Imaginary-emission method for modeling disturbances of all magnetoplasma species: Reflecting and absorbing objects in motion through a rarefied plasma at different angles to the ambient magnetic field

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(Received 31 October 1995)

Disturbances of all plasma species are simulated simultaneously by modeling charged particle emissions from imaginary and additional sources. These sources have been defined for arbitrarily shaped objects. The complex processes of interactions of the ambient plasma with the object surfaces have been included as reflections, secondary particle emissions for any particle hitting an object surface, and photoemission effects of additional sources. The special cases of direct reflection and absorption of any particle impinging on the object surface are considered in detail. The collisionless plasma is dealt with. As applications of the method, simple analytical expressions have been obtained for the far disturbances of the ion concentration due to the reflecting plate motion at an arbitrary angle to the ambient magnetic field. Different directions of the motion with respect to the plate normal are considered. [S1063-651X(96)11807-9]

PACS number(s): 52.35.−g, 47.27.Vf, 52.40.Hf, 95.40.+s

I. BASIC THEORY

This paper is concerned with the kinetic modeling of collisionless charged particle flows in the presence of the ambient electric and magnetic fields. The expansion of an ionized cloud, the flow in a jet, and the time-dependent disturbances in the rarefied plasma due to its interactions with surfaces are three examples of this kind of problem. These problems are not only of practical interest in the space sciences and in astrophysics, but also are of some fundamental interest in their own right, as they enable one to concentrate on the effect of the ambient electric and magnetic fields.

The basic unknown in a kinetic description of a plasma is the charged particle velocity distribution $F_\alpha(\vec{r},\vec{v},t)$ (of plasma species α). This function is governed by the Boltzmann equation

$$\frac{\partial F_\alpha}{\partial t}+\vec{v}\frac{\partial F_\alpha}{\partial \vec{r}}+\frac{q_\alpha}{m_\alpha}(\vec{v}\times\vec{B}+\vec{E})\frac{\partial F_\alpha}{\partial \vec{v}} = Q_\alpha(\vec{r},\vec{v},t)+\sum_\beta C_{\alpha\beta}(F_\alpha,F_\beta), \qquad (1)$$

which is valid for each of the plasma components indicated by the α index, where q_α is the electric charge of the α particles, m_α is the α-particle mass, $\vec{B}$ is the magnetic field strength, and $\vec{E}$ is the electric field strength. The last term in Eq. (1) refers to particle collisions, where $Q_\alpha(\vec{r},\vec{v},t)$ is the intensity of the α-particle sources. In general, $\vec{E}$, $\vec{B}$, and $F_\alpha(\vec{r},\vec{v},t)$ are related by Maxwell's equations with appropriate boundary conditions. Due to the complexity of all these correlated equations, it is nearly impossible to obtain their solutions in the general case. So, different simplifications have been made in the definition of existing theoretical models. Rokhlenko [1] investigated spatially homogeneous stationary solutions of the Boltzmann equation describing the electron component of a gas plasma in a homogeneous electric field. The ion distribution function in a weakly collisional sheath is obtained analytically for different electric field configurations by Hamaguchi *et al.* [2]. In both these papers the effects of an ambient magnetic field are disregarded. However, there are many interesting cases where the magnetic field effects are dominant (see, for instance, Greaves *et al.* [3]). In the present work we are specifically concerned with the time-dependent processes in a low-density plasma with emphasis on the ambient magnetic field effects. This paper extends the previous results of Ponomarjov and Gunko [4], where the special cases of emission and charged particle cloud expansion have been considered in the ambient electric and magnetic fields. In the paper of Ponomarjov [5], the imaginary-emission method has been proposed for the simulation of plasma disturbances due to an object motion. Below, this method is generalized and its applications are discussed.

Any relative motion between a solid surface and its ambient plasma results in a complex interaction that modifies both the electrodynamic characteristics of the surface and the flow field of the plasma. A set of processes that needs to be considered in a complete description of the plasma-surface interactions includes the effects of differential charging, the ambient magnetic field, ion shock wave disturbances ahead of the surface, charged particle emissions from the surface (and photoemission), and neutral particle emission and subsequent ionization.

Due to the complexity of all these correlated effects, its theoretical models must involve considerable simplifications. Three basic simplifications have been made in the definition

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of the existing theoretical models for bodies moving through a thin plasma (see also Al'pert *et al.* [6], Gurevich *et al.* [7], Senbetu and Henley [8], and Samir *et al.* [9]):

(1) In the case of the very small size of the Debye length compared to the body radius, the quasineutral condition of equal local density for ions and electrons in the plasma is assumed to be valid.

(2) If the object velocity is much greater than the average thermal velocity of the ions, a common practice is to neglect ion thermal motion and to replace the unknown ion distribution function in front of the object by a beam moving in a straight line with the constant object velocity.

(3) It has been suggested that plasma expansion processes may have a direct relation to the problem of the flow of a rarefied plasma past a rapidly moving object.

