Self-bending photorefractive solitons

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We found stationary intensity profiles of photorefractive screening solitons in the presence of the diffusion processes. We show that these solitons are stable and in a certain range of parameters they propagate with negligible radiation losses along exactly parabolic trajectories. We studied the collision of screening solitons using numerical simulations. We found that even a small contribution of the diffusion effect leads to strong energy exchange between the colliding photorefractive solitons. [S1063-651X(96)03011-5]

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The self-induced spatial soliton is the central idea of the light-guiding-light concept [1]. Optical spatial solitons in photorefractive media have attracted much attention recently due to the possibility of their observation at very low laser power levels and great potentials for applications in all-optical switching and processing [2]. Three different types of photorefractive solitons have been identified so far—quasi-steady-state [2], screening [3–5], and photovoltaic [6]. Among them the so-called screening solitons are the most promising. In this case, the optical beam propagates in photorefractive crystal biased with external dc electric field. The presence of the optical beam in the crystal leads to photoexcitation of electric charges, which migrate and are trapped by the defects. In a steady-state regime, the spatial distribution of charges screens the externally applied electric field and this process results in the decreasing of the total electric field in the illuminated area of the crystal. The spatial modulation of the static electric field modifies the index of refraction via the Pockels effect, in such a way that the beam becomes self-trapped and propagates in a form of a spatial soliton. One- and two-dimensional screening solitons have been observed recently in experiments with strontium barium niobate crystals [7,8]. The space-charge redistribution in photorefractive crystals is caused mainly by the drift of photoexcited charges in a biasing electric field. This mechanism leads directly to a *local* change of the refractive index and the self-focusing [9]. In addition, transport of the photoexcited charges occurs also due to the diffusion. This process results in the *nonlocal* contribution to the refractive index change. The strength of the diffusion effect is determined by the width of the beam. In the case of a strong biasing field and relatively wide beams, the diffusion term is often neglected. However, its contribution can become significant for very narrow beams. It is a well known fact that this diffusion process leads to a strong bending of the trajectory of optical beams in photorefractive crystals [10–13]. It has also been shown [5,8,14] that the diffusion bends the trajectories of the photorefractive solitons. Variational and numerical analyses have shown that the soliton propagates along an approximately parabolic trajectory with a constant intensity profile [14].

In this paper we analyze stationary profiles of the photorefractive solitons in the presence of the diffusion effect. We show, in particular, that these solitons, with asymmetric intensity profile, propagate along exactly parabolic trajectories.

Steady-state propagation of the one-dimensional optical beam (in planar geometry) in biased photorefractive crystal can be described by the following normalized equation [3,14]:

$$i\frac{\partial u}{\partial z}+\frac{1}{2}\frac{\partial u}{\partial x^2}-\frac{u}{1+|u|^2}+\gamma\frac{\partial \ln(1+|u|^2)}{\partial x}u=0, \qquad (1)$$

where $u(x,z)$ is the slowly varying amplitude of the field normalized to dark or background intensity, while z and x are propagation and transverse coordinates, respectively. This is the so-called modified nonlinear Schrödinger equation with saturable nonlinearity. The term $1/(1+|u|^2)$ represents *local* nonlinear (saturable) change of the refraction index of the crystal induced by the presence of the optical beam. The last term on the left-hand side of Eq. (1) describes the contribution from the diffusion process. The parameter

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γ is responsible for the strength of the diffusion effects (relative to the external electric field),

$$\gamma \propto \left(\frac{r_{\rm eff}}{E_0}\right)^{1/2}, \tag{2}$$

where $r_{\rm eff}$ and E_0 denote an effective electro-optic coefficient and dc biasing field, respectively.

Equation (1) has a conserved quantity which we call power

$$Q = \int_{-\infty}^{\infty} |u|^2 dx. \tag{3}$$

Translational invariance in z suggests [15] that another conserved quantity exists which is analogous to Hamiltonian. However, its analytic expression cannot be found.

We are interested here in stationary solutions of Eq. (1). Using symmetry reductions [15] (for a particular example similar to our problem see [16]), it can be shown that the propagation equation has a stationary solution in the form

$$u(z,x) = f(\xi)\exp\left[-iz\left(\frac{b^2}{3}z^2 - bx + q\right)\right], \tag{4}$$

where $\xi = x - (b/2)z^2$, and q and b are free parameters. The function $f(\xi)$ must be real for the profile to be stationary. It satisfies the equation

$$\frac{1}{2}f_{\xi\xi} + qf - b\xi f - \frac{f}{1+f^2} + 2\gamma_0\frac{ff_\xi}{1+f^2} = 0. \tag{5}$$

Therefore, if a stationary solution to Eq. (1) exists, it will propagate along an exactly parabolic trajectory given by the relation

$$x - \frac{b}{2}(z - z_0)^2 = x_0,$$

where x_0 and z_0 are arbitrary constants.

