Ion acoustic solitons in a weakly relativistic magnetized warm plasma

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The oblique propagation of nonlinear ion acoustic solitary waves (solitons) in a magnetized low-β, weakly relativistic warm plasma is examined by the Korteweg–de Vries equation. Two types of modes (fast and slow ion acoustic modes) exist in the plasma. The fast mode corresponds to the propagation of compressive solitons, whereas the rarefactive solitons exist for the slow mode. The amplitude and energy of both types of solitons increase with the angle between the wave vector and magnetic field and also with the relativistic ion drift velocity. The effect of finite ion temperature is to decrease the amplitude of the soliton and to increase its width. The strength of the magnetic field weakens the soliton energy and the width becomes smaller. [S1063-651X(96)07910-X]

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I. INTRODUCTION

The ion acoustic solitary waves arising from the balance of the nonlinearity and dispersion in plasmas have been investigated over a period approaching a score of years. Their one-dimensional propagation is described with the Korteweg–de Vries (KdV) equation [1]. Using this equation, the characteristics of the ion acoustic solitons have been studied by several researchers in nonrelativistic [2–6] as well as in relativistic plasmas [7–11]. In the relativistic plasma, a finite ion temperature was shown to influence the soliton characteristics [7]. A limit on the ion drift velocity for the existence of ion acoustic solitons was obtained by Malik, Singh, and Dahiya [8] in the relativistic plasma by taking the combined effect of electron mass and ion temperature. Large amplitude ion acoustic waves have also been analyzed in a two-fluid plasma by assuming the electron and ion flow velocities to be relativistic [11].

The ion acoustic solitons have also been studied in the magnetized plasmas. Shukla and Yu [12] have shown that finite-amplitude ion acoustic solitary waves propagating obliquely to an external magnetic field can occur in a cold-ion plasma. Three types of nonlinear waves including ion acoustic solitons have also been observed by Lee and Kan [13] in a magnetized plasma with cold ions. Later, finite ion temperature was included in the analysis to examine the fully nonlinear ion acoustic solitary waves in a magnetized plasma [14]. Finite-amplitude ion acoustic waves, ion cyclotron waves, and the ion acoustic solitons have also been studied in a warm-ion magnetized plasma [15].

As is evident from the above-mentioned references, most of the investigations on ion acoustic solitons are either for unmagnetized relativistic plasmas or for magnetized plasmas having no relativistically drifting ions. Since the magnetic field modifies the soliton characteristics, it is of interest to determine how a magnetic field affects the propagation of ion acoustic solitary waves in a relativistic plasma. Therefore, in the present analysis, the ion acoustic solitons have been examined in a weakly relativistic magnetized warm plasma. It is found that the fast ion acoustic mode corresponds to the propagation of compressive solitons, whereas the rarefactive solitons exist for the slow ion acoustic mode.

II. BASIC FLUID EQUATIONS

A small but finite amplitude ion acoustic wave propagating in a magnetized, collisionless, weakly relativistic plasma consisting of warm ions and hot electrons is considered. The propagation of the wave is assumed to be in the (x,z) plane and inclined at an angle to the direction of the magnetic field **B**, which is along the z axis. The characteristic frequency is assumed to be much smaller than the ion gyrofrequency and the ratio β of the particle pressure to the magnetic pressure is small. Since the perturbations are of low frequency, the electron mass is neglected and their usual Boltzmann distribution is assumed. Under these conditions, the basic fluid equations in the normalized form can be written as [14–16]

$$n_t+\nabla\cdot(n\mathbf{v})=0, \tag{1a}$$

$$(\gamma\mathbf{v})_t+(\mathbf{v}\cdot\nabla)\gamma\mathbf{v}+\nabla\phi-(\Omega_i/\omega_{pi})\mathbf{v}\times\mathbf{z}+(2\sigma/n)\nabla n=\mathbf{0}, \tag{1b}$$

$$n_\epsilon=\exp(\phi), \tag{1c}$$

$$\nabla^2\phi-n_e+n=0. \tag{1d}$$

In these equations n and n_e are, respectively, the ion and electron densities normalized by the unperturbed plasma density n_0. **v** is the ion drift velocity normalized by the ion acoustic speed. The potential ϕ is normalized by kT_e/e, where k is the Boltzmann constant, T_e, the electron temperature, and e, the electronic charge. The subscript t denotes differentiation and the time t and space coordinates are, respectively, normalized by the ion plasma period, i.e., ω_{pi}^{-1} and the electron Debye length. **z** is the unit vector along the direction of magnetic field $\mathbf{B}(=B_0\mathbf{z})$. Ω_i is the ion gyrofrequency. σ is the ion to electron temperature ratio and $\gamma=(1-v^2/c^2)^{-1/2}$ is the relativistic factor with $v\ll c$, with c the speed of light. It can be noted from the ion momentum equation that the specific heat ratio is taken as 2 because the number of degrees of freedom for the present case (two-dimensional motion) is 2.

