Morphological instabilities of dynamic fractures in brittle solids

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We present a study of the stress fields in the neighborhood of a moving crack tip in the framework of linear elastic fracture mechanics. This approach is found to be physically relevant for a large range of the crack speeds. We show that the stability analyses based on conditions of attainment of a critical tensile stress on some plane are inadequate to describe the instabilities of the crack path. A study of the largest principal stress in the neighborhood of the crack surface is reported. We show that at ''low'' crack velocities the path of the crack extension is an opening mode. However, this property disappears when the crack speed exceeds a critical velocity V_c and reappears again beyond a faster speed V_B, but at a different orientation from that of pure opening mode. These variations have been interpreted as the onset of roughening and branching instabilities. [S1063-651X(96)05511-0]

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I. INTRODUCTION

Dynamic brittle fracture experiments have always shown many puzzling phenomena [1–3]. However, the recent series of experiments [4–7] have clearly and definitely established that these phenomena are related to fundamental physical processes, such as crack speed, crack branching, surface roughening transition, and dynamic instability.

More precisely, it has been shown that a dynamic instability controls the crack advance when its velocity v exceeds a critical velocity V_c. This threshold has been evaluated as a fraction of the Rayleigh wave speed V_R, the speed at which elastic waves travel across a flat surface. At that point, the initially flat fracture surface of the material becomes *rough*. For speeds higher than V_c, the crack dynamics change dramatically: the acoustic emission from the crack increases [5,6], the velocity oscillations are amplified, and a pattern, which is more or less correlated with the velocity oscillations, is created on the fracture surface [4,6]. Recent experiments [7] have also distinguished an intervening transition region of fine-scale fracturing. Indeed, beyond a velocity $V_b(<V_c)$, the straight crack branches locally. This *microbranching* instability may be the origin of the weak sound emission for $v<V_c$ [6]. These different patterns that characterize the crack surface were already known in materials science as the *mirror-mist-hackle* zones [2]. Finally, at higher velocities $V_B(>V_c)$, a *macroscopic branching* instability occurs: the crack tip splits or deviates from its original orientation [8].

Crack branching and/or roughening has also been observed in recent simulations of crack motion using molecular dynamics [9], using a numerical resolution of constitutive equations on a lattice [10] or by modeling the elastic medium as a two-dimensional lattice of coupled springs [11]. From the theoretical side, although much work has been done in the field for over 70 years [1], the mechanisms that govern the dynamics of cracks are not well understood and a theory of instability does not exist yet. Consequently, it has been argued [12–14] that the quasistatic, far-field assumptions upon which most conventional theories are based are inherently inadequate for detecting these instabilities, and that it is necessary to study complete dynamic models of deformation and decohesion at the crack tip [15] in order to understand the experimental observations. Recent theoretical improvements have been achieved in this direction [12–14]. However, the point that has remained unclear until now is the relevance of these analytical and numerical models for the observed crack *morphologies*. Particularly in the numerical simulations, privileged directions of propagation are often imposed, and the lattice sizes are not ''microscopic'' compared to the size of the system. Consequently, the comparison with experimental results, where the microbranching and the roughening instabilities are of microscopic scales, is limited.

In this paper, we will determine some properties of the crack extension by examining the singular stress fields in the neighborhood of a geometrically sharp crack tip. In Sec. II, we will discuss the validity and the limitations of this approach to fracture mechanics. We will investigate the roughening and the branching transitions and show that they can be described, at least to a first approximation, by the same approach. These instabilities are the second and third transitions in the morphology diagram of the crack surface. However, we will assume that *the three instabilities are uncorrelated*. Therefore, their origins can be deduced separately. This assumption can be justified by observing that the appearance of a new zone does not lead to the disappearance of the old one, although it may even amplify it. For instance, the microbranches are more dense when the crack becomes rough [6,7] and there is no reason why the macroscopic crack branches have to be smooth.

In Sec. III, we study the largest principal stress near the tip, in conjunction with symmetry considerations on the broken surface [16]. We correlate the behavior of this quantity before and after cracking and study the curves of constant largest principal stress. In this way, we extract a property that shows that at low crack velocities, the path of crack extension is that of a pure opening mode (mode I) [17,18]. How-

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ever, this property disappears when the crack speed exceeds V_c and reappears again beyond a speed V_B, but in a direction different from that for low velocities ($v<V_c$). These changes are interpreted as being related to the roughening and branching transitions, respectively. The thresholds V_c and V_B are below the well-known Yoffe critical velocity V_y [19] and are in the range of values obtained in both experiments and numerics.

