Energy balance equation for electromagnetic waves in bianisotropic media

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A macroscopic treatment of the energy relations for quasimonochromatic fields in bianisotropic media is realized by taking into account moderate absorption and effects of temporal and spatial dispersion. It is shown that spatial dispersion in bianisotropic media provides additional power flow similar to such effects in anisotropic media. A special feature of bianisotropic media is that the energy transport of quasimonochromatic fields is defined not only by the constitutive parameters of a medium, but also by the structure of the field. The group velocity correlates with the energy transport velocity only for a certain configuration of the quasimonochromatic field. [S1063-651X(96)06310-6]

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I. INTRODUCTION

At present, we are witnessing a great and continuous interest in electromagnetic-wave–material interactions. Well known problems of wave interaction with isotropic and anisotropic media are enriched now by problems concerning wave interactions with chiral and bi(an)isotropic media. One of the powerful tools to investigate the electromagnetic-wave propagation in media is an analysis of energy balance equations. Such an analysis lets us understand the mechanisms of storage and absorption of the energy as well as the character of the energy flow.

A macroscopic treatment of the energy relations for quasimonochromatic field in anisotropic media was realized by taking into account moderate absorption and effects of temporal and spatial dispersion [1,2]. In chiral and bi(an)isotropic media, temporal dispersion is a subject of many investigations [3–6]; meanwhile the problem of electromagnetic-wave propagation in such media with spatial dispersion is poorly developed. Spatial dispersion is a well known phenomenon in plasma [7,8], ferromagnetics [9,10], and optical crystals [11,12]. As an initial study of this problem in bianisotropic media, we can point out Hornreich and Shtrikman's work [13], where the effects of spatial dispersion in natural magnetoelectric materials at optical frequencies were considered.

Taking spatial dispersion into account implies an assumption that electric and magnetic dipole moments are defined not only by the fields in the given point, but also by the fields in the vicinity of the point. At optical frequencies, the wavelength is essentially greater than spatial scales in a medium (for example, parameters of a lattice) and therefore, the effects of spatial dispersion in chiral and bi(an)isotropic media may not be so strong. At the same time, in artificial chiral and bi(an)isotropic materials at microwave and millimeter-wave regions, the effect of spatial dispersion may be considerable. In these media sizes of small inclusions and distances between them may be comparable with the wavelength and one has to take into account the effects of spatial dispersion in the constitutive relations as a first-order assumption for homogeneous media. These problems may especially arise in guide-wave structures based on artificial chiral and bi(an)isotropic media [14].

The aim of this paper is to obtain and analyze the energy balance equation for quasimonochromatic waves in bianisotropic media in the most general form by taking into consideration moderate absorption and effects of temporal and spatial dispersion. To the best of the author's knowledge, this is the first time such an analysis has been realized. Until now, we have had investigations of energetic relations only for time-harmonic electromagnetic waves in chiral and bi(an)isotropic lossless media [3–5,15]. In Ref. [6], a general form of energetic relations was obtained for a time domain field in bianisotropic media with temporal dispersion. These general relations do not permit, however, the analysis of the mechanisms of storage and absorption of the energy and the character of the energy transport.

We will show that spatial dispersion in bianisotropic media provides additional power flow similar to such effects in anisotropic media [1,2]. A very interesting result is that the energy transport of quasimonochromatic fields may be defined, not only by parameters of a medium, but also by the structure of the electromagnetic field. Such an effect, which is a special feature of bianisotropic media, causes the correspondence between the group velocity and the velocity of energy transport only for a certain structure of quasimonochromatic field.

For further general consideration of bianisotropic media with moderate absorption and temporal and spatial dispersion, we will use the term *complex media*. Necessary analysis has to be done for constitutive relations in complex media with the quasimonochromatic field.