Solutions of Eq. (5) cannot be found in analytic form although different approximations are possible. They can be found using, e.g., the variational approach. We have to bear in mind that exact profiles necessarily have to have an oscillatory tail at one side of the soliton which extends to infinity and decays slowly in ξ. This follows from the analysis of the linearized equation

$$\frac{1}{2}f_{\xi\xi} + qf - b\xi f - f = 0, \tag{6}$$

which has a solution in the form of an Airy function. Asymptotic behavior of the solution is different at plus and minus infinity due to the asymmetric term $b\xi f$. This means also that the intensity profile of the soliton is asymmetric. The oscillatory tail indicates that there will always be radiation phenomena which can be interpreted physically as bending losses. Consequently, when bending is small ($b \ll 1$), radiation can be ignored.

The solitonlike solutions of Eq. (5) can be found using numerical integration (shooting technique). For a given value of γ there is a one-parameter family of soliton solutions characterized by the parameter b. Soliton amplitude and the parameter q depend on b. We can add one more parameter to the solution using Galilean transformation

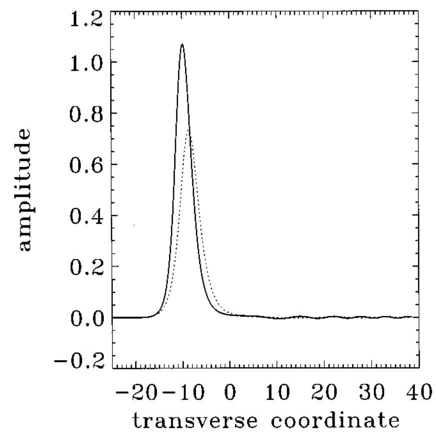

FIG. 1. Examples of intensity profiles of the self-bending photorefractive solitons characterized by different values of the parameter b: solid line, $b = 0.026\,3435$; dashed line, $b = 0.018\,82$; $\gamma = 0.15$ in both cases.

$$u'(x,z) = u(x - vz, z)\exp(ivx - iv^2 z/2). \tag{7}$$

This transformation adds ''velocity'' v (initial slope of the trajectory) as a new parameter of the solution. Existence of the symmetry (4) allows us to consider (7) as transformation, shifting the whole solution in x and z.

In Fig. 1 we present an intensity profile corresponding to two solutions with different amplitudes. In both cases the parameter γ is equal to 0.15. As is evident, the solitons feature an asymmetric intensity profile. The steeper tail of the soliton (at negative ξ) is a monotonically decaying function of ξ. The other tail (at positive ξ) has an oscillatory behavior at infinity. By choosing parameter b to be small, the oscillatory part of the tail can be made arbitrarily small and ignored. In this case, the solution of the problem can be separated effectively into two parts: finding the shape of the soliton and calculating radiation effects. The solutions of the second part can be performed analogously to the calculation of bending losses in waveguide theory [17].

In the above approximation, the radiation does not influence much the propagation of the soliton itself. To show this, we used the profiles obtained from Eq. (5) as an initial condition in Eq. (1) which was subsequently integrated using the split-step fast Fourier transform (FFT) method. Results of these simulations are shown in Fig. 2. The solid line in Fig. 2(a) is the initial intensity profile of the soliton. The dashed line shows the soliton profile after propagation over distance $z = 80$. It is clear that the soliton propagates without visible changes. The radiation field is, in this particular case, below the level 10^{-4}. The contour plot in Fig. 2(b) shows parabolic trajectory of the moving soliton.

In separate numerical simulations we studied stability of these solitons with respect to perturbation of the initial profile. We found that self-bending solitons are stable and can withstand relatively large perturbations. If the initial profile is not exactly a solution of Eq. (5), the beam quickly adjusts its parameters and transforms itself into the stationary one. In