II. ASYMPTOTIC STRESS FIELDS NEAR A MOVING CRACK TIP

Consider a body of nominally elastic material that contains a crack. Under the action of applied loads on the boundary of the body or on the crack faces, the crack edge is a potential site for stress concentration. The theory that has been developed so far [1] for describing the relationships between crack tip fields and the loads applied to a solid of specified configuration is linear elastic fracture mechanics. In this approach, the local analysis in the neighborhood of the crack tip shows that the asymptotic stress tensor field $\bar{\bar{\sigma}}$ exhibits a *universal* square root singularity [1]. When the local deformation field is an in-plane opening mode, the components of the stress field near the tip are expressed as

$$\sigma_{ij}(\theta,v,t)=\frac{K_I(t)}{\sqrt{2\pi r}}\Sigma_{ij}(\theta,v) \tag{1}$$

for $r\to 0$, where (r,θ) are polar coordinates in a plane perpendicular to the crack edge. The point $r=0$ coincides with the crack edge and the line $\theta=0$ is the tangent to the crack surface at the crack edge (in the forward direction). The dimensionless function $\Sigma_{ij}(\theta,v)$ represents the angular variation of each component of the stress near the crack tip. It is a universal function, independent of the configuration of the body and the details of the applied loads [20]. Moreover, Σ_{ij} depends on the material constants only through c_d and c_s, which are the elastic dilatational and shear wave speeds. These quantities are related to the Poisson ratio ν, through κ, by

$$\left(\frac{c_d}{c_s}\right)^2\equiv\kappa=\begin{cases}\dfrac{2}{1-\nu} & \text{for a plane stress}\\[2ex] \dfrac{2-2\nu}{1-2\nu} & \text{for a plane strain.}\end{cases} \tag{2}$$

It is important to emphasize that Σ_{ij} depends on the motion of the crack tip only through *the instantaneous crack tip speed* $v(t)$. All information about loading and configuration are embedded in the scalar multiplier $K_I(t)$ called the dynamic stress intensity factor.

The variation of stress components near the crack tip is often represented in polar coordinates

$$\sigma_{rr}+\sigma_{\theta\theta}=\sigma_{xx}+\sigma_{yy}, \tag{3}$$

$$\sigma_{rr}-\sigma_{\theta\theta}+2i\sigma_{r\theta}=e^{-2i\theta}(\sigma_{xx}-\sigma_{yy}+2i\sigma_{xy}). \tag{4}$$

It is also useful to compute the hydrostatic and the maximum shear stresses P and $\tau_{\max}$, which are given by [21]

$$2P=\sigma_1+\sigma_2=\sigma_{xx}+\sigma_{yy}, \tag{5}$$

$$2\tau_{\max}=\sigma_1-\sigma_2=\sqrt{(\sigma_{xx}-\sigma_{yy})^2+4\sigma_{xy}^2}, \tag{6}$$

where σ_1 and σ_2 are the principal stresses. The direction of the largest principal stress σ_1 is found from the condition that the shear stress on a plane whose normal makes an angle β with respect to the x axis vanishes:

$$\tan 2\beta=\frac{2\Sigma_{xy}}{\Sigma_{xx}-\Sigma_{yy}}. \tag{7}$$

In 1951, Yoffe [19] observed that for crack speeds less than a critical velocity V_y, the transverse tensile stress $\sigma_{\theta\theta}$ is maximum along $\theta=0$, that is, in the direction of crack growth. For crack speeds greater than V_y, this component of the stress develops a maximum in a direction $\theta\neq 0$. Yoffe [19] suggested that this modification of the local singular stress field could account for the observation that rapidly growing cracks in very brittle materials bifurcate into branched cracks. However, Williams [21] pointed out that the transverse tensile stress is not the most fundamental quantity for understanding crack advance in brittle materials. Instead, if the local condition for fracture is the attainment of a critical tensile stress on some plane, then the maximum principal stress σ_1 is the relevant measure of stress to consider [21,22].

Suppose that one can construct such a condition based on a maximization of the largest principal stress. In this case, the direction β of σ_1 must satisfy $\beta=\theta_1\pm\pi/2$, where θ_1 is the location of the maximum of σ_1 [16]. This is a direction perpendicular to a plane that contains the crack edge ($r=0$). This stress configuration is never satisfied for any velocity v [16]. So this approach is inadequate even in the quasistatic limit, where $\theta=0$ is known to be the privileged direction of crack extension [17]. Note, however, that Baker [22] observed that for low crack speeds, the angular variation of σ_1 has a shallow maximum at an angle $\theta\neq 0$. For very high speeds, on the other hand, the variation of this stress shows local maxima at both $\theta=0$ and some value of θ larger than $\pi/2$.