If the ion drift velocity is considered to be relativistic only in the direction of the magnetic field (i.e., along the z axis) and its other components (i.e., perpendicular to the magnetic field) are nonrelativistic, then the following basic equations are obtained from the original equations:

$$n_t+(nv_x)_x+(nv_z)_z=0, \tag{2a}$$

$$(v_x)_t+v_x(v_x)_x+v_z(v_x)_z+\phi_x-Av_y+(2\sigma/n)n_x=0, \tag{2b}$$

$$(v_y)_t+v_x(v_y)_x+v_z(v_y)_z+Av_x=0, \tag{2c}$$

$$(\gamma v_z)_t+v_x(\gamma v_z)_x+v_z(\gamma v_z)_z+\phi_z+(2\sigma/n)n_z=0, \tag{2d}$$

$$n_e=\exp(\phi), \tag{2e}$$

$$\phi_{xx}+\phi_{zz}-n_e+n=0. \tag{2f}$$

Here $A=\Omega_i/\omega_{pi}$ and $\gamma\approx 1+v_0^2/2c^2$, with v_0 the ion drift velocity in the z direction.

III. KORTEWEG–DE VRIES EQUATION

To study the propagation of the ion acoustic wave, the basic fluid equations are solved for stationary nonlinear solutions. For this purpose, the following stretched coordinate system is introduced [16]

$$\xi=\epsilon^{1/2}(\mathbf{k}\cdot\mathbf{r}-\lambda_0 t)=\epsilon^{1/2}(x\ \sin\theta+z\ \cos\theta-\lambda_0 t), \tag{3a}$$

$$\tau=\epsilon^{3/2}t, \tag{3b}$$

where $\mathbf{k}$ is the unit vector along the direction of wave propagation that makes an angle θ with the direction of the magnetic field and λ_0 is the phase velocity of the ion acoustic wave in (ξ,τ) space. ϵ is small dimensionless expansion parameter.

The quantities n, n_e, ϕ, and v are expanded around the equilibrium state in terms of ϵ to balance between nonlinear and dispersive terms. Their expansion is given by [8,16]

$$n=1+\epsilon n_1+\epsilon^2 n_2+\cdots, \tag{4a}$$

$$n_e=1+\epsilon n_{e1}+\epsilon^2 n_{e2}+\cdots, \tag{4b}$$

$$\phi=\epsilon\phi_1+\epsilon^2\phi_2+\cdots, \tag{4c}$$

$$v_x=\epsilon^{3/2}v_{x1}+\epsilon^2 v_{x2}+\cdots, \tag{4d}$$

$$v_y=\epsilon^{3/2}v_{y1}+\epsilon^2 v_{y2}+\cdots, \tag{4e}$$

$$v_z=v_0+\epsilon v_{z1}+\epsilon^2 v_{z2}+\cdots. \tag{4f}$$

Equations (3a), (3b), and (4a)–(4f) are used in the basic fluid equations to obtain the K-dV equation. The terms of order $\epsilon^{1/2}$ give

$$\cos\theta\ v_{0\xi}=0. \tag{5a}$$

The following relations are obtained at the order of ϵ:

$$n_{e1}=n_1=\phi_1. \tag{5b}$$

The terms of order $\epsilon^{3/2}$ yield

$$-\lambda_0 n_{1\xi}+\cos\theta\ v_{z1\xi}+v_0\cos\theta\ n_{1\xi}=0, \tag{5c}$$

$$\sin\theta\ \phi_{1\xi}-Av_{y1}+2\sigma\ \sin\theta\ n_{1\xi}=0, \tag{5d}$$

$$Av_{x1}=0, \tag{5e}$$

$$-\lambda_0\gamma v_{z1\xi}+v_0\gamma\ \cos\theta\ v_{z1\xi}+\cos\theta\ \phi_{1\xi}+2\sigma\ \cos\theta\ n_{1\xi}=0. \tag{5f}$$