II. CONSTITUTIVE RELATIONS FOR COMPLEX MEDIA WITH QUASIMONOCHROMATIC FIELD

It is known that to describe electromagnetic fields in media, Maxwell's equations have to be added with constitutive relations relating the electric field $\vec{E}$, the magnetic induction $\vec{B}$, the displacement field $\vec{D}$, and the magnetic field $\vec{H}$ to each other. The constitutive relations in their most general form are usually given as a relationship between pairs of fields $\{\vec{D},\vec{H}\}$ and $\{\vec{E},\vec{B}\}$ or $\{\vec{D},\vec{B}\}$ and $\{\vec{E},\vec{H}\}$.

For the case of nonmagnetic materials and with the assumption of the linear response function, the constitutive relation has the form of the integral relation:

$$D_i(\vec{r},t)=\int_{-\infty}^{t}dt'\int d\vec{r}'\,\epsilon_{ij}(t,\vec{r},t',\vec{r}')E_j(\vec{r}',t'). \quad (1)$$

Here only the causality principle (that is, the displacement field $\vec{D}$ at the time t is defined by the electric field $\vec{E}$ at the time $t'\leqslant t$) is taken into account. For the time-invariant and spatially homogeneous medium, the constitutive relation (1) has the form of temporal and space convolution [2,11].

One can extend the above formulation to the case of linear complex media. By taking into account the causality principle, two forms of constitutive relations are possible:

$$D_i(\vec{r},t)=\int_{-\infty}^{t}dt'\int d\vec{r}'\,\alpha_{ij}(t,\vec{r},t',\vec{r}')E_j(\vec{r}',t') + \int_{-\infty}^{t}dt'\int d\vec{r}'\,\beta_{ij}(t,\vec{r},t',\vec{r}')B_j(\vec{r}',t'), \quad (2)$$

$$H_i(\vec{r},t)=\int_{-\infty}^{t}dt'\int d\vec{r}'\,\gamma_{ij}(t,\vec{r},t',\vec{r}')E_j(\vec{r}',t') + \int_{-\infty}^{t}dt'\int d\vec{r}'\,\nu_{ij}(t,\vec{r},t',\vec{r}')B_j(\vec{r}',t'),$$

and

$$D_i(\vec{r},t)=\int_{-\infty}^{t}dt'\int d\vec{r}'\,\epsilon_{ij}(t,\vec{r},t',\vec{r}')E_j(\vec{r}',t') + \int_{-\infty}^{t}dt'\int d\vec{r}'\,\xi_{ij}(t,\vec{r},t',\vec{r}')H_j(\vec{r}',t'), \quad (3)$$

$$B_i(\vec{r},t)=\int_{-\infty}^{t}dt'\int d\vec{r}'\,\zeta_{ij}(t,\vec{r},t',\vec{r}')E_j(\vec{r}',t') + \int_{-\infty}^{t}dt'\int d\vec{r}'\,\mu_{ij}(t,\vec{r},t',\vec{r}')H_j(\vec{r}',t').$$

It should be noted that ϵ_{ij} in Eq. (3) is not the same as the permittivity tensor in Eq. (1).

For time-invariant and spatially homogeneous bianisotropic medium, the constitutive relations (2) and (3) may have the form of temporal and space convolution. The temporal convolution form of the constitutive relations in bianisotropic media was considered in [6] and [16]. We now wish to extend such a formulation to the general case of the temporal and space convolution form of the constitutive relations. For this purpose, we have to consider an influence of short-time and short-space electrostatic and magnetostatic interactions between particles on the polarization properties of a medium. The functions α_{ij}, β_{ij}, γ_{ij}, and ν_{ij} in Eq. (2) and the functions ϵ_{ij}, ξ_{ij}, ζ_{ij}, and μ_{ij} in Eq. (3) are real tensor response functions. For a given time moment, the reaction of a medium is dependent on previous values of the fields because of a finiteness of a time for reorganization of all the system of dipoles. In fact, such a "memory" is retained during the time of system relaxation T_r. Therefore, the response functions decrease rapidly for $t-t'\gg T_r$. On the other hand, we may have a nonlocal connection between the fields and the system reaction. The response functions have to decrease when the difference $|\vec{r}-\vec{r}'|$ increases. In these assumptions, we have the convergence of integrals in Eqs. (2) and (3) and the constitutive relations have the form of temporal and space convolution.