Along $\theta=0$, the in-plane principal stresses within the singular field are equal for zero crack speed and nearly equal for low crack speeds. However, the maximum shear stress $\tau_{\max}$ is a rapidly increasing function of the velocity for ''high'' values of v and diverges as v approaches V_R. Therefore, it has been argued [1] that this effect may explain the observed tendency for rapidly growing cracks in brittle materials to develop rough fracture surfaces. However, this change from the corresponding equilibrium result in the nature of the asymptotic field also shows that σ_1 always exceeds σ_2 for any velocity $v\neq 0$. This result may imply that this class of fracture model is intrinsically unstable [13,14].

Nevertheless, one knows that the asymptotic expansion of the stress field near the tip is not really singular. In fact, the normal stress at the tip must be exactly equal to the yield stress σ_0 since the condition for fracture is the attainment of a critical stress. This constraint is taken into account in cohesive-zone models of the kind introduced by Barenblatt [15]. In these models, an isotropic, ideally brittle solid obeys linear elasticity everywhere outside sharply defined fracture

surfaces and a finite-ranged cohesive stress of amount σ_0 opposes the separation of these surfaces near the crack tip. These models are, however, simplified pictures of the complex, nonlinear phenomena occurring within the process zone near the tip of a real crack [23].

A qualitatively different attempt to describe the vicinity of the crack tip has been proposed [1]. One knows that plasticity occurs before the yield stress σ_0 is attained. Therefore, the purely linear elastic approach is strictly valid only when the components of the stress field are smaller than a certain critical stress $\sigma_{\max}(\leqslant\sigma_0)$. Since σ_1 is the largest stress, one must impose $\sigma_1\leqslant\sigma_{\max}$. This condition gives an estimate of the size of the cohesive zone near the crack tip. The linear elastic theory is valid in the region $r\geqslant r_{\min}$, with

$$r_{\min}(\theta,v,t)\approx\frac{K_I^2(t)}{2\pi}\left(\frac{\Sigma_I(\theta,v)}{\sigma_{\max}}\right)^2. \tag{8}$$

Of course, the asymptotic expansion of the stress field (1) remains valid for $r\geqslant r_{\min}$ only if $r_{\min}$ is very small. When the plastic effects are introduced in this way, the stress singularity is avoided and the tractions have no reason to favor motion perpendicular to the original direction of propagation.

It is plausible to take $\sigma_{\max}$ as a material constant. Then, the plastic zone defined by Eq. (8) depends on $(K_I\Sigma_1)^2$. It is known [1] that the stress intensity factor K_I is a decreasing function of v for a given geometry and loading conditions. However, the behavior of $r_{\min}$ is essentially controlled by that of Σ_1. Thus the size of the nonelastic zone is nearly constant for low velocities. It is almost equal to the size of the quasistatic cohesive zone, which is, by definition, very small. However, according to the behavior of Σ_1 [1], $r_{\min}$ is a rapidly increasing function for large values of v and diverges when $v\rightarrow V_R$. Thus, to be physically relevant, our analysis relying upon linear elastic theory is limited to velocities definitely less than these large crack speeds.

III. STABILITY OF A MODE-I MOVING CRACK

We have seen that analyses based on conditions of attainment of a critical tensile stress on some plane [19,22] fail to describe the instabilities of the crack path. In fact, in order to do a stability analysis, one must first define how the moving crack chooses its direction of propagation. In the quasistatic case, the crack extension must satisfy a principle of local symmetry [17], which states that the path taken by a crack in a brittle homogeneous isotropic material is the one for which the local stress field at the tip is of mode-I type. It is not obvious that the crack propagation still satisfies this strong criterion in the dynamic case [14], where the inertial effects increase with velocity [1]. When the crack dynamics are taken into account, the response of the stress field to a velocity-dependent extension of the crack tip is effectively delayed [18]. However, if the configuration satisfies the stability criterion of [24], the quasistatic analysis imposes the crack tip to begin its motion towards a line of principal stress [17,18].