When considering magnetoplasma disturbances due to object motion, in general four flow regions should be separated: the far and near regions on the upstream side of the body, and the near and far regions downstream of the body. In the near regions, which includes the boundary sheath, the effects of self-consistent fields are more important than the ambient magnetic field effects. In the far regions (i.e., at distances from the surface which are of order or more than $2\pi V/\omega$, where V is a plasma drifting velocity and ω the gyrofrequency) the ambient magnetic field effects become important. Usually the near regions have been considered in the self-consistent electric field. Magnetic field effects are disregarded. It is impossible, however, to overlook these effects for the far regions. Due to this circumstance and others, a general method for simulating all the regions simultaneously does not exist. The present work is devoted to the analytic description of the ambient magnetic field effects on the dynamics of disturbances in the far regions. Some results in this direction have been obtained by Al'pert *et al.* [6], where the far disturbances have been calculated in a rarefied plasma due to the rapid motion of the circular disk and right-angled plate whose velocities were parallel and perpendicular to the ambient magnetic field (respectively). The velocities were parallel to the normals of the plate and disk. The case of absorption of any particle hitting the object surface is considered. Three-dimensional surfaces of constant disturbances are plotted using computational methods. In addition, an effect of induced electric field is estimated in the far wake. This estimation supports the assumption of a vanishingly small value of this effect compared to that of the ambient magnetic field on the motion of ions in the far wake due to their shielding by faster electrons. In addition, the effect of an induced field shielding by faster electrons of the ambient plasma has been described by Chrien *et al.* [10].

So in the present work, we are specifically concerned with the low-density charged-particle fluxes, for which a kinetic, rather than fluid description is appropriate. We consider charged-particle fluxes whose currents are so small that the flux-generated magnetic field is very small compared to the ambient field. Further, we also assume that effects of any polarized electric fields present in the far regions are very small compared to the ambient field effects on the charged-particle motion. It should be noted that these conditions are also satisfied if $V/\omega_{pi} \gg \rho_i$, where V is a plasma drifting velocity, ω_{pi} the ion plasma frequency, and ρ_i the ion gyroradius. The condition outlined above is known as the condition of a tenuous plasma [10,11].

II. CHARGED-PARTICLE DISTRIBUTION DUE TO EMISSION IN ELECTRIC AND MAGNETIC FIELDS

We will use a Cartesian coordinate system $oxyz$ with the x axis directed parallel to the homogeneous magnetic field $\vec{H}$, where $\vec{r}=(x,y,z)$ and $\vec{r}_0=(x_0,y_0,z_0)$ are radius vectors of certain original points, $\vec{v}=(v_x,v_y,v_z)$ and $\vec{v}_0=(v_{x_0},v_{y_0},v_{z_0})$ are velocities of particles in certain original points, and $\vec{E}=(E_x,E_y,E_z)$. In the case that electric and magnetic fields are homogeneous and time-independent, $C_{\alpha\beta}\equiv 0$, the following analytical solution of Eq. (1) has been obtained by Ponomarjov and Gunko [4]:

$$F_\alpha=\int_{t_0}^{t} Q_\alpha(\vec{r}_s,\vec{v}_s,t-s)ds+F_{\alpha 0}(\vec{r},\vec{v}), \tag{2}$$

where $F_{\alpha 0}(\vec{r},\vec{v})=F_\alpha(\vec{r},\vec{v},t)|_{t=t_0}$ is an initial α-particle distribution and

$$x_s=x-v_{x_s}\tau-\frac{e}{m}E_x\frac{\tau^2}{2},$$

$$y_s=y-\frac{1}{\omega}[v_{z_s}(1-\cos\varphi)+v_{y_s}\sin\varphi]+\frac{e}{m\omega^2}[E_z(\sin\varphi-\varphi)-E_y(1-\cos\varphi)],$$

$$z_s=z-\frac{1}{\omega}[v_{z_s}\sin\varphi+v_{y_s}(\cos\varphi-1)]+\frac{e}{m\omega^2}[E_y(\varphi-\sin\varphi)-E_z(1-\cos\varphi)],$$

$$v_{x_s}=v_x-\frac{e}{m}E_x\tau,$$

$$v_{y_s}=\left(v_y-\frac{e}{\omega m}E_z\right)\cos\varphi-\left(v_z+\frac{e}{\omega m}E_y\right)\sin\varphi+\frac{e}{\omega m}E_z,$$

$$v_{z_s}=\left(v_y-\frac{e}{\omega m}E_z\right)\sin\varphi+\left(v_z+\frac{e}{\omega m}E_y\right)\cos\varphi-\frac{e}{\omega m}E_y,$$

$$\omega=\frac{He}{mc},\quad \tau=t-s,\quad \varphi=\omega\tau.$$

In the paper by Ponomarjov [5], it was proposed to use this solution for the simulation of disturbances in the ambient magnetoplasma due to its interactions with surfaces. Namely, the method of imaginary emission has been proposed. Imaginary sources have been defined previously for the simulation of one plasma component only. In the present paper imaginary and additional sources are defined for modeling disturbances of all plasma species simultaneously. Different applications of the imaginary emission method are considered.

III. GENERAL PROBLEM OF MAGNETIZED PLASMA FLOW PAST AN OBJECT: IMAGINARY-EMISSION METHOD

The collisionless ionospheric and space plasma is described in phase space by the Vlasov equation,

$$\frac{\partial F_i}{\partial t}+\vec{v}\frac{\partial F_i}{\partial \vec{r}}+\frac{q_i}{m_i}(\vec{v}\times\vec{B}+\vec{E})\frac{\partial F_i}{\partial \vec{v}}=0\ , \tag{3}$$

which is valid for each of the plasma components indicated by the i index, $F_i(\vec{r},\vec{v},t)$ is the distribution function of plasma species i, q_i is the electric charge of the i-particles, m_i is the i-particle mass, $\vec{B}$ is the magnetic field strength, and $\vec{E}$ is the electric field strength. In general, $\vec{E}$, $\vec{B}$, and $F_i(\vec{r},\vec{v},t)$ are related by Maxwell's equations with the corresponding boundary conditions (see, for instance, Feynman *et al.* [12]).

