Synchronization of a chaotic laser pulsation with its prerecorded history

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We show that a chaotic pulsation in the passive Q-switched CO_2 laser system can be made to synchronize to its prerecorded history. A signal based only on the prerecorded history is used to modulate the saturable absorber instead of the actual difference between the prerecorded signal and the current signal. Numerical calculation shows that the synchronization also occurs even if the modulation signal is modified to be the on-off signal. [S1063-651X(96)01010-0]

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A chaotic system is known to show a variety of oscillation patterns even in a low-dimensional system, although we cannot realize a particular pattern intentionally in a real system because of its sensitive dependence on the initial conditions and external noises. This variety characteristic of chaotic behavior is advantageous in applications in the fields of communications and neural networks if we can control a chaotic system and change its behavior into any desired aperiodic or periodic behaviors intrinsic to the system. In fact, there has been much research to demonstrate the change of a chaotic behavior into a periodic behavior [1–5] and the synchronization of a chaotic system with another chaotic system [6–10]. Recently, Kittel *et al.* showed that the present state of a chaotic system can be synchronized to its prerecorded history in the electric circuit [11]. This is achieved by a small self-controlling feedback perturbation in the form of the difference between the prerecorded signal and the current signal. The perturbation transforms an unpredictable chaotic behavior into a predictable aperiodic or periodic behavior. This method was successfully applied to the chaotic response of a yttrium iron garnet film in ferromagnetic resonance [12]. In real applications, how far we can reduce the information necessary to control a chaotic system is an important question. So far few studies have focused on this subject.

A single-mode laser that contains a saturable absorber inside its cavity exhibits self-induced pulsation, known as passive Q switching (PQS). In a CO_2 laser, chaotic PQS pulsation appears in some limited region of the parameters [13]. Sugawara *et al.* showed that chaotic pulsations in identical laser systems can be synchronized by modulating the saturable absorber of one laser with the output of the other laser [9]. In the present paper, it is demonstrated that the synchronization also occurs between the present chaotic pulsation and its prerecorded history, that is, the past chaotic pulsation can be reproduced intentionally at any time. In contrast to Kittel's method, which uses the actual difference between the prerecorded signal and the current signal as a controlling signal, a signal based only on the prerecorded history is used to modulate the saturable absorber. We also try to modify the modulation signal to reveal the mechanism of synchronization. It is found that a pattern of the sequence of the dark time, the separation between the successive pulses in the modulation signal, is essential for the synchronization.

The experimental setup of the laser system is described in detail in Ref. [13]. In brief, the CO_2 laser consists of a 2.5-m-long discharge gain tube and a 35-cm-long intracavity absorption cell. The total cavity length is 3.5 m. A gas mixture of CO_2, N_2, and He (1:1:8) flows through the laser tube at a total pressure of 8.2 Torr. The laser oscillates on a single mode of TEM_{00} at the 9-μm $P(34)$ line.

The intracavity cell contains a gaseous saturable absorber (CH_3OH) and buffer gas (SF_6) of 45 and 95 mTorr, respectively. The absorption coefficient of the saturable absorber is controlled by applying dc and ac voltages to the Stark electrodes in the cell. The electrodes consist of two aluminum-coated glass plates that are placed parallel to each other. Each electrode has a dimension of 9.5 cm in length and 3.5 cm in width. The gap between the electrodes is 1.0 cm.

The experimental procedure is as follows. First, we observe a chaotic time sequence at the discharge current of 13.6 mA and dc Stark voltage of 130 V and we store the sequence in the computer in each 0.1-μs interval. The total recorded length is 2000 μs. Next, we convert the history into the modulation signal. In the present case, the absorption coefficient B_a is a function of the Stark voltage ϵ and is calculated to be [14]

$$B_a(\epsilon)=B_a(0)\sum_{M,\Delta M} A_{M,\Delta M}\exp\left(-\frac{(\nu_0+\Delta\nu(\epsilon)-\nu_L)^2}{\Delta\nu_D^2}\right), \tag{1}$$

where $A_{M,\Delta M}$ is proportional to the transition probability between the M and $M+\Delta M$ sublevels. $\Delta\nu_D$, ν_L, and ν_0 are the Doppler width, the laser frequency, and the center frequency of the absorption line [15], respectively. $\Delta\nu(\epsilon)$ represents the Stark shifts of the relevant energy levels. On the other hand, for a virtual light intensity of $\alpha I_m(t)$, the absorption is reduced to

$$B_a=B_{a0}/[1+\alpha I_m(t)/I^*], \tag{2}$$

where B_{a0}, α, $I_m(t)$, and I^* are the values of the absorption without the modulation, the coupling constant, the prerecorded history, and the saturation parameter, respectively. By equating the right-hand-sides of Eqs. (1) and (2), the Stark voltage of the modulation signal $\epsilon(t)$ is calculated for