The following relations between the first-order quantities are obtained at the order of ϵ^2:

$$\sin\theta\ v_{x1\xi}=0, \tag{5g}$$

$$-\lambda_0 v_{x1\xi}+v_0\cos\theta\ v_{x1\xi}-Av_{y2}=0, \tag{5h}$$

$$-\lambda_0 v_{y1\xi}+v_0\cos\theta\ v_{y1\xi}+Av_{x2}=0. \tag{5i}$$

Equations (5a)–(5i) yield the phase velocity relation

$$\lambda_0=\cos\theta[v_0\pm\{(1+2\sigma)\gamma^{-1}\}^{1/2}]. \tag{6}$$

It is evident from Eq. (6) that the two types of modes, namely, a fast ion acoustic mode [corresponding to the plus sign in Eq. (6)] and a slow ion acoustic mode [corresponding to the negative sign in Eq. (6)], are possible in the plasma. It is also clear that the phase velocity λ_0 depends on the angle of propagation of the wave, relativistic ion drift velocity, and ion to electron temperature ratio. It can also be seen that the phase velocity of both types of modes increases with the ion drift velocity, but decreases for the larger angle θ. The dependence of the fast and slow modes on the ion to electron temperature ratio has the opposite nature. The phase velocity of the fast mode goes up, whereas that of slow mode goes down for the higher ion temperatures.

The second-order quantities are presented by the following equations, obtained from the basic fluid equations at the order of ϵ^2:

$$n_{e2}-\phi_2-\phi_1^2/2=0, \tag{7a}$$

$$\phi_{1\xi\xi}-n_{e2}+n_2=0. \tag{7b}$$

The terms of order $\epsilon^{5/2}$ give rise to

$$-\lambda_0 n_{2\xi}+n_{1\tau}+\sin\theta\ v_{x2\xi}+\cos\theta\ v_{z2\xi}+\cos\theta\ n_1 v_{z1\xi}+\cos\theta\ v_{z1}n_{1\xi}+v_0\cos\theta\ n_{2\xi}=0, \tag{7c}$$

$$-\lambda_0 v_{x2\xi}+v_0\cos\theta\ v_{x2\xi}+\sin\theta\ \phi_{2\xi}+n_1\sin\theta\ \phi_{1\xi}-An_1v_{y1}+2\sigma\ \sin\theta\ n_{2\xi}=0, \tag{7d}$$

$$-\lambda_0 v_{y2\xi}+v_0\cos\theta\ v_{y2\xi}=0, \tag{7e}$$

$$-\lambda_0\gamma v_{z2\xi}-\lambda_0 n_1\gamma v_{z1\xi}+\gamma v_{z1\tau}+v_0\gamma\ \cos\theta\ v_{z2\xi}+v_{z1}\gamma\ \cos\theta\ v_{z1\xi}+n_1v_0\gamma\ \cos\theta\ v_{z1\xi}+\cos\theta\ \phi_{2\xi}+n_1\cos\theta\ \phi_{1\xi}+2\sigma\ \cos\theta\ n_{2\xi}=0. \tag{7f}$$

Now all the second-order quantities n_2, n_{e2}, ϕ_2, v_{x2}, v_{y2}, and v_{z2} appearing in Eqs. (7a)–(7f) are eliminated by using the phase velocity relation [Eq. (6)]. The following equation in the first-order quantity ϕ_1 is obtained after the elimination of the second-order quantities:

$$2\sec\theta\phi_{1\tau}+2\sec\theta(\lambda_0-v_0\cos\theta)\phi_1\phi_{1\xi}+[\{A^2+(1+2\sigma)^2\sin^2\theta\}\cos\theta/(\lambda_0-v_0\cos\theta)\gamma A^2]\times\phi_{1\xi\xi\xi}=0. \tag{8}$$

Equation (8) gives the following KdV equation in ϕ_1:

$$\phi_{1\tau}+\alpha\phi_1\phi_{1\xi}+\beta\phi_{1\xi\xi\xi}=0. \tag{9}$$

The coefficients appearing in Eq. (9) are given by

$$\alpha=(\lambda_0-v_0\cos\theta), \tag{10a}$$

$$\beta=\cos^2\theta\{A^2+(1+2\sigma)^2\sin^2\theta\}/2(\lambda_0-v_0\cos\theta)\gamma A^2. \tag{10b}$$

Here α is the coefficient of nonlinear term and β that of dispersive term of the KdV equation (9).