For convenience, let us assume that a straight crack is created under the action of a mode-I loading on the boundary of the body. In brittle fracture mechanics, the crack must be a stress-free surface where the conditions

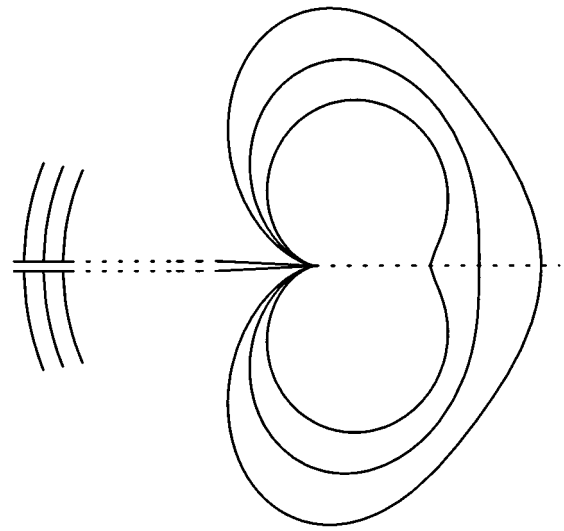

FIG. 1. Shape of the surfaces of constant σ_1 near a moving crack tip, when $\kappa=3$ and for different values of the crack tip speed. The inner (middle and outer) curve corresponds to $v/c_s=$ 0.3 (0.5 and 0.6). For clarity, we have plotted the surfaces $\sigma_1=K_I/\sqrt{2\pi}$. The behavior of the lines of constant σ_1 in the neighborhood of the cracked surface and far from the edge are also represented qualitatively (curves to the left).

$$\sigma_{ij}n_j=0 \tag{9}$$

are satisfied; $\vec{n}$ is the vector normal to the crack surface and repeated indices indicate summation. So in the two-dimensional case, the principal stress σ_2 vanishes along the surface of the fracture, in both quasistatic and dynamic propagation. From the boundary conditions (9) and the symmetry of the mode-I loading, another definition of the condition satisfied by the crack shape can be proposed. The crack is a line perpendicular to the curves of constant largest principal stress σ_1. Moreover, the stress field components increase when approaching the crack edge. Thus, near the crack surface and far from its edge, the sign of the curvature of the lines of constant σ_1 is also known (see Fig. 1). These observations are obvious in the vicinity of the broken surface, but they allow an approach of the stress field near the whole surface of the crack, including its edge.

Now consider the largest principal stress field ahead of the crack edge. In Fig. 1, we have plotted the lines of constant σ_1, for different values of crack tip speeds. The vector field $\vec{s}$ normal to these lines is

$$\vec{s}=\vec{\nabla}\sigma_1|_{\sigma_1=\text{const}}\propto\frac{1}{2\Sigma_1^3}\left(-\Sigma_1\vec{e}_r+2\frac{\partial\Sigma_1}{\partial\theta}\vec{e}_\theta\right), \tag{10}$$

where $(\vec{e}_r,\vec{e}_\theta)$ are the unit vectors in polar coordinates. The orientation ψ of $\vec{s}$ with respect to the initial direction of crack propagation is thus given by

$$\psi=\pi+\theta-\tan^{-1}\left[\frac{2}{\Sigma_1}\frac{\partial\Sigma_1}{\partial\theta}\right]. \tag{11}$$

The angle π is introduced to recall that the stress field increases when one approaches the crack tip. Note that the crack edge lies along the $\vec{s}$ direction only when

$$\psi = \theta + \pi. \tag{12}$$

Suppose that, in addition to being parallel to the vector field $\vec{s}$, the crack has to propagate in a direction that satisfies this condition. Then one has at least three possibilities of crack tip propagation: $\theta = 0$ and $\theta = \pm\theta_0$, given by the solutions of Eq. (12). However, it is consistent to assume that at low crack speeds, the principle of local symmetry [17,18] still holds. So, unless it is initially unstable [24], the crack is expected to grow in the direction $\theta = 0$. The conclusion is that one has to define a mechanism that selects, at low velocities, the solution $\theta = 0$ and prevents other orientations.