One can use the field renormalization to describe electrical and magnetic properties of a bianisotropic medium only by the tensor of the permittivity without separation of electric polarization and magnetization currents. In such a case, we introduce a vector of generalized electric displacement [2,11]. This procedure was used in [13] to analyze the effects of spatial dispersion in bianisotropic media. We, however, will analyze the dispersion effects by taking into consideration electrical and magnetic properties of a medium.

Let us represent the variable field, in Eq. (2) as quasimonochromatic quantities:

$$\vec{E}=\vec{E}_m(\vec{r},t)e^{i(\omega t-\vec{k}\cdot\vec{r})}, \quad \vec{B}=\vec{B}_m(\vec{r},t)e^{i(\omega t-\vec{k}\cdot\vec{r})}, \quad (4)$$

where the amplitudes $\vec{E}_m$ and $\vec{B}_m$ are smooth functions of a coordinate and a time, so that

$$|(k^{-1}\vec{\nabla})E_{m_i}|\ll E_m, \quad \left|\left(\omega^{-1}\frac{\partial}{\partial t}\right)E_{m_i}\right|\ll E_m, \quad (5)$$

and

$$|(k^{-1}\vec{\nabla})B_{m_i}|\ll B_m, \quad \left|\left(\omega^{-1}\frac{\partial}{\partial t}\right)B_{m_i}\right|\ll B_m. \quad (6)$$

One can express the slowly varying amplitudes $\vec{E}_m(\vec{r}',t')$ and $\vec{B}_m(\vec{r}',t')$ as a sum of the first few terms of the Taylor series

$$\vec{E}_m(\vec{r}',t')\simeq\vec{E}_m(\vec{r},t)+[(\vec{r}'-\vec{r})\cdot\vec{\nabla}]\vec{E}_m(\vec{r},t) + (t'-t)\frac{\partial\vec{E}_m(\vec{r},t)}{\partial t}, \quad (7)$$

$$\vec{B}_m(\vec{r}',t')\simeq\vec{B}_m(\vec{r},t)+[(\vec{r}'-\vec{r})\cdot\vec{\nabla}]\vec{B}_m(\vec{r},t) + (t'-t)\frac{\partial\vec{B}_m(\vec{r},t)}{\partial t}. \quad (8)$$

On the basis of expressions (7) and (8), using some transformations (see Appendix A), we can represent Eq. (2) as

$$D_i(\vec{r},t)=D_{m_i}e^{i(\omega t-\vec{k}\cdot\vec{r})}, \quad H_i(\vec{r},t)=H_{m_i}e^{i(\omega t-\vec{k}\cdot\vec{r})}, \quad (9)$$

where

$$D_{m_i}=\alpha_{ij}(\omega,\vec{k})E_{m_j}(\vec{r},t)+i\,\frac{\partial\alpha_{ij}(\omega,\vec{k})}{\partial\vec{k}}\frac{\partial E_{m_j}(\vec{r},t)}{\partial\vec{r}}-i\,\frac{\partial\alpha_{ij}(\omega,\vec{k})}{\partial\omega}\frac{\partial E_{m_j}(\vec{r},t)}{\partial t}+\beta_{ij}(\omega,\vec{k})B_{m_j}(\vec{r},t)+i\,\frac{\partial\beta_{ij}(\omega,\vec{k})}{\partial\vec{k}}\frac{\partial B_{m_j}(\vec{r},t)}{\partial\vec{r}}-i\,\frac{\partial\beta_{ij}(\omega,\vec{k})}{\partial\omega}\frac{\partial B_{m_j}(\vec{r},t)}{\partial t},\tag{10}$$

$$H_{m_i}=\gamma_{ij}(\omega,\vec{k})E_{m_j}(\vec{r},t)+i\,\frac{\partial\gamma_{ij}(\omega,\vec{k})}{\partial\vec{k}}\frac{\partial E_{m_j}(\vec{r},t)}{\partial\vec{r}}-i\,\frac{\partial\gamma_{ij}(\omega,\vec{k})}{\partial\omega}\frac{\partial E_{m_j}(\vec{r},t)}{\partial t}+\nu_{ij}(\omega,\vec{k})B_{m_j}(\vec{r},t)+i\,\frac{\partial\nu_{ij}(\omega,\vec{k})}{\partial\vec{k}}\frac{\partial B_{m_j}}{\partial\vec{r}}-i\,\frac{\partial\nu_{ij}(\omega,\vec{k})}{\partial\omega}\frac{\partial B_{m_j}(\vec{r},t)}{\partial t}.\tag{11}$$