At distances far from the object the plasma will be considered undisturbed and in a state of thermodynamic equilibrium, so that at great distances from the object surface

$$F_i|_{|\vec{r}|\to+\infty}\to N\left(\frac{\beta_i}{\pi}\right)^{3/2}\exp[-\beta_i(\vec{v})^2], \tag{4}$$

where $\vec{v}$ is the velocity of a particle with reference to the coordinate system at rest. It should be noted that the symbol $|\vec{r}|\to+\infty$ denotes the distances from the object surface those are greater than the object sizes and these distances are small compared to the mean free paths of the corresponding particles in the ambient plasma. The boundary condition at the object surface depends on complex interactions of particles with surface materials. In fact, these interactions include a number of complex effects such as reflection, secondary particle emission, photoemission, and absorption. So that, in general the boundary condition at the object surface S is as follows:

$$(\vec{n}\cdot\vec{u})F_i(\vec{r},\vec{u},t)|_{\vec{r}\in S,(\vec{n}\cdot\vec{u})>0} = -\int_{(\vec{n}\cdot\vec{u_1})<0}\sum_k w_k^i(\vec{u},\vec{u_1},\vec{r},t)(\vec{n}\cdot\vec{u_1})F_k(\vec{r},\vec{u_1},t)d\vec{u_1}\ , \tag{5}$$

where $\vec{u}$, $\vec{u_1}$ are the velocities of particles with reference to the coordinate system in which the moving object is at rest, $\vec{n}$ is the outside normal to the object surface S at the point $\vec{r}$, $w_k^i(\vec{u},\vec{u_1},\vec{r},t)$ is the probability that the k particle, impinging on the object surface S at the point $\vec{r}$ with the velocity $\vec{u_1}$, will produce the i particle emitting from the S at the $\vec{r}$ with the velocity $\vec{u}$ at time t.

In the case of absorption of any particle impinging on the object surface S, $w_k^i\equiv 0$ and we have

$$F_i(\vec{r},\vec{u})|_{\vec{r}\in S,(\vec{n}\cdot\vec{u})>0}=0. \tag{6}$$

In the case of the direct reflection of any particle impinging on the object surface S, $w_k^i(\vec{u},\vec{u_1},\vec{r})=\delta(\vec{u_1}-\vec{u}+2(\vec{u}\cdot\vec{n})\vec{n})$ if $i=k$ ($w_k^i=0$ if $i\neq k$) and we have

$$F_i(\vec{r},\vec{u},t)|_{\vec{r}\in S,(\vec{n}\cdot\vec{u})>0}=F_i(\vec{r},\vec{u}-2(\vec{u}\cdot\vec{n})\vec{n},t). \tag{7}$$

Due to the complexity of these correlated equations, it is nearly impossible to obtain their solutions in analytical form, so that, usually, different simplifications have been proposed. It has been assumed that a plasma is completely ionized and composed of electrons and singly ionized positive ions of one species. Furthermore, it has been suggested that the flux-generated magnetic field is very small compared to the ambient magnetic field. The particle flow past an object has been considered time-independent, and electrostatic models have been proposed.

In some cases an introduction of imaginary sources allows a simple analytic description of plasma disturbances taking into account effects of electric and magnetic fields. Besides, this method can be very useful in more complex computer simulations of plasma disturbances due to reflections, secondary particle emissions for any particle hitting an object surface, photoemission effects, and the effects of absorption.

The imaginary-emission method is based on an introduction of the imaginary sources, so that

$$f_i^{im}+f_i^{re}=f_i\ , \tag{8}$$

where f_i^{im} is the i-particle distribution due to imaginary emission or, perhaps, a perturbation of the ambient state caused by object surface interactions with the ambient plasma, f_i is the i-particle distribution in the ambient plasma (undisturbed distribution), and f_i^{re} is the i-particle disturbed distribution. By using this method, the problem of finding plasma disturbances can be reduced to a problem of emission from imaginary sources or to an equivalent problem of cloud expansion. So that, for $f_i^{im}(\vec{r},\vec{v},t)$, we have the collisionless Boltzmann equation

$$\frac{\partial f_i^{im}}{\partial t}+\vec{v}\frac{\partial f_i^{im}}{\partial \vec{r}}+\left(\frac{q_i}{m_i c}[\vec{v}\times\vec{H}]+\frac{q_i}{m_i}\vec{E}\right)\frac{\partial f_i^{im}}{\partial \vec{v}}=Q_i^{im}, \tag{9}$$

where q_i is the electric charge of the i particles, m_i is the i-particle mass, $\vec{H}$ is the magnetic field strength, $\vec{E}$ is the electric field strength, and $Q_i^{im}(\vec{r},\vec{v},t)$ is the intensity of imaginary sources.

The central point of the method of imaginary emission is to find $Q_i^{im}(\vec{r},\vec{v},t)$. In general, the intensity Q_i^{im} must be considered in the form which allows the boundary conditions to be satisfied both at the object surfaces and at great distances from the surfaces, so that the boundary conditions define $Q_i^{im}(\vec{r},\vec{v},t)$ or the particle distributions for the corresponding imaginary clouds. It should be noted that the imaginary-emission method allows an introduction of transient imaginary sources that can be located both near the object surface and at certain distance from the surface. These advantages of the imaginary-emission method make it possible to simulate plasma disturbances due to interactions of arbitrary shape objects with the ambient plasma, and the object (or a set of objects) can be in motion with time-dependent velocities or at rest.