To find the stationary solution of the KdV equation (9), the transformation [8,10,16]

$$\nu=\xi-U\tau, \tag{11}$$

where U is a constant velocity, is used. With the use of this transformation, the KdV equation is converted into an equation with a single variable ν. Now this equation is integrated under boundary conditions that $\phi_1(\nu)$ and all its derivatives vanish as $|\nu|\to\infty$, which gives the solution

$$\phi_1(\nu)=(3U/\alpha)\mathrm{sech}^2\{(U/4\beta)^{1/2}\nu\}. \tag{12}$$

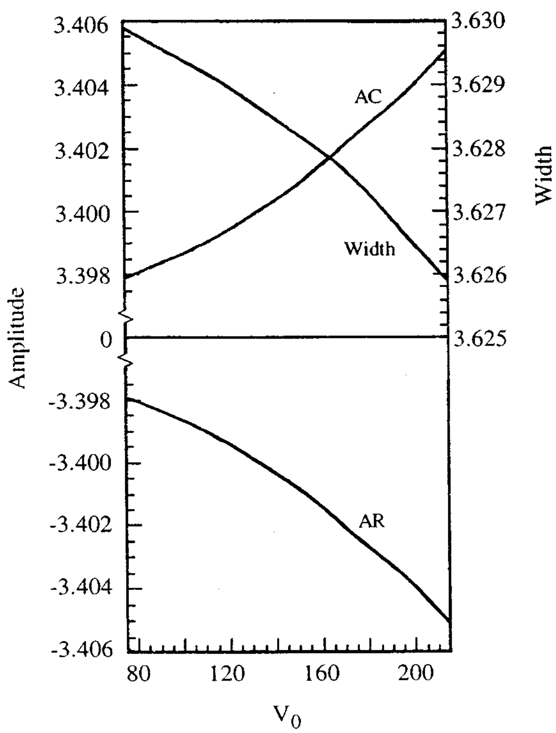

FIG. 1. Weak dependence of the amplitude (ϕ_m/U) and the width ($LU^{1/2}$) of the compressive and rarefractive solitons on the relativistic ion drift velocity v_0. Here σ=0.02, θ=30°, A=0.2, and AC and AR represent, respectively, the amplitudes of the compressive and the rarefractive solitons.

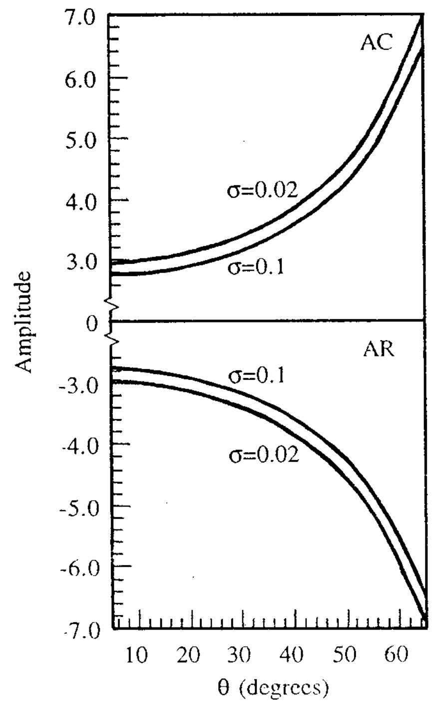

FIG. 2. Variation of the amplitude (ϕ_m/U) of the compressive and rarefractive solitons with the wave propagation angle θ. Here A=0.2 and v_0=115.