The coefficients $\alpha_{ij}(\omega,\vec{k})$ and other coefficients in Eqs. (10) and (11), as well as derivatives of these coefficients are defined on the basis of the Fourier transformation (see Appendix A).

Now let us express the fields in Eq. (3) as quasimonochromatic quantities:

$$\vec{E}=\vec{E}_m(\vec{r},t)e^{i(\omega t-\vec{k}\cdot\vec{r})},\quad \vec{H}=\vec{H}_m(\vec{r},t)e^{i(\omega t-\vec{k}\cdot\vec{r})}.\tag{12}$$

On the basis of the analogous procedure, we can rewrite Eq. (3) as

$$D_i(\vec{r},t)=D_{m_i}e^{i(\omega t-\vec{k}\cdot\vec{r})},\quad B_i(\vec{r},t)=B_{m_i}e^{i(\omega t-\vec{k}\cdot\vec{r})},\tag{13}$$

where

$$D_{m_i}=\epsilon_{ij}(\omega,\vec{k})E_{m_j}(\vec{r},t)+i\,\frac{\partial\epsilon_{ij}(\omega,\vec{k})}{\partial\vec{k}}\frac{\partial E_{m_j}(\vec{r},t)}{\partial\vec{r}}-i\,\frac{\partial\epsilon_{ij}(\omega,\vec{k})}{\partial\omega}\frac{\partial E_{m_j}(\vec{r},t)}{\partial t}+\xi_{ij}(\omega,\vec{k})H_{m_j}(\vec{r},t)+i\,\frac{\partial\xi_{ij}(\omega,\vec{k})}{\partial\vec{k}}\frac{\partial H_{m_j}(\vec{r},t)}{\partial\vec{r}}-i\,\frac{\partial\xi_{ij}(\omega,\vec{k})}{\partial\omega}\frac{\partial H_{m_j}(\vec{r},t)}{\partial t},\tag{14}$$

$$B_{m_i}=\zeta_{ij}(\omega,\vec{k})E_{m_j}(\vec{r},t)+i\,\frac{\partial\zeta_{ij}(\omega,\vec{k})}{\partial\vec{k}}\frac{\partial E_{m_j}(\vec{r},t)}{\partial\vec{r}}-i\,\frac{\partial\zeta_{ij}(\omega,\vec{k})}{\partial\omega}\frac{\partial E_{m_j}(\vec{r},t)}{\partial t}+\mu_{ij}(\omega,\vec{k})H_{m_j}(\vec{r},t)+i\,\frac{\partial\mu_{ij}(\omega,\vec{k})}{\partial\vec{k}}\frac{\partial H_{m_j}(\vec{r},t)}{\partial\vec{r}}-i\,\frac{\partial\mu_{ij}(\omega,\vec{k})}{\partial\omega}\frac{\partial H_{m_j}(\vec{r},t)}{\partial t}.\tag{15}$$

The coefficients $\epsilon_{ij}(\omega,\vec{k})$ and other coefficients in Eqs. (14) and (15) as well as their derivatives are also defined on the basis of the Fourier transformation.

In our analysis, we introduce the fields $\vec{E}_m$ and $\vec{B}_m$ in Eqs. (10) and (11) and the fields $\vec{E}_m$ and $\vec{H}_m$ in Eqs. (14) and (15), independently one from the other. Such an assumption is possible due to conditions Eqs. (5) and (6) for $\vec{E}_m$ and $\vec{B}_m$ and analogous conditions for $\vec{E}_m$ and $\vec{H}_m$. This independence of the fields will give us the possibility to impose certain conditions on the structure of the complex envelopes of the quasimonochromatic field.