Let us consider a very thin plate with maximal sizes A, B (respectively) and $A\sim B$. Let $\vec{n}$ be a normal to a plane of the plate and dS be an area of the plate. If $F_i(\vec{r},\vec{v},t)$ is a perturbation of the distribution function of the i particles caused by the plate dS, then

$$\lim_{dS\to 0} \frac{F_i^{im}}{dS} = f_i^{im} \tag{10}$$

is referred to as a perturbation of the distribution of the i particles caused by the surface element (the unit of area of the small plate) dS with the normal $\vec{n}$. Let us approximate the object surface by a number of surface elements dS with the corresponding normals $\vec{n}(dS)$. The problem is thus reduced to finding a wake from any surface element.

IV. WAKE OF SURFACE ELEMENT: IMAGINARY SOURCES FOR ARBITRARILY SHAPED OBJECTS

According to the imaginary-emission method, first let us define the imaginary sources for the small plate dS with the normal $\vec{n}$. According to the boundary conditions (4) and (6), in the case of absorption of any particle impinging on the surface element dS ($w_k^i \equiv 0$) we have to consider the point-like imaginary source Q^{im} which is placed at the center of the surface element dS with the radius vector $\vec{r_S}$,

$$Q_i^{im} = Q_i^{im^+} + Q_i^{im^-}, \tag{11}$$

where

$$Q_i^{im^+}(\vec{r},\vec{u},t) = \delta(\vec{r}-\vec{r_S})\times \begin{cases} 0, & (\vec{n}\cdot\vec{u})>0 \\ -(\vec{n}\cdot\vec{u})F_i(\vec{r},\vec{u},t), & (\vec{n}\cdot\vec{u})<0, \end{cases}$$

$$Q_i^{im^-}(\vec{r},\vec{u},t) = \delta(\vec{r}-\vec{r_S})\times \begin{cases} 0, & (\vec{n}\cdot\vec{u})<0 \\ (\vec{n}\cdot\vec{u})F_i(\vec{r},\vec{u},t), & (\vec{n}\cdot\vec{u})>0. \end{cases}$$

More complex processes such as reflections, secondary particle emissions for any particle hitting an object surface, and photoemission effects can be included using the additional source Q_i^w, which is placed at the center of the surface element dS with the radius vector $\vec{r_S}$,

$$Q_i^w = Q_i^{w^+} + Q_i^{w^-}, \tag{12}$$

where

$$Q_i^{w^+}(\vec{r},\vec{u},t) = \delta(\vec{r}-\vec{r_S})\times \begin{cases} \int_{(\vec{n}\cdot\vec{u_1})<0} \sum_k w_k^i(\vec{u},\vec{u_1},\vec{r},t)(\vec{n}\cdot\vec{u_1})F_k(\vec{r},\vec{u_1},t)d\vec{u_1}, & (\vec{n}\cdot\vec{u})>0 \\ 0, & (\vec{n}\cdot\vec{u})<0, \end{cases}$$

$$Q_i^{w^-}(\vec{r},\vec{u},t) = \delta(\vec{r}-\vec{r_S})\times \begin{cases} -\int_{(\vec{n}\cdot\vec{u_1})>0} \sum_k w_k^i(\vec{u},\vec{u_1},\vec{r},t)(\vec{n}\cdot\vec{u_1})F_k(\vec{r},\vec{u_1},t)d\vec{u_1}, & (\vec{n}\cdot\vec{u})<0 \\ 0, & (\vec{n}\cdot\vec{u})>0. \end{cases}$$

$w_k^i(\vec{u},\vec{u_1},\vec{r},t)$ is the probability that the k particle, impinging on the object surface S at the point $\vec{r}$ with the velocity $\vec{u_1}$, will produce the i particle emitting from the S at the $\vec{r}$ with the velocity $\vec{u}$ at time t.

In the case of the direct reflection of any particle impinging on the surface element dS, $w_k^i(\vec{u},\vec{u_1},\vec{r}) = \delta(\vec{u_1}-\vec{u}+2(\vec{u}\cdot\vec{n})\vec{n})$ if $i=k$ ($w_k^i=0$ if $i\neq k$) and we obtain

$$Q_i^{im^+} + Q_i^{w^+} = \delta(\vec{r}-\vec{r_S})\times \begin{cases} -(\vec{n}\cdot\vec{u})F_i(\vec{r},\vec{u}-2(\vec{u}\cdot\vec{n})\vec{n},t), & (\vec{n}\cdot\vec{u})>0 \\ -(\vec{n}\cdot\vec{u})F_i(\vec{r},\vec{u},t), & (\vec{n}\cdot\vec{u})<0, \end{cases} \tag{13}$$

$$Q_i^{im^-} + Q_i^{w^-} = \delta(\vec{r}-\vec{r_S})\times \begin{cases} (\vec{n}\cdot\vec{u})F_i(\vec{r},\vec{u}-2(\vec{u}\cdot\vec{n})\vec{n},t), & (\vec{n}\cdot\vec{u})<0 \\ (\vec{n}\cdot\vec{u})F_i(\vec{r},\vec{u},t), & (\vec{n}\cdot\vec{u})>0, \end{cases} \tag{14}$$

