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Switching between orbits in a periodic window

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This paper shows the use of chaos control techniques within a periodic window making use of the fact that the infinite variety of unstable periodic orbits coexist with a stable periodic orbit. The original stable periodic orbit can be distorted into a chaoslike state induced by small external excitations. When this perturbed state approaches an unstable periodic orbit of the original system, control methods can be applied to stabilize the system onto the particular orbit. This technique is applied in the period-3 window of the logistic map using constant excitations and a self-locating control scheme. [S1063-651X(96)12112-7]

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An advantage of controlling chaos is that the technique is able to stabilize and switch between different orbits utilizing only small controls making use of the sensitive property of chaos and an infinite number of naturally existing unstable periodic orbits embedded within a chaotic attractor [1,2]. The technique of chaos control has been applied to mechanical [3], chemical [4], electronic [5], laser [6], communication [7], and biological systems [8] and these achievements indicate a vast potential for implementations that may result in new technologies to solve important problems in diverse fields. In most previous studies, the technique of chaos control has been limited to systems that are necessarily set within a chaotic regime. Until very recently it has generally been regarded that the same advantage cannot be achieved in nonchaotic systems (i.e., nonlinear dynamical systems set beyond chaotic regimes) [2]. However, this paper seeks to adjust this perception, thus broadening the possible applications.

A very recent report [9] by Christini and Collins showed that the technique of chaos control can be applied to nonchaotic systems. The basic idea was as follows. A nonlinear dynamical system is defined at some parameter setting beyond the chaotic regime, referred to as a nonchaotic system. With this configuration, the system has a stable periodic motion though some unstable periodic orbits may coexist. To change the system from the stable periodic orbit to one of the unstable periodic orbits, one deliberately adds external noise into the nonchaotic system. The excitation of small noise may destabilize the stable periodic motion and induce a chaoslike state. When the state is close to a desired unstable orbit the Ott-Grebogi-Yorke control technique [1] can be applied to stabilize the system onto the orbit. Switching between the unstable orbits can be carried out by combining excitations and control techniques. Thus flexible selection of different performances for a system by the use of only small control efforts is achievable in nonchaotic systems. It is particularly significant in engineering where many systems cannot necessarily be set in a chaotic regime.

In Ref. [9], the feasibility of this idea was shown in a parameter range where only a few unstable periodic orbits were available. In this paper we are particularly interested in activating control within one of the periodic windows that typically exist between chaotic regimes. As an illustration, a bifurcation diagram for the logistic map

$$x_{i+1}=ax_i(1-x_i) \tag{1}$$

is shown in Fig. 1, where a is a parameter. This diagram displays stable periodic orbits (some of them labeled by P1, P2, P4), unstable periodic orbits (broken curves), chaos (dots), and periodic windows (one of which is marked as a P3 window). A closeup of the region $3.8<a<3.9$ is included showing a detailed view of the P3 window where a stable period-3 orbit exists. Each bifurcation in the cascade of period-doubling bifurcations produces an unstable periodic orbit (broken curves), which, as the parameter increases, extends into the regimes of chaos and importantly also into the periodic windows. It is known that there are an infinite number of such unstable periodic orbits that coexist with stable periodic orbits of the periodic windows. Thus a system, set in a periodic window (such as the period-3 window shown in Fig. 1), possesses a rich resource of different periodic orbits. The system can be maneuvered onto any of these unstable periodic motions provided that they can be approached and stabilized by control.

Outside the periodic windows, for example, if one sets the system in the parameter range of the period-4, orbit say at $a=3.5$ in Fig. 1, then the system has only three orbits available: the period 1, 2, and 4. However, for the logistic map, within the period-3 window, there are an infinite number of unstable periodic orbits of all periods. The period-3 window presents one of the largest reserves for selection of orbits for control. In addition this window is a predominant window, which is stable over the widest parameter range compared with other periodic windows.