In the approach presented here, the properties of the field σ_1 are responsible for the crack path selection. Thus one must relate the behavior of the largest principal stress before and after breaking the medium. For this purpose, let us define the stress components in the frame $(\vec{s}, \vec{t})$, where $\vec{t}$ is the tangent vector to the lines of constant σ_1. Near the crack tip, these components can be written as

$$\sigma_{ss} + \sigma_{tt} \approx (\Sigma_{xx} + \Sigma_{yy}) \frac{\sigma_1}{\Sigma_1}, \tag{13}$$

$$\sigma_{ss} - \sigma_{tt} + 2i\sigma_{st} \approx e^{-2i\psi} (\Sigma_{xx} - \Sigma_{yy} + 2i\Sigma_{xy}) \frac{\sigma_1}{\Sigma_1}. \tag{14}$$

By definition, the process of crack growth in a certain direction is essentially the negation, in this direction, of the traction distribution ahead of the crack tip induced by the applied loads. The cracked surface becomes a principal stress line, independently of the direction of propagation. Therefore, if the crack path follows a direction parallel to $\vec{s}$, the stress components σ_{tt} and σ_{st} in this direction are canceled and σ_{ss} becomes, after breaking, the largest principal stress. Moreover, in the neighborhood of the future crack surface, the opening and eventually the shear loading must be applied on opposite planes with respect to this path. Our construction evidently satisfies this condition at both $\theta = 0$ and $\theta = \pm\theta_0$.

As can be seen in Fig. 1, for low crack speeds, the in-plane stress σ_{ss} in the neighborhood of $\theta = 0$ acts on planes off the direction $\vec{s}$ of this orientation. The resultant loadings are dilational. On the other hand, near $\pm\theta_0$, σ_{ss} acts on planes towards the direction $\vec{s}$ of these orientations. This leads to a compression effect. As a consequence, one can state that the plane $\theta = 0$ is ''prepared'' to break, but on $\theta = \pm\theta_0$ the loadings act to prevent breaking. Moreover, if the crack propagates along the direction $\theta = 0$, the lines of constant σ_1 near the newly broken surface will have the same sign of curvature as that of the already fractured surface. This is not the case if $\pm\theta_0$ were the directions of propagation. At low velocities, $\theta = 0$ is thus the selected direction for crack propagation.

The dilational or compressive effect of the in-plane stress σ_{ss} is our basic concept that leads to a selection mechanism of the crack direction of propagation. In fact, by examining the stress field in the neighborhood of the already broken surface, we conjectured that the new crack surface must be created following the behavior of the local stress field σ_{ss}. If opening (compressive) mode tractions exist in certain directions with respect to the coordinates system $(\vec{s}, \vec{t})$, the crack propagation is favorable (unfavorable) there. On the other hand, the absence of privileged directions of propagation will be related to the absence of opening mode tractions in the frame $(\vec{s}, \vec{t})$.

If the arguments presented above are adopted for any crack speed, one can study now the behavior of the lines of constant σ_1 at higher velocities. Beyond a critical velocity V_c, the in-plane stress field σ_{ss} near $\theta = 0$ changes its main property. Effectively, in this case and as shown in Fig. 1 for a crack speed of $0.5c_s$, one still has three roots for Eq. (12), but all of these directions have the same properties. Now the loading near $\theta = 0$ acts, similarly to the directions $\pm\theta_0$, on compressive planes with respect to the central plane $\theta = 0$. Thus it is expected that there is not a privileged direction of propagation. The crack can open anywhere: this is a manifestation of a roughening phase, in which the crack has difficulties extending since no direction is prepared to break. This explains qualitatively why dissipative mechanisms become important beyond V_c.

However, when the crack speed exceeds a second critical velocity $V_B > V_c$, a new situation occurs. Equation (12) now has five roots (see Fig. 1): $\theta = 0, \pm\theta_0$, and $\pm\theta_B$, with $0 \leqslant \theta_B < \theta_0$. The loadings σ_{ss} in the neighborhood of $\theta = 0$ and $\pm\theta_0$ are still acting in a way that counteracts breaking. However, the in-plane stresses σ_{ss} act on planes off the new directions $\pm\theta_B$. The resultant loadings are thus dilational. This is a manifestation of the possibility for the crack to branch towards the velocity-dependent orientations $\pm\theta_B$.

It is easy to demonstrate that these morphological transitions of the crack pattern occur when $[\partial\psi/\partial\theta]_{\theta=0} = 0$ for the first instability and when $[\partial\psi/\partial\theta]_{\theta=0} = 1$ for the second one. Therefore, the roughening and branching thresholds are given by the solutions of

$$V_c \rightarrow \left[\Sigma_1 - 2\frac{\partial^2 \Sigma_1}{\partial \theta^2} \right]_{\theta=0} = 0, \tag{15}$$

$$V_B \rightarrow \left[\frac{\partial^2 \Sigma_1}{\partial \theta^2} \right]_{\theta=0} = 0, \tag{16}$$

V_c can be seen as a quantitative criterion and a physical interpretation of the threshold for which the stress fields become rapidly increasing functions of v [1]. On the other hand, V_B corresponds to the critical velocity where the largest principal stress exhibits local maxima at both $\theta = 0$ and some value of θ beyond $\pi/2$ [22]. Note that V_B is probably larger than speeds for which the purely elastic description was supposed to be valid. We believe, however, that the physical origin of the branching phenomenon is given by such an analysis and that only the precise value of V_B is sensitive to the cohesive processes that occur near the crack tip.