III. POYNTING'S THEOREM

Let us represent real vectors in Poynting's theorem as a composition of complex vectors:

$$\vec{E}\to\tfrac{1}{2}(\vec{E}+\vec{E}^*),\quad \vec{H}\to\tfrac{1}{2}(\vec{H}+\vec{H}^*),\tag{16}$$

$$\vec{D}\to\tfrac{1}{2}(\vec{D}+\vec{D}^*),\quad \vec{B}\to\tfrac{1}{2}(\vec{B}+\vec{B}^*),$$

where complex vectors are defined on the basis of Eqs. (4) and (9) or Eqs. (12) and (13). One obtains Poynting's theorem for amplitudes of complex vectors in accordance with the procedure of time averaging:

$$-\vec{\nabla}\cdot[\mathrm{Re}(\vec{E}_m\times\vec{H}_m^*)]=\mathrm{Re}\left[\vec{E}_m^*\cdot\left(\frac{\partial\vec{D}}{\partial t}\right)_m+\vec{H}_m^*\cdot\left(\frac{\partial\vec{B}}{\partial t}\right)_m\right].\tag{17}$$

Two types of constitutive relations [Eqs. (2) and (3)] give two types of Poynting's equations. On the basis of expressions (9)–(11) by neglecting the terms containing second-order derivatives (an assumption of small dispersion), we have after some transformations:

$$-\vec{\nabla}\cdot[\mathrm{Re}(\vec{E}_m\times\vec{H}_m^*)]=\mathrm{Re}\Bigg[i\omega\alpha_{ij}E_{m_j}E_{m_i}^*+\frac{\partial(\omega\alpha_{ij})}{\partial\omega}\frac{\partial E_{m_j}}{\partial t}E_{m_i}^*-\omega\frac{\partial\alpha_{ij}}{\partial\vec{k}}\frac{\partial E_{m_j}}{\partial\vec{r}}E_{m_i}^*+i\omega\beta_{ij}B_{m_j}E_{m_i}^*+\frac{\partial(\omega\beta_{ij})}{\partial\omega}\frac{\partial B_{m_j}}{\partial t}E_{m_i}^*$$
$$-\omega\frac{\partial\beta_{ij}}{\partial\vec{k}}\frac{\partial B_{m_j}}{\partial\vec{r}}E_{m_i}^*+i\omega\gamma_{ij}^*E_{m_j}^*B_{m_i}-\omega\frac{\partial\gamma_{ij}^*}{\partial\omega}\frac{\partial(E_{m_j}^*B_{m_i})}{\partial t}+\frac{\partial(\omega\gamma_{ij}^*)}{\partial\omega}E_{m_j}^*\frac{\partial B_{m_i}}{\partial t}+\omega\frac{\partial\gamma_{ij}^*}{\partial\vec{k}}\frac{\partial E_{m_j}^*}{\partial\vec{r}}B_{m_i}$$
$$+i\omega\nu_{ij}^*B_{m_j}^*B_{m_i}-\omega\frac{\partial\nu_{ij}^*}{\partial\omega}\frac{\partial(B_{m_j}^*B_{m_i})}{\partial t}+\frac{\partial(\omega\nu_{ij}^*)}{\partial\omega}B_{m_j}^*\frac{\partial B_{m_i}}{\partial t}+\omega\frac{\partial\nu_{ij}^*}{\partial\vec{k}}\frac{\partial B_{m_j}^*}{\partial\vec{r}}B_{m_i}\Bigg]. \tag{18}$$