Christini and Collins [9] showed that a stable periodic motion of a nonchaotic system can be excited into the chaoslike motion under the influence of certain levels of noise. We here consider the use of constant perturbations to a nonchaotic system to stimulate a chaotic state. Thus, for the logistic map, a term of constant perturbation δ can be added into the original system Eq. (1), which is rewritten in the form

$$x_{i+1}=ax_i(1-x_i)+\delta. \tag{2}$$

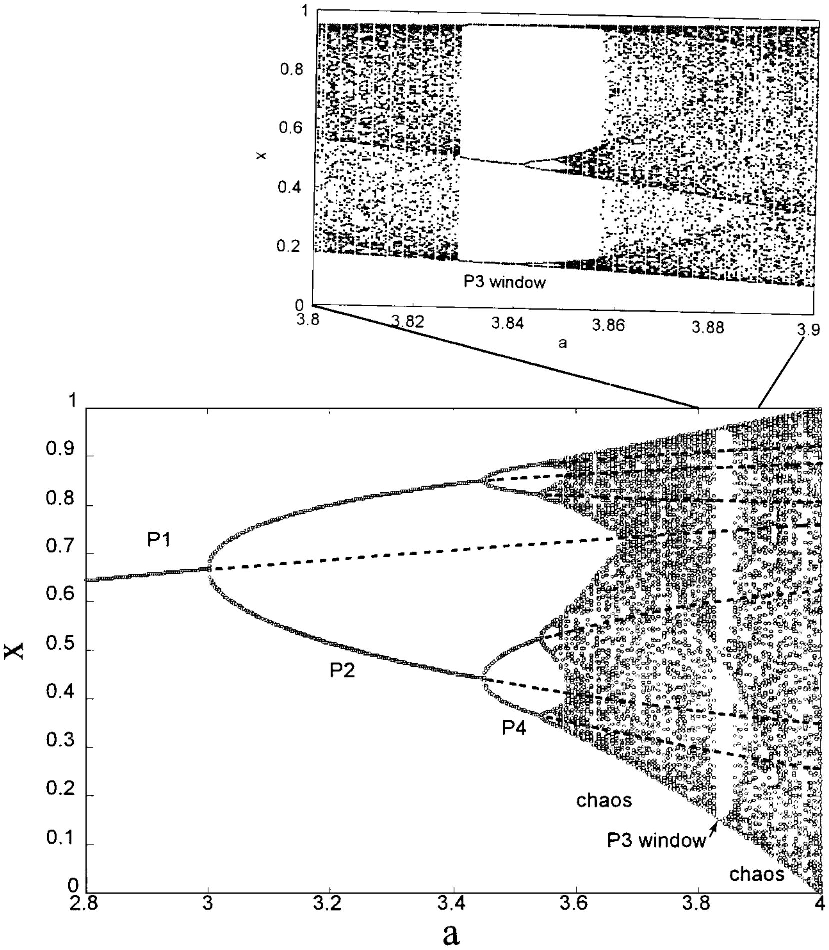

FIG. 1. A cascade of period-doubling bifurcations of the logistic map with a variation of the parameter a from 2.8 to 4. The solid curves marked P1, P2, and P4 correspond to stable period-1, period-2, and period-4 orbits, respectively, within the different parameter ranges. The broken curves beyond the stable P1, P2, and P4 orbits correspond to unstable orbits, which extend into the chaotic regimes as the parameter increases. Between the chaotic regimes there exist periodic windows, one of which is denoted as a P3 window. Within the P3 window, there is a stable period-3 orbit (solid curves), which coexists with these unstable periodic orbits (broken curves).

In other works an additive term was used by Shinbrot in synchronization of coupled maps [10] in which the constant term plays the role of suppressing a chaotic state into a periodic state, while Bradley used parametric excitations to destabilize a stable periodic state in a phase locked loop [11]. In this paper, constant excitations are applied in order to destabilize a stable periodic state resulting in a chaotic state that approaches a desired unstable orbit. Constant excitations may efficiently generate a chaotic state and are much simpler than the use of external noise for applications.