so that, in fact, we have two kinds of sources $Q_i^{im^+}$, $Q_i^{w^+}$ and $Q_i^{im^-}$, $Q_i^{w^-}$ for the surface element dS. This is because we represent now the surface element as a part of the ''very thin'' plate dS, and this ''very thin'' plate is bounded by two surfaces both interacted with the ambient plasma [for both $(\vec{n}\cdot\vec{u})<0$, and $(\vec{n}\cdot\vec{u})>0$ where $\vec{n}$ is a normal to the plate dS, $\vec{u}$ is the particle velocity near the plate dS with reference to the coordinate system in which the moving plate is at rest]. If we will represent the surface element dS as a part of the surface S of a ''more thick'' object (as opposed to the ''very thin'' plate), we must believe $Q_i^{im^-}\equiv 0$ and $Q_i^{w^-}\equiv 0$ because any surface element dS interacts with the ambient plasma if $(\vec{n}\cdot\vec{u})<0$ only, where $\vec{n}$ is the outside normal to the object surface S at the point $\vec{r_S}$ that corresponds to the center of the surface element dS, $\vec{u}$ is the particle velocity near the surface element dS with reference to the coordinate system in which the moving object is at rest, so that, for the objects with the boundary surface S, we have

$$Q_i^{imS} = \oint_S Q_i^{im} dS \ , \tag{15}$$

where $Q_i^{im} = Q_i^{im^+}$.

$$Q_i^{wS} = \oint_S Q_i^{w} dS \ , \tag{16}$$

where $Q_i^{w} = Q_i^{w^+}$.

V. FAR DISTURBANCES OF THE CHARGED-PARTICLE CONCENTRATION DUE TO MOTION OF REFLECTING PLATE

Let the plate velocity be $\vec{V}$. We choose a Cartesian coordinate system $OXYZ$ with the $\overrightarrow{OX}$ axis directed parallel to the homogeneous magnetic field $\vec{H}$, where $\vec{V} = (V_x, 0, V_z)$. Let $\vec{r_S}$ be the radius vector of the center of the small plate dS with the normal $\vec{n}$, $\vec{r_S} = \vec{V}t$, i.e., at times $(0,\tau)$ the small plate dS moves from the point (0,0,0) to the point $(V_x\tau, 0, V_z\tau)$. We will consider the concentration of ions under assumption of small their mean thermal velocity compared to the plate velocity $\vec{V}$ (i.e., $\beta V^2 \gg 1$). Then we have for the disturbance of the charged-particle concentration by the unit of area of the reflecting plate dS $(\vec{V} \| \vec{n})$,

$$n^{im}(\vec{r},\tau) = \left(\frac{\beta}{\pi}\right)^{3/2} \int_{-\infty}^{\tau} \frac{N_p^1 \exp[-L^2(\theta)] - N_p^2 \exp[-M^2(\theta)]}{4\theta \sin^2(\omega\theta/2)/\omega^2} d\theta, \tag{17}$$

where

$$N_p^1 = \begin{cases} N^+ & \text{if } X\breve{V}_x + Z\breve{V}_z < \breve{V}^2\omega\tau \\ N^- & \text{if } X\breve{V}_x + Z\breve{V}_z > \breve{V}^2\omega\tau, \end{cases}$$

$$N_p^2 = \begin{cases} N^+ & \text{if } X\breve{V}_x + Z\breve{V}_z > \breve{V}^2\omega\tau \\ N^- & \text{if } X\breve{V}_x + Z\breve{V}_z < \breve{V}^2\omega\tau. \end{cases}$$

Ion fluxes onto the unit of area of the small plate dS with the normal $\vec{n}$ are calculated as follows:

$$N^+ = -\int_{(\vec{n}\cdot\vec{u})<0} (\vec{n}\cdot\vec{u}) F(\vec{r},\vec{u},t) d\vec{u}, \tag{18}$$

$$N^- = \int_{(\vec{n}\cdot\vec{u})>0} (\vec{n}\cdot\vec{u}) F(\vec{r},\vec{u},t) d\vec{u}. \tag{19}$$

Using the assumption that $\beta V^2 \gg 1$ we obtain

$$N^+|_{\beta V^2 \gg 1} \approx NV, \tag{20}$$

$$N^-|_{\beta V^2 \gg 1} \approx \frac{N\exp(-\beta V^2)}{2\sqrt{\beta\pi}} \ , \tag{21}$$

where $N = N_0$ is the charged-particle undisturbed concentration, i.e., the concentration existing at distances far from the object surface.

$$L^2(\theta) = \left(\frac{\hat{X}}{\omega\theta}\right)^2 + \frac{Y^2 + \hat{Z}^2}{4\sin^2(\omega\theta/2)} \ ,$$

$$\begin{aligned} M^2(\theta) = {} & \frac{Y^2 + \hat{Z}^2}{4\sin^2(\omega\theta/2)} - \left(\breve{V}_x \frac{\hat{X}}{\omega\theta} + \frac{\breve{V}_z}{2}(Y + \phi)\right. \\ & \times \left(\frac{2\breve{V}_z}{\breve{V}^2}\phi + \frac{4\breve{V}_x}{\breve{V}^2}\frac{\hat{X}}{\omega\theta} + 8\right) \\ & + \frac{\breve{V}_z^2}{\breve{V}^2}\phi^2 + 2\breve{V}_z\phi\left(\breve{V}_x\frac{\hat{X}}{\omega\theta} + 1\right) + \left(\frac{\hat{X}}{\omega\theta}\right)^2 \\ & \times \left(1 + 4\frac{\breve{V}_x^2}{\breve{V}^2}\right) + 4\left(\breve{V}^2 + \breve{V}_x\frac{\hat{X}}{\omega\theta}\right), \end{aligned}$$