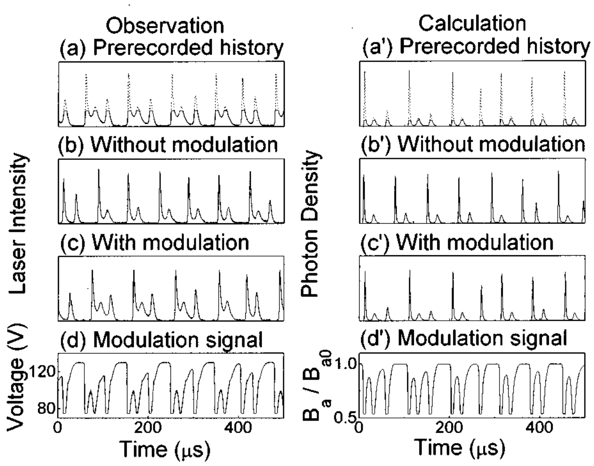

FIG. 1. (a) Prerecorded history of the chaotic pulsation observed at the discharge current of 13.6 mA before (dashed line) and after (solid line) clipping. (b) and (c) Time sequences of the present chaotic pulsation observed at the discharge current of 13.4 mA (b) without and (c) with the modulation. (d) Time sequence of the Stark modulation voltage. (a′) History of the chaotic pulsation calculated at the pumping rate $P_m = 16.70$ Hz before (dashed line) and after (solid line) clipping. (b′) and (c′) Time sequences of the present chaotic pulsations calculated at the pumping rate $P = 16.45$ Hz (b′) without and (c′) with the modulation. (d′) Time sequence of the modulated absorption.

each step of 0.1 μs, after all laser intensities in the prerecorded history larger than a certain clipping level are set equal to the level. The calculated modulation signal is stored in the wave generator. Finally, we observe the present chaotic pulsation at the discharge current of 13.4 mA. If the value of α/I^* is properly chosen, we can observe synchronization.

In Fig. 1(a) the dashed line and the solid line show a part of the prerecorded chaotic history before and after clipping, respectively. Figures 1(b) and 1(c) show the time sequences of the present chaotic pulsation without and with the modulation, respectively. To examine the correlation in the amplitude between the prerecorded history and the present pulsation, the pulse height of the present pulsation X is plotted against the corresponding height in the prerecorded history X_m [Figs. 2(a) and 2(b)]. In the absence of the modulation, the present system exhibits the independent chaotic time sequence as shown in Fig. 1(b). In the correlation plots, data points are scattered in two dimensions in an erratic manner [Fig. 2(a)], indicating that the laser is pulsating independently of the prerecorded history. In the limited range of the coupling constant, the present chaotic pulsation is synchronized to the prerecorded history [Fig. 1(c)]. The pulse heights and the time intervals in Fig. 1(c) correspond well to those in Fig. 1(a). A linear line in the correlation plots in Fig. 2(b) also shows the occurrence of synchronization. When the laser intensity of the prerecorded history increases, the Stark modulation voltage is decreased to reduce the absorption as shown in Fig. 1(d). The cross correlation analysis reveals that the present pulsation is delayed from the prerecorded history by 7 μs. At the smaller or larger coupling constant, the present chaotic pulsation is less correlated to the prerecorded history.

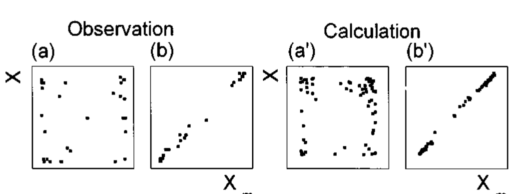

FIG. 2. Correlation plots between the pulse heights without [(a) and (a′)] and with [(b) and (b′)] the modulation.

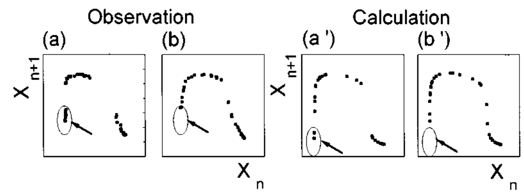

FIG. 3. Return maps of the prerecorded histories of the chaotic pulsations [(a) and (a′)] and those of the present chaotic pulsations without the modulation [(b) and (b′)].