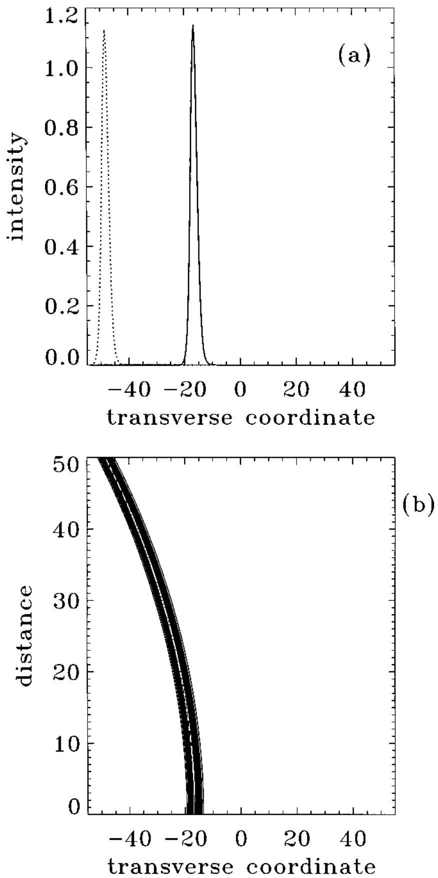

FIG. 2. Propagation of stationary self-bending photorefractive screening soliton with $b = 0.010\,882$ ($\gamma_0 = 0.15$). (a) Intensity profile (solid line—initial profile, dashed line—final profile); (b) contour plot illustrating soliton trajectory.

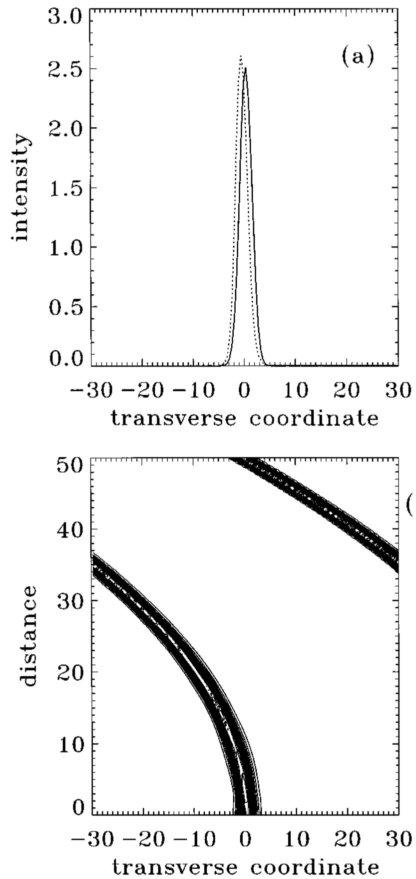

FIG. 3. Gaussian beam excitation of photorefractive soliton in the presence of the diffusion term ($\gamma = 0.15$). (a) Initial (solid line) and final (dashed line) profile of the beam. (b) Contour plot illustrating propagation of the beam. After a short transient region the beam propagates as a self-bending soliton.

experiments, it is very unlikely that the initial beam has exactly the right soliton profile. Typically, the Gaussian beam from the laser is launched into the photorefractive crystal. This beam subsequently adjusts itself to form the soliton [18].

In order to mimic experimental conditions, we performed numerical simulation of self-bending solitons using the Gaussian intensity distribution as the initial condition. Examples of these simulations are shown in Figs. 3 and 4. The input intensity distribution is a Gaussian function $I(x) = 2.51\exp(-0.35x^2)$. The plots in Figs. 3 and 4 correspond to two different values of the bending parameter, $\gamma = 0.15$ and $\gamma = 0.30$, respectively. One can see that except for a relatively short initial distance when the beam width and intensity vary, the beam propagates in the form of a stationary solution as a self-bending soliton. The plots in Fig. 3(a) and Fig. 4(a) also illustrate the development of the asymmetry of the beam which increases with an increase in the values of γ.

It is interesting to see how the presence of the diffusion term affects the collision of photorefractive solitons. Soliton collision is important for their application in optical switching and for use in optical circuitry. The unique property of Kerr solitons is that they collide elastically so that a few crossings of optical channels can simultaneously operate without the cross talks. The soliton collisions can also be used to create passive waveguide structures such as lossless X junctions [19]. In media with saturable nonlinearity such as are considered here, photorefractive materials, the propagation of solitons is described by the nonintegrable nonlinear Schrödinger equation. Nonintegrability manifests itself in the inelasticity of the soliton collision [20], i.e., radiation is emitted from the impact region and solitons change their parameters due to the collision. In order to investigate the influence of the diffusion term on the soliton collision we solved numerically Eq. (1) using as an initial condition two spatially separated soliton solutions (with $\gamma = 0$ and maximum intensity $I_0 = 0.6$). These solutions have been launched with different initial velocities [using the transformation (7)] in order to introduce relative transverse velocity of the beams. The results of these simulations are presented in Figs. 5(a) and 5(b). Figure 5(a) shows the collision of two solitons without the diffusion term ($\gamma = 0$). Both solitons are initially in phase and collide with low losses in the impact area. However, the plot in Fig. 5(b) shows that inclusion of even a very small nonlocal term ($\gamma = 0.02$) leads to an energy exchange between the beams. This behavior is analogous to the reported earlier interaction of very narrow temporal solitons in the presence of the Raman effect [21]. Strong energy exchange between the colliding solitons has been observed [21]. The diffusion term in Eq. (1) is analogous to the Raman term in the equation describing temporal solitons. It has the same