$$\phi = (Y + \hat{Z})\cot(\omega\theta/2),$$

$$\hat{X} = X - \breve{V}_x\omega(\tau - \theta),$$

$$\hat{Z} = Z - \breve{V}_z\omega(\tau - \theta),$$

$$X^2 = \beta\omega^2 x^2, \quad Y^2 = \beta\omega^2 y^2, \quad Z^2 = \beta\omega^2 z^2, \quad \breve{V}^2 = \beta V^2.$$

It should be noted that if the reflecting plate moves perpendicular to its normal $\vec{n}$ $(\vec{V} \perp \vec{n})$, then, according to our analytic results, there are no disturbances in the ambient Maxwellian plasma. If $N_p^2 \equiv 0$ [in Eq. (17)], then we obtain the disturbances of the concentration for the absorption of any particle impinging on the object surface (when $\vec{V} \| \vec{n}$). For the absorbing plate in motion perpendicular to its normal $\vec{n}$, the disturbances of the concentration are described by Eq. (17), where $N_p^1 \equiv N/2\sqrt{\beta\pi}$ and $N_p^2 \equiv 0$.

A. Small reflecting plate in motion parallel to $\vec{H}$

We will use a Cartesian coordinate system $OXYZ$ with the $\overrightarrow{OX}$ axis directed parallel to the homogeneous magnetic field $\vec{H}$. The velocity of the surface element dS with the normal $\vec{n}$ is chosen in the form $\vec{V} = (V, 0, 0)$. Let $\vec{r_S}$ be the radius vector of the center of the small plate dS, and $\vec{r_S} = \vec{V}t$, i.e., the small plate dS moves from the point (0,0,0) to the point $(V\tau, 0, 0)$. Then, for the disturbance of the charged-particle concentration by the unit of area of the small plate dS $(\vec{V} \| \vec{n})$, we have

$$n^{im}(\vec{r},\tau)=\left(\frac{\beta}{\pi}\right)^{3/2}\int_{-\infty}^{\tau}\frac{1}{4\theta\sin^2(\omega\theta/2)/\omega^2}$$
$$\times\left\{N_p^1\exp\left[-\left(\frac{X-\check{V}\omega(\tau-\theta)}{\omega\theta}\right)^2\right]\right.$$
$$\left.-N_p^2\exp\left[-\left(\frac{X-\check{V}\omega(\tau+\theta)}{\omega\theta}\right)^2\right]\right\}$$
$$\times\exp\left(-\frac{R^2}{4\sin^2(\omega\theta/2)}\right)d\theta, \qquad (22)$$

where

$$N_p^1=\begin{cases}N^+ & \text{if } X<\check{V}\omega\tau\\ N^- & \text{if } X>\check{V}\omega\tau,\end{cases}$$

$$N_p^2=\begin{cases}N^+, & \text{if } X>\check{V}\omega\tau\\ N^-, & \text{if } X<\check{V}\omega\tau,\end{cases}$$

$X^2=\beta\omega^2x^2$, $R^2=\beta\omega^2(y^2+z^2)$, $\check{V}^2=\beta V^2$. For brevity, here we will drop the superscript in the symbol $\check{V}$. In general, the integral in Eq. (22) cannot be calculated in terms of elementary functions. Assuming that $V^2\gg 1$, the asymptotic expression of Eq. (22) may be found by using the Laplace method

$$n^{im}(\alpha,V)\approx(N_p^1-N_p^2)\left(\frac{\beta}{\pi}\right)^{3/2}\pi^{1/2}\frac{\exp\left(-\frac{R^2}{4\sin^2(\alpha/2)}\right)}{4V\sin^2(\alpha/2)/\omega^2}, \qquad (23)$$

where $\alpha=\omega\tau-X/V$. The surfaces of constant disturbances are

$$\frac{R^2}{4\sin^2(\alpha/2)}=\ln\left(\frac{|N_p^1-N_p^2|\omega^2}{4CV\sin^2(\alpha/2)}\right). \qquad (24)$$

The constant C characterizes a current value of the concentration disturbance. According to Eqs. (23) and (24), the surfaces of constant disturbances of the concentration are formed from the circles of different radii R whose centers lie on the X axis (see Fig. 1). If $\omega\tau-X/V=2\pi n$ (n is integer) these circles degenerate to points which are defined by $(R=0,\ X/V=\omega\tau-2\pi n)$, so that we obtain the flute structure of the surfaces of constant disturbances of charged-particle concentration.