The synchronization is realized when the discharge current of the present system (13.4 mA) is slightly smaller than the current used to store the chaotic time sequence (13.6 mA) in the first step of our experiment. A similar relation between the pumping rate of the master and slave lasers was also reported in Ref. [9]. This difference in the discharge current is clearly observed in the first return maps, where adjacent pulse heights of the time sequence X_{n+1} vs X_n are plotted [Figs. 3(a) and 3(b)]. The return map of the highly excited laser (the prerecorded history) has a longer tail than that of the less excited laser (the present chaotic pulsation without modulation) at the left-hand-side of the unimodal curve as indicated by circles and arrows. This shows that the modulation on the absorber causes entrainment of the laser dynamics towards a different strange attractor.

We carry out a rate-equation analysis to reproduce the observed dynamics of the modulated CO_2 laser system. According to the three-level–two-level model, which was successfully employed to analyze the PQS dynamics in our previous publications [9,13,16–18], the present laser system is described by the following rate equations for the population densities in the upper and lower laser levels M_1 and M_2, the difference in the population density between the upper and lower rotational-vibrational states of the saturable absorber N, and the photon density in the lasing mode I:

$$\dot{I} = B_g f_g(J) I (M_1 - M_2) l_g / L - B_a I N l_a / L - kI, \tag{3}$$

$$\dot{M}_1 = -B_g f_g(J) I (M_1 - M_2) + PM - R_1 M_1, \tag{4}$$

$$\dot{M}_2 = B_g f_g(J) I (M_1 - M_2) - R_2 M_2, \tag{5}$$

$$\dot{N} = -2 B_a I N - r(N - N^*), \tag{6}$$

where B_g and B_a are the cross section multiplied by the light velocity c for the induced emission in the gain medium and the absorption in the absorbing medium, respectively. The length of the gain tube, the absorption cell, and the laser

TABLE I. Parameters used in the numerical analysis.

$B_{a0}N^*l_a/L$	$B_{a0}/[B_g f_g(J)]$	$B_g M f_g(J) l_g/L$
1.5 MHz	900	3820 MHz
R_1	R_2	P_m
300 Hz	380 kHz	16.70 Hz
k	r	P
2.0 MHz	6.0 MHz	16.45 Hz

cavity are denoted by l_g, l_a, and L, respectively. The fraction of CO_2 molecules in the rotational level with the quantum number J is represented by $f_g(J)$, the cavity loss rate by k, and the pumping rate by P. The relaxation rate from the ith level to the ground state in the laser medium is written as R_i and r is the collisional relaxation rate of the absorbing gas. The population density of CO_2 molecules is denoted by M and the thermal equilibrium value of N by N^*. A detailed formulation of the rate equations is described in Ref. [13]. In this paper, the cross section of the saturable absorber is reduced with the Stark voltage according to Eq. (2).

The observations are successfully reproduced with the parameter values listed in Table I. These values are reasonable for our CO_2 laser system [13]. Only the pumping rate is different between the calculation of the prerecorded history and the present pulsation. Figure 1(a′) shows the history of a chaotic pulsation calculated at the pumping rate $P_m = 16.70$ Hz. The present chaotic pulsations are calculated without and with the modulation at the pumping rate $P = 16.45$ Hz [Figs. 1(b′) and 1(c′)]. The correlation plots between the pulse heights are shown in Figs. 2(a′) and 2(b′).

In the absence of the modulation, the first return map of the present chaotic pulsation is slightly different from that of the prerecorded history as shown in Figs. 3(a′) and 3(b′), which agree well with the observation [Figs. 3(a) and 3(b)]. In the limited range of the coupling constant (3.3–4.8 %), the present chaotic pulsation is synchronized to the prerecorded history. This coupling constant agrees with the value used in the experiment (3.2 %), which is estimated on the assumption that $I^*/I_{\max}$ is same in the experiment and in the calculation, where $I_{\max}$ is the maximum pulse height. Figure 1(d′) shows the time sequence of the modulated absorption, which is similar to that of the actually applied Stark modulation voltage [Fig. 1(d)]. This relation also shows that the coupling constants used in the experiment and in the calculation are nearly equal because the time sequence of the modulated absorption is sensitive to the coupling constant. A delay of 4 μs is found between the prerecorded history and the present chaotic pulsation, which agrees fairly well with the experimental result.