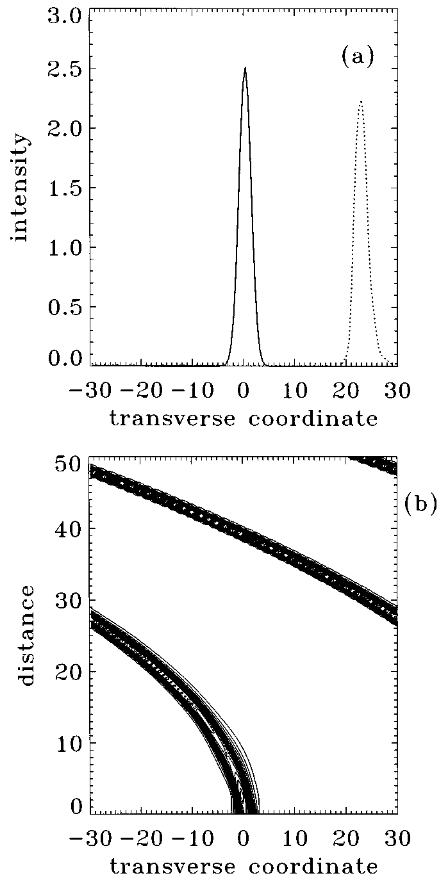

FIG. 4. The same as Fig. 3 but with $\gamma = 0.30$. Notice development of the asymmetry of the beam.

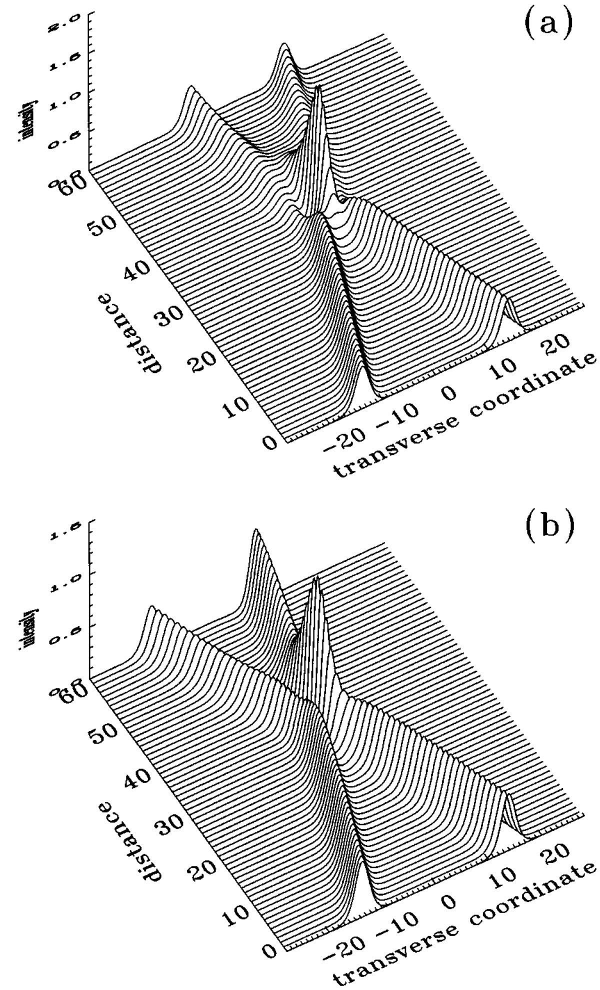

FIG. 5. Collision of two identical, initially in-phase photorefractive solitons (a) without and (b) with small contribution of the diffusion term; in this latter case $\gamma = 0.02$.

dependence (as a transverse or temporal derivative of the beam intensity) for low saturation regime when nonlinearity follows the Kerr law. The energy exchange during soliton collision represents quite generic behavior of solitons governed by the nonintegrable nonlinear Schrödinger equation. This property can be useful in modeling optical components such as asymmetric *X* junctions with desirable transmission characteristics.

In conclusion, we studied propagation of the photorefractive screening solitons in the presence of the diffusion effects. We found the intensity profile of the solitons and showed that they propagate as stationary solutions along parabolic trajectories. We have also shown that the nonlocal contribution to the refractive index change induced by the diffusion term leads to the energy exchange between the two colliding solitons. It should also be noted that, while we have considered here the specific case of screening solitons, our results are also applicable to the case of solitons which exist in photovoltaic photorefractive crystals [6].

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