Finally, we determined a *universal estimate* of the roughening and branching critical velocities (see Fig. 2). This is an advantage of the purely elastic approach. For $\kappa \approx 3$, which corresponds to a wide range of elastic materials, we found

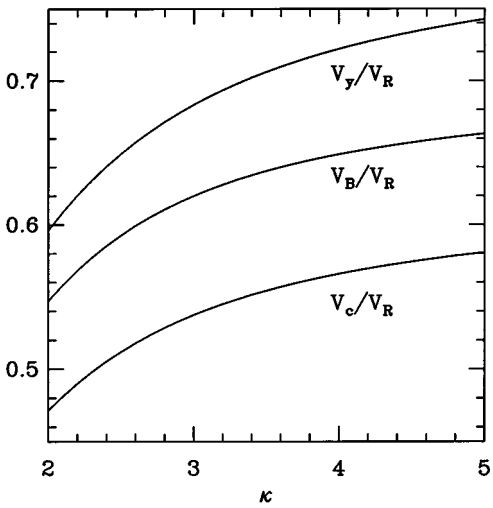

FIG. 2. Plot of the roughening (V_c) and the branching (V_B) critical velocities, in units of the Rayleigh wave speed V_R and in terms of the material parameter κ [Eq. (2)]. For comparison, the Yoffe critical speed V_y, which is the solution of $[\partial^2\Sigma_{\theta\theta}/\partial\theta^2]_{\theta=0}=0$, is also plotted.

$V_c \approx 0.54V_R$ and $V_B \approx 0.62V_R$, which is below the critical Yoffe velocity $V_y \approx 0.68V_R$. These values are in qualitative agreement with experiments [5,6] for the roughening threshold and with both the experimental [6] and numerical [11] estimations of the branching threshold. However, the comparison between theses diferent results relies on the hypothesis that the crack morphologies may be classified into mirror-mist-hackle-branched zones [8].

IV. CONCLUSION

We have classified the morphological instabilities of a crack into three regimes that occur upon increasing the crack speed: the microbranching, the surface roughening, and the crack branching [8]. We have proposed a model that exhibits both roughening and branching transitions. Our analysis is based on the study of the shape of the surfaces (lines) of constant largest principle stress in the neighborhood of the whole surface discontinuity of the crack. We determined the property that favors the mode-I extension at low speeds. We studied this property for higher crack velocities and interpreted its variations as manifestations of crack pattern transitions. This approach shows that roughening and branching are intrinsic instabilities that are displayed at the level of the linear elastic theory. The addition of cohesive zone treatments near the crack edge should not change the instability processes themselves. However, the velocity thresholds V_c and V_B may be readjusted, depending on the geometry of the problem considered, the crack dynamics, and, of course, the nature of the cohesive zone model.

Although we have put into evidence some mechanisms of crack instability, we did not make quantitative predictions on the post-threshold manifestations of these instabilities. Moreover, the microbranching instability, and thus the real origin of the velocity oscillations observed in experiments, cannot be studied in the framework of such a simple local model. In fact, microbranching and acoustic emission are manifestations of dissipative mechanisms [6,7]: microbranches are created once the crack tip begins to emit sound [6]; they do not extend throughout the entire thickness of the material [7] and they are more dense when the fracture becomes rough. Therefore, these phenomena cannot be revealed without taking into account the inevitable nonlinearities that occur in the vicinity of a crack tip [23].

Another major theoretical challenge is the determination of the dynamics of the crack itself. According to the present theory of fracture mechanics [1], the crack tip should smoothly accelerate until it reaches the Rayleigh wave speed V_R. Experiments, however, seldom show crack speeds exceeding half this speed [2,3]. This limitation on the terminal velocity does not seem to be always related to the dissipative mechanisms at the crack tip. Indeed, experiments [4–7,11] show that the final crack speed depends on the potential energy fed in by the external loading. In addition, if the instabilities, which induce the velocity oscillations of the crack tip, can be treated as a perturbative process, the crack motion of the corresponding unperturbed problem must be known.

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