Analogous procedure based on expressions (13)–(15) gives for Poynting's theorem:

$$-\vec{\nabla}\cdot[\mathrm{Re}(\vec{E}_m\times\vec{H}_m^*)]=\mathrm{Re}\Bigg[i\omega\epsilon_{ij}E_{m_j}E_{m_i}^*+\frac{\partial(\omega\epsilon_{ij})}{\partial\omega}\frac{\partial E_{m_j}}{\partial t}E_{m_i}^*-\omega\frac{\partial\epsilon_{ij}}{\partial\vec{k}}\frac{\partial E_{m_j}}{\partial\vec{r}}E_{m_i}^*+i\omega\xi_{ij}H_{m_j}E_{m_i}^*+\frac{\partial(\omega\xi_{ij})}{\partial\omega}\frac{\partial H_{m_j}}{\partial t}E_{m_i}^*$$
$$-\omega\frac{\partial\xi_{ij}}{\partial\vec{k}}\frac{\partial H_{m_j}}{\partial\vec{r}}E_{m_i}^*+i\omega\zeta_{ij}E_{m_j}H_{m_i}^*+\frac{\partial(\omega\zeta_{ij})}{\partial\omega}\frac{\partial E_{m_j}}{\partial t}H_{m_i}^*-\omega\frac{\partial\zeta_{ij}}{\partial\vec{k}}\frac{\partial E_{m_j}}{\partial\vec{r}}H_{m_i}^*+i\omega\mu_{ij}H_{m_j}H_{m_i}^*$$
$$+\frac{\partial(\omega\mu_{ij})}{\partial\omega}\frac{\partial H_{m_j}}{\partial t}H_{m_i}^*-\omega\frac{\partial\mu_{ij}}{\partial\vec{k}}\frac{\partial H_{m_j}}{\partial\vec{r}}H_{m_i}^*\Bigg]. \tag{19}$$

One can see that in spite of formal equivalence of the constitutive relations (2) and (3), these relations may give different forms of Poynting's equations and therefore different macroscopic treatment of the energy relations. The paper [17] is an example of how a form of constitutive relations may provide proper macroscopic treatment of the energy relations in anisotropic media.

In this paper, we restrict our analysis to only Eq. (19). Every term on the right-hand side of Eq. (19) is a contraction of two tensors: a constitutive tensor (or its derivatives) and a dyadic product of two electromagnetic field vectors. Any tensor of the second rank (or any dyad) A_{ij} can be written as a sum of Hermitian A_{ij}^{h} and anti-Hermitian A_{ij}^{ah} tensors (or dyads) [18,19]. After some transformations (see Appendix B) we rewrite Eq. (19) as

$$-\vec{\nabla}\cdot\overline{\vec{S}}=\overline{Q(\omega,t)}+\overline{R(\vec{k},\vec{r})}+\overline{\mathcal{P}}, \tag{20}$$

where

$$\overline{\vec{S}}=\tfrac{1}{4}(\vec{E}\times\vec{H}^*+\vec{E}^*\times\vec{H}), \tag{21}$$

$$\overline{Q(\omega,t)}=\frac{1}{4}\frac{\partial(\omega\epsilon_{ij}^{h})}{\partial\omega}\frac{\partial}{\partial t}(E_i^*E_j)+\frac{1}{4}\frac{\partial(\omega\mu_{ij}^{h})}{\partial\omega}\frac{\partial}{\partial t}(H_i^*H_j)+\frac{1}{2}\frac{\partial(\omega\xi_{ij}^{h})}{\partial\omega}\left(E_{m_i}^*\frac{\partial H_{m_j}}{\partial t}\right)^{h}+\frac{1}{2}\frac{\partial(\omega\xi_{ij}^{ah})}{\partial\omega}\left(E_{m_i}^*\frac{\partial H_{m_j}}{\partial t}\right)^{ah}$$
$$+\frac{1}{2}\frac{\partial(\omega\zeta_{ij}^{h})}{\partial\omega}\left(H_{m_i}^*\frac{\partial E_{m_j}}{\partial t}\right)^{h}+\frac{1}{2}\frac{\partial(\omega\zeta_{ij}^{ah})}{\partial\omega}\left(H_{m_i}^*\frac{\partial E_{m_j}}{\partial t}\right)^{ah}, \tag{22}$$