Figure 2 shows that the stable period-3 motion of the P3 window can be distorted into a ''fuzzy'' periodic motion or even a chaotic motion under different levels of constant excitations. Comparing the results of two different levels of excitations $\delta=0.008$ ($I=200$–600) and $\delta=0.01$ ($i=800$–1200), the more intensive excitation induces more ''fuzzy'' periodic motion. It can be seen that the area of scattered dots is enlarged with an increase of the level of excitations. Initiating the excitation of $\delta=0.02$ during the time interval $i=1400$–1800, the P3 orbit is completely destabilized resulting in a chaoslike state. A phase portrait of the induced chaotic state is shown in Fig. 3 using the delay coordinates x_i and x_{i+1}. The induced chaotic state forms a stable attractor, which lies nearby the unstable periodic orbits (labeled by asterisks, plusses, open circles, and crosses indicating P1, P2, P4, and P5 orbits, respectively) of the original map Eq. (1). The induced chaotic state can visit close to any of these unstable periodic orbits of the original system. Thus the techniques of chaos control [1,12,13] can be applied to stabilize the system onto a desired orbit with small controls.

In this paper, to stabilize orbits of the logistic map, the self-locating control method [12] is used. The control algorithm is based on the Newton method to pinpoint a desired periodic solution utilizing the feedback of an output se-

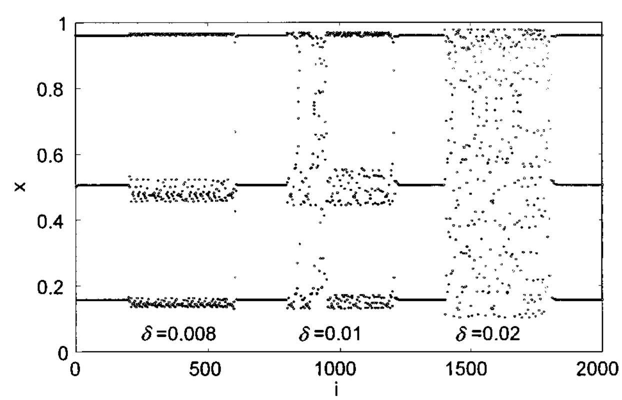

FIG. 2. The logistic map behaves in the manner of period-3 motion as the parameter is set at $a=3.83$ within the period-3 window. The fuzzy period-3 motion and chaoslike motion can be induced by introducing small constant excitations at the levels of $\delta=0.008$ ($i=200$–600), $\delta=0.01$ ($i=800$–1200), and $\delta=0.02$ ($i=1400$–1800). The influence of destabilizing a stable periodic orbit increases as the level of the excitation increases.

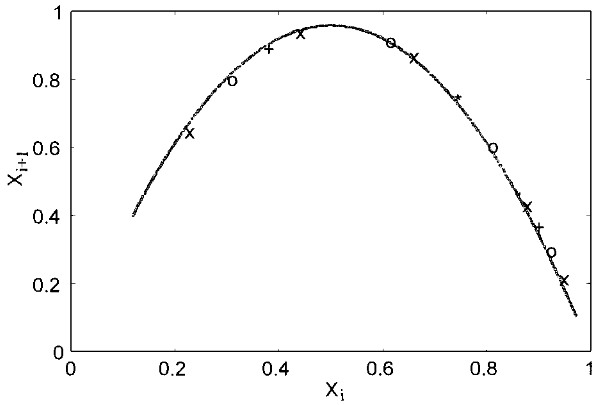

FIG. 3. A phase portrait of the induced chaotic state resulting from the excitation level of δ=0.02 is shown using the delay coordinates x_i and x_{i+1}. The chaotic state forms a stable attractor, which lies nearby the unstable periodic orbits of the original system, the location of which are denoted by *, period-1 orbit; +, period-2 orbit; ○, period-4 orbit; and ×, period-5 orbit.