B. Small reflecting plate in motion oblique to $\vec{H}$

Let the plate velocity be $\vec{V}$. We choose a Cartesian coordinate system $OXYZ$ with the $\overrightarrow{OX}$ axis directed parallel to the homogeneous magnetic field $\vec{H}$, where $\vec{V}=(V_x,0,V_z)$. Let $\vec{r_S}$ be the radius vector of the center of the small plate dS with the normal $\vec{n}=(1,0,0)$ and $\vec{r_S}=\vec{V}t$, i.e., the small plate dS moves from the point (0,0,0) to the point $(V_x\tau,0,V_z\tau)$. Then, for the disturbance of the charged-particle concentration by the unit of area of the small plate dS, we have

$$n^{im}(\vec{r},\tau)=\left(\frac{\beta}{\pi}\right)^{3/2}\int_{-\infty}^{\tau}\frac{\omega^2}{4\theta\sin^2(\omega\theta/2)}$$
$$\times\left\{N_p^1\exp\left[-\left(\frac{X-\check{V}_x\omega(\tau-\theta)}{\omega\theta}\right)^2\right]\right.$$
$$\left.-N_p^2\exp\left[-\left(\frac{X-\check{V}_x\omega(\tau+\theta)}{\omega\theta}\right)^2\right]\right\}$$
$$\times\exp\left(-\frac{Y^2+[Z-\check{V}_z\omega(\tau-\theta)]^2}{4\sin^2(\omega\theta/2)}\right)d\theta, \qquad (25)$$

where

$$N_p^1=\begin{cases}N^+ & \text{if } X<\check{V}_x\omega\tau\\ N^- & \text{if } X>\check{V}_x\omega\tau,\end{cases}$$

$$N_p^2=\begin{cases}N^+ & \text{if } X>\check{V}_x\omega\tau\\ N^- & \text{if } X<\check{V}_x\omega\tau,\end{cases}$$

$X^2=\beta\omega^2x^2$, $Y^2=\beta\omega^2y^2$, $Z^2=\beta\omega^2z^2$, $\check{V}^2=\beta V^2$. For brevity, here we will drop the superscript in the symbol $\check{V}$. Charged particle fluxes onto the unit of area of the small plate dS N^+ and N^- are described by Eqs. (18) and (19). For a rapidly moving plate ($\beta V_x^2\gg 1$) we obtain

$$N^+|_{\beta V_x^2\gg 1}\approx NV_x\ , \qquad (26)$$

$$N^-|_{\beta V_x^2\gg 1}\approx\frac{N\exp(-\beta V_x^2)}{2\sqrt{\beta\pi}}\ . \qquad (27)$$

In general, the integral in Eq. (25) cannot be calculated in terms of elementary functions. In the case that the plate rapidly moves approximately parallel to $\vec{H}$, i.e., $V_x\gg V_z$ and

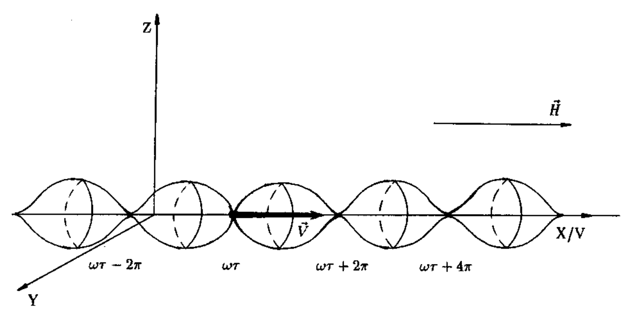

FIG. 1. The flute structure of isodensity surfaces in the case where a reflecting small plate moves parallel to $\vec{H}$. The point $(X/V=\omega\tau,\ Y=0,\ Z=0)$ represents the position of the plate with normal $\vec{n}$. The plate moves with velocity $\vec{V}=(V,0,0)$, which is parallel to $\vec{n}$ [cf. Eqs. (23) and (24)]. If $X>V\omega\tau$, then $N_p^2\gg N_p^1$ and we obtain the increase of the disturbed concentration in comparison with the ambient concentration $N=N_0$. In other cases ($X<V\omega\tau$, $N_p^1\gg N_p^2$) the disturbance results in the decrease of the concentration. The surfaces correspond to the density $0.95N_0$ (if $X<V\omega\tau$) or $1.05N_0$ (if $X>V\omega\tau$). If $X<V\omega\tau$, then this figure shows the isodensity surface of $0.95N_0$ for the rapidly moving absorbing object, too.

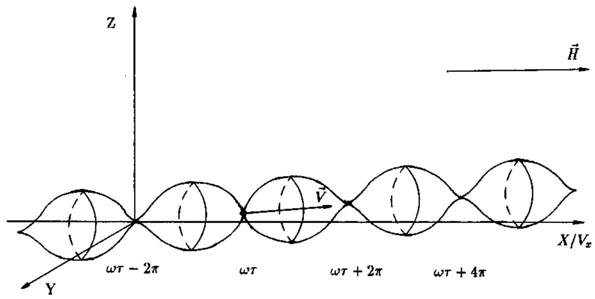

FIG. 2. The flute structure of isodensity surfaces in the case that a reflecting small plate moves aproximately parallel to $\vec{H}$. The point $(X/V_x=\omega\tau,\ Y=0,\ Z=V_z\omega\tau)$ represents the position of the plate with the normal $\vec{n}=(1,0,0)$. The plate moves with velocity $\vec{V}=(V_x,0,V_z)$, $V_x \gg V_z$ [cf. Eq. (28)]. If $X>V_x\omega\tau$, then $N_p^2 \gg N_p^1$ and we obtain the increase of the disturbed concentration in comparison with the ambient concentration $N=N_0$. In other cases $(X<V_x\omega\tau,\ N_p^1 \gg N_p^2)$ the disturbance results in the decrease of the concentration. The surfaces correspond to the density $0.95N_0$ (if $X<V_x\omega\tau$) or $1.05N_0$ (if $X>V_x\omega\tau$). If $X<V_x\omega\tau$, then this figure shows the isodensity surface of $0.95N_0$ for the rapidly moving absorbing object, too.