The phenomenon of synchronization is investigated in more detail by a numerical analysis with modified modulation signal. The variance of the correlation plot from the best-fit linear relation σ^2 is measured for 100 data points sampled from the calculated time sequences. It is given as $\sigma^2 = \Sigma_i (X_i - aX_{m,i})^2 / \Sigma_i X_i^2$ where X_i, $X_{m,i}$, and a are the ith pulse height of the present pulsation, the corresponding height in the prerecorded history, and the coefficient of the best-fit linear relation. For each value of the clipping level $I_{\rm clip}$, σ^2 is a function of the coupling constant, whose minimum value is shown by the solid line in Fig. 4. The modulation signal is further modified to be the on-off signal, where all laser intensities in the prerecorded history larger (smaller) than the clipping level are set equal to the level (zero). The dashed line in Fig. 4 shows the minimum variance in the case of the on-off signal. In both cases, the rapid decrease of the variance is observed around the clipping level $I_{\rm clip}/I^* \sim 4$, which corresponds to the largest minimum photon density between successive pulses. In the case of the clipped signal, the variance converges to the value without clipping when $I_{\rm clip}$ further increases. Synchronization occurs for sufficiently large value of $I_{\rm clip}$, which is clearly shown in the correlation plots in Fig. 4. However, in the case of the on-off signal, the variance increases with the increase of $I_{\rm clip}$ because some pulses that are smaller than $I_{\rm clip}$ are neglected. Thus, in the limited range of the clipping level, synchronization occurs for the on-off signal. The almost linear relation in the correlation plots shows the occurrence of the synchronization. It is necessary that all pulses in the modulation signal can be distinguished.

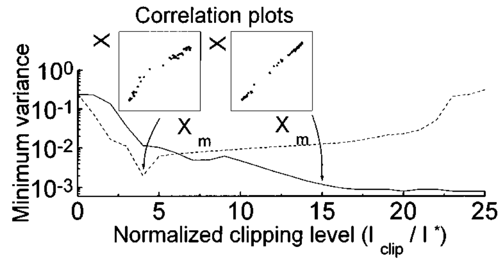

FIG. 4. Minimum of the variance as a function of the normalized clipping level when all pulses taller than the clipping level are set equal to the level (solid line) and when the modulation signal is further modified to be the on-off signal (dashed line). Correlation plots in both cases are also shown: left, on-off signal ($I_{\rm clip}/I^* = 4$); right, clipped signal ($I_{\rm clip}/I^* = 15$).

The above numerical simulation implies that, for synchronization, the information on the signal amplitude can be omitted if the phase information is preserved. In Fig. 5 we plot a correlation between the pulse height and the preceding dark time (the time interval between the preceding peak and the present peak). The definite one-dimensional correlation between the pulse height and the dark time suggests that the amplitude information is involved in the phase behavior, which leads to the successful synchronization with the clipped signal. It is interesting that the laser is working as a signal processor that reproduces lost information.

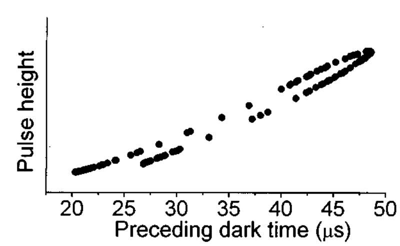

FIG. 5. Pulse height as a function of the preceding dark time obtained from the calculated PQS pulsation without modulation.

This synchronization of the present chaotic pulsation with its prerecorded history is also applied to stabilize the unstable periodic pulsation characteristic of the prerecorded history. This is achieved if we select an unstable periodic pulsation between two peaks of nearly the same height, convert the selected history into the modulation signal, and apply the modulation signal repeatedly. Figure 6 shows the observed time sequences before and after the modulation signal tailored from the prerecorded history is applied. The pulse train consisting of eight pulses, which is denoted by a horizontal bar, is regularly repeated in Fig. 6(b). Further details of this stabilization will be reported elsewhere.

In conclusion, we demonstrated that a chaotic pulsation of a CO_2 laser system can be made to synchronize to its prerecorded history. A signal based only on the prerecorded history is used to modulate the saturable absorber instead of the actual difference between the prerecorded signal and current signal. Numerical calculation shows that the synchronization also occurs even if the modulation signal is modified to be the on-off signal. In other words, for this synchronization, it is essential to reduce the intracavity absorption with the proper pattern of the sequence of the dark time between the successive pulses. We believe this technique is applicable to low-dimensional dissipative chaotic systems.

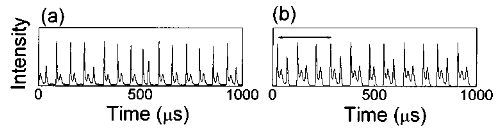

FIG. 6. Observed time sequences of PQS pulsation (a) before and (b) after the modulation based on the unstable periodic pulsation is applied.

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