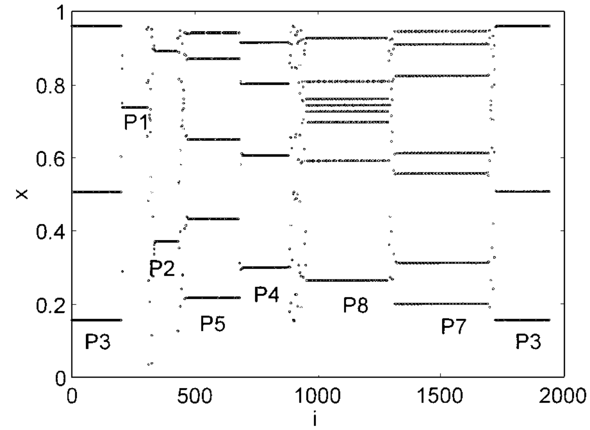

FIG. 4. At the parameter setting $a=3.83$ within the P3 window, the logistic map is originally in the motion of period-3 orbit. The excitation of δ=0.02 is activated to the map resulting in a chaotic motion (just after $i=200$). When the chaotic state is close to P1 the perturbation is switched off and at the same time control is activated. Thus the system is locked onto P1. Using a similar process the transition for the orbits from P3→P1→P2→P5→P4→P8→P7→P3 can be carried out within the period-3 window of the logistic map. The locations of these orbits are precisely detected during the control process using the self-locating control scheme.

quence on accessible parameters. The method can automatically detect the location of the orbit with high accuracy during the process of stabilization. The application of this method only requires an approximate location of the desired orbit so that it is not necessary to find the precise location of the orbit before control.

For the logistic map we use a as the control parameter and the system is initially set within the range of the period-3 window by choosing $a=3.83$. The value of the constant perturbations is set to δ=0.02. Figure 4 demonstrates the stabilization and switching between the period orbits (indicated by P1, P2, P3, P4, P5, P7, and P8 where the integer stands for the periodicity of the orbit) by plotting the variable x against iteration number i. These periodic orbits are all unstable except for the P3 orbit. Before control, the approximate locations of these unstable orbits need to be estimated. An easy way to find this information for the logistic map can be achieved by the use of the bifurcation diagram (Fig. 1), with an approximate estimation made via the observation of the projection of a stable orbit with an increase of the parameter a. By so doing, we obtain the approximate locations for P1, ≅0.75; P2, ≅0.4; P4, ≅0.3; P5, ≅0.5, P7, ≅0.92, and P8, ≅0.7 (at $a=3.83$), which are close to one of points of the unstable periodic orbits. The precise location for P3 is very easy to obtain since the system converges to this stable period-3 orbit after some transient time, here the P3 location of 0.504 6665 is one of points of the period-3 orbit.

Initially, the original system behaves in a regular motion of period-3 ($i=1$–200) without perturbation (δ=0) and control. To change the state of the system from P3 to P1, first a constant perturbation δ=0.02 is applied to the map resulting in a chaotic motion (just after $i=200$). When the induced chaotic state is close to P1 (within a distance of 0.1), the perturbation is turned off (i.e., δ=0) and simultaneously the control is activated. Thus the system is locked onto P1 and stabilization is maintained for a further 100 iterates ($i=206$–306). To switch the motion from P1 to P2, the control is turned off while concurrently turning on the excitation ($\delta=0.02$). Once again a chaotic state results, which visits close to P2. As before the excitation is ceased and control applied to maintain the state on the P2 orbit. When both control and excitation are turned off the system converges towards the naturally stable period-3 orbit as shown from P7 to P3 (after $i=1700$). Figure 4 demonstrates the process of the transition of these orbits from P3→P1→P2→P5→P4→P8→P7→P3 in the period-3 window of the logistic map. The locations of these orbits are P1, 0.7389034; P2, 0.3691614; P4, 0.2991621; P5, 0.4318443; P7, 0.9102810; and P8, 0.7265884 precisely detected during the control process using the self-locating control scheme.

In conclusion, an infinite number of unstable periodic orbits coexist in a periodic window. Taking advantage of this feature, one can flexibly manage a system among the infinite variety of periodic motions by means of external excitations and chaos controls. In this paper, the feasibility for this technique is shown in a period-3 window using chaotic states induced by constant excitations and the self-locating control scheme. Switching among many different naturally existing orbits only using small controls is achievable in periodic windows where a system is originally configured as a non-chaotic system.

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