This is the soliton solution of the KdV equation. If the peak soliton amplitude is denoted by ϕ_m and the soliton width by L, then

$$\phi_m=3U/\alpha, \tag{13a}$$

$$L=2\beta^{1/2}U^{-1/2}. \tag{13b}$$

IV. DISCUSSION

It can be noted from Eqs. (10a), (13a), and (6) that the coefficient α of the nonlinearity remains positive for the fast ion acoustic mode and hence compressive solitons exist in the plasma. However, as the slow ion acoustic mode is considered, rarefractive solitons are allowed to propagate in the plasma, since the coefficient α becomes negative (it has been proved that the KdV equation with a negative coefficient for the nonlinear term governs the propagation of the rarefractive solitons [17,18]). The weak dependence of the peak soliton amplitude and the width of both types of solitons on the relativistic ion drift velocity, for other fixed parameters (θ =30°, A=0.2, and σ=0.02) [8], is shown in Fig. 1. It is obvious from the figure that the amplitude increases with the relativistic ion drift velocity, whereas the soliton width decreases. It may also be noted that the magnitude of the amplitudes of compressive and rarefractive solitons is the same.

Variation of the peak soliton amplitude with the angle of propagation of the wave, for different values of ion to electron temperature ratio, is shown in Fig. 2 for v_0=115 and A=0.2. It is evident from the figure that the angle of propagation reinforces the amplitude, whereas it goes down for the higher ion temperatures. It can be observed that as the wave approaches the direction perpendicular to the magnetic field,

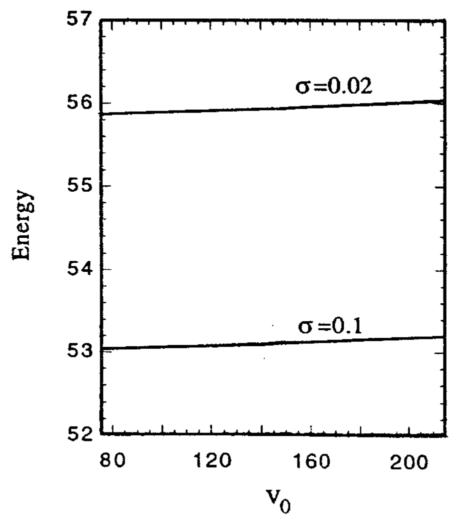

FIG. 3. Weak dependence of the soliton energy ($EU^{-3/2}$) on the relativistic ion drift velocity v_0. Here $A=0.2$ and $\theta=30°$.

its amplitude suddenly becomes very high and finally the wave disappears. This can be understood from Eqs. (6), (10a), and (13a), which show that the phase velocity and the coefficient α (which appears in the denominator of the amplitude expression) tends to zero as $\theta\rightarrow 90°$. It should also be noted from Eqs. (10a) and (13a) that the peak soliton amplitude is independent of the magnetization and hence it is not affected by the strength of the magnetic field. However, the soliton width becomes smaller with the strong magnetic field [Eqs. (10b) and (13b)].

The energy of the soliton is one of its characteristics. By using the soliton solution, one can obtain an expression for the soliton energy in terms of the phase velocity, angle of the wave propagation, and strength of the magnetic field. The integral $\int_{-\infty}^{\infty}\phi_1^2(\nu)d\nu$ gives the relation for the soliton energy [8,10]

$$E=12\sqrt{2}U^{3/2}\cos\theta\{A^2+(1+2\sigma)^2\times\sin^2\theta\}^{1/2}/(\lambda_0-v_0\cos\theta)^{5/2}\gamma^{1/2}A. \tag{14}$$

Figure 3 shows the weak dependence of the soliton energy on the relativistic ion drift velocity for $\theta=30°$, $A=0.2$, and different values of ion temperature. It is evident from the figure that the soliton energy increases with the relativistic ion drift velocity but decreases for the higher ion temperature. It can also be seen from Eq. (14) that the soliton becomes more energetic with the angle of propagation of the wave but loses energy for the stronger magnetic field.

CONCLUSION

The Korteweg–de Vries equation, which describes the propagation of the ion acoustic solitons in a weakly relativistic magnetized warm plasma, is derived. Two types of modes, namely, a fast ion acoustic mode and a slow ion acoustic mode, exist in the plasma. Fast and slow modes correspond respectively, to the propagation of compressive and rarefractive solitons of the amplitudes with the same magnitude. The energy of these solitons is calculated in terms of the phase velocity, the strength of the magnetic field, and the propagation angle of the wave. The peak amplitude and energy of both types of solitons increase with the relativistic ion drift velocity and also with the propagation angle of the wave. The strength of the magnetic field weakens the soliton energy. It is found that with increasing ion temperature, the amplitude and the energy of the solitons go down but the width becomes wider.

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