maintains its symmetrical pattern for a long period of time. Although the DU transition has not been explicitly treated before, the phenomenon seems to occur commonly for granules of a certain size range. One of the reasons why this phenomenon has not been systematically studied to date is probably because it is difficult to see what is happening in three-dimensional setups, such as cylindrical containers, since a pair of valleys becomes oppositely curved under the influence of boundary conditions.

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