$V_x^2 \gg 1$, the asymptotic expression of Eq. (25) may be found by using the Laplace method (see Fig. 2)

$$n^{im}(\vec{r},\tau) \approx \left(\frac{\beta}{\pi}\right)^{3/2} (N_p^1 - N_p^2)\sqrt{\pi}\omega^2 \times \frac{\exp\left(-\frac{\left(Z - X\frac{V_z}{V_x}\right)^2 + Y^2}{4\sin^2(\alpha_1/2)}\right)}{4V_x \sin^2(\alpha_1/2)}, \qquad (28)$$

where $\alpha_1=\omega\tau - X/V_x$. Assuming that $V_x^2, V_z^2 \gg 1$, we obtain

$$n^{im}(\vec{r},\tau) \approx \left(\frac{\beta}{\pi}\right)^{3/2} (N_p^1 - N_p^2)\sqrt{\pi}\omega^2 \times \left(\frac{\exp\left(-\frac{\left(Z - X\frac{V_z}{V_x}\right)^2 + Y^2}{4\sin^2(\alpha_1/2)}\right)}{4V_x \sin^2(\alpha_1/2)} + \frac{\exp\left[-\left(\frac{Z\frac{V_x}{V_z} - X}{\alpha_2}\right)^2\right] \exp\left(-\frac{Y^2}{4\sin^2(\alpha_2/2)}\right)}{2V_z|\alpha_2 \sin(\alpha_2/2)|} \right), \qquad (29)$$

where $\alpha_1=\omega\tau - X/V_x$, $\alpha_2=\omega\tau - Z/V_z$ (see Fig. 3).

VI. BASIC CONCLUSIONS

The method of imaginary emission is developed. This method allows a simple analytical description of all plasma species distributions taking into account both effects of particle interactions with surfaces and effects of electric and magnetic fields. Besides, this method can be very useful in more complex computer simulations of plasma disturbances due to reflections, secondary particle emissions for any particle hitting an object surface, and photoemission effects.

According to this method, imaginary and additional sources have been defined for arbitrarily shaped objects. The complex processes of interactions of the ambient plasma with the object surface have been included. The special cases of the direct reflection and absorption of any particle impinging on the object surface are considered in detail. The collisionless plasma is dealt with. As applications of the imaginary-emission method, simple analytical expressions for the far disturbances of charged-particle concentration have been obtained for the small reflecting plate motion at an arbitrary angle to the ambient magnetic field. Different directions of the motion with respect to the plate normal are considered.

The main objective of the present work is to attract the

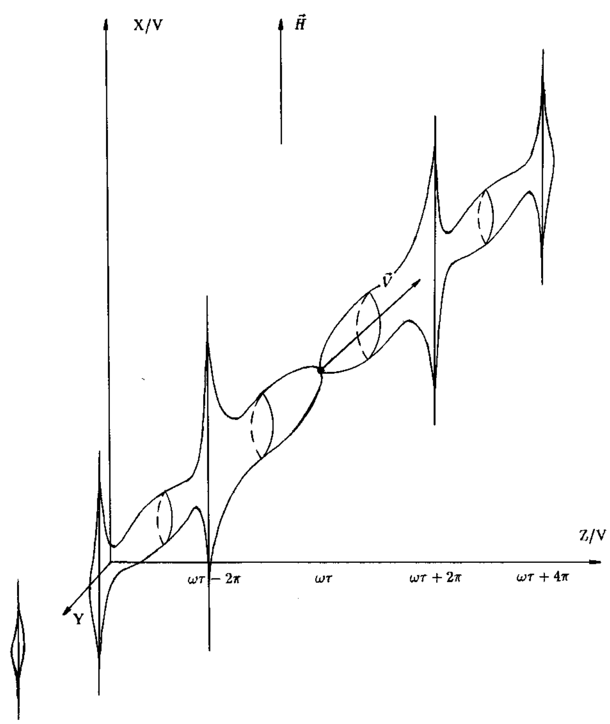

FIG. 3. Isodensity surface for a reflecting plate in motion with velocity $\vec{V}=(V,0,V)$, $V^2 \gg 1$ [cf. Eq. (29)]. The point $(X/V=\omega\tau,\ Y=0,\ Z/V=\omega\tau)$ represents the position of the small plate dS with the normal $\vec{n}=(1,0,0)$. If $X>V\omega\tau$, then $N_p^2 \gg N_p^1$ and we obtain the increase of the disturbed concentration in comparison with the ambient concentration $N=N_0$. In other cases $(X<V\omega\tau,\ N_p^1 \gg N_p^2)$ the disturbance results in the decrease of the concentration. The surface corresponds to the density $0.95N_0$ (if $X<V\omega\tau$) or $1.05N_0$ (if $X>V\omega\tau$). If $X<V\omega\tau$, then this figure shows the isodensity surface of $0.95N_0$ for the rapidly moving absorbing object, too.

attention of specialists to the method presented here and to the possibility of its further elaboration as applications to modeling interactions of bodies with ionospheric and space magnetoplasma. This advanced topic can be the subject for further research with specialists in related fields.

ACKNOWLEGMENTS

This work has been made possible by the Tomalla Foundation and is carried out under the research program of the International Center for Fundamental Physics in Moscow.

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