$$\overline{R(\vec{k},\vec{r})}=-\Bigg[\frac{1}{4}\omega\vec{\nabla}\cdot\left(\frac{\partial\epsilon_{ij}^{h}}{\partial\vec{k}}E_i^*E_j\right)+\frac{1}{4}\omega\vec{\nabla}\cdot\left(\frac{\partial\mu_{ij}^{h}}{\partial\vec{k}}H_i^*H_j\right)+\frac{1}{2}\omega\frac{\partial\xi_{ij}^{h}}{\partial\vec{k}}\left(E_{m_i}^*\frac{\partial H_{m_j}}{\partial\vec{r}}\right)^{h}+\frac{1}{2}\omega\frac{\partial\xi_{ij}^{ah}}{\partial\vec{k}}\left(E_{m_i}^*\frac{\partial H_{m_j}}{\partial\vec{r}}\right)^{ah}$$
$$+\frac{1}{2}\omega\frac{\partial\zeta_{ij}^{h}}{\partial\vec{k}}\left(H_{m_i}^*\frac{\partial E_{m_j}}{\partial\vec{r}}\right)^{h}+\frac{1}{2}\omega\frac{\partial\zeta_{ij}^{ah}}{\partial\vec{k}}\left(H_{m_i}^*\frac{\partial E_{m_j}}{\partial\vec{r}}\right)^{ah}\Bigg], \tag{23}$$

$$\overline{\mathcal{P}}=\tfrac{1}{2}\omega[\epsilon_{ij}^{ah}E_i^*E_j+\mu_{ij}^{ah}H_i^*H_j+(\zeta_{ij}^{h}-\xi_{ij}^{h})(H_i^*E_j)^{ah}+(\zeta_{ij}^{ah}+\xi_{ij}^{ah})(H_i^*E_j)^{h}]. \tag{24}$$

For monochromatic fields, we have $\overline{Q(\omega,t)}=0$ and $R(\vec{k},\vec{r})=0$ and therefore

$$-\vec{\nabla}\cdot\overline{\vec{S}}=\overline{\mathcal{P}}. \tag{25}$$

The term $\overline{\mathcal{P}}$ describes dissipative losses. The lossless case is characterized by the following two systems of relations for the constitutive tensors. The first system is

$$\overleftrightarrow{\epsilon}=\overleftrightarrow{\epsilon}^{+},\quad \overleftrightarrow{\mu}=\overleftrightarrow{\mu}^{+},\quad \overleftrightarrow{\xi}=\overleftrightarrow{\zeta}^{+}, \tag{26}$$

where the superscript + denotes the transpose and the complex conjugate procedure. The second one is

$$\begin{gathered}\overleftrightarrow{\epsilon}=\overleftrightarrow{\epsilon}^{+},\quad \overleftrightarrow{\mu}=\overleftrightarrow{\mu}^{+},\\ (\zeta_{ij}^{h}-\xi_{ij}^{h})(H_i^*E_j)^{ah}+(\zeta_{ij}^{ah}+\xi_{ij}^{ah})(H_i^*E_j)^{h}=0.\end{gathered} \tag{27}$$

The relations (26) are well known [3,15]. These relations mean that the term $\mathcal{P}$ vanishes for all possible $\vec{E}$ and $\vec{H}$ fields. In contrast, the relations (27) demonstrate a certain correlation between the field structure and the constitutive parameters of a medium which provides nondissipative propagation of electromagnetic waves.

We can assume that for quasimonochromatic fields in a weakly absorbing bianisotropic medium with temporal and spatial dispersion, the term $\mathcal{P}$ also describes dissipative losses and the lossless case is also described by the relations (26) or (27).

Let us consider the lossless case which is characterized by the relations (26). The tensors $\overleftrightarrow{\epsilon}$ and $\overleftrightarrow{\mu}$ are Hermitian and $\overleftrightarrow{\xi}^{h}=\overleftrightarrow{\zeta}^{h}$, $\overleftrightarrow{\xi}^{ah}=-\overleftrightarrow{\zeta}^{ah}$. For this case, we have

$$\begin{aligned}\overline{Q(\omega,t)}=&\frac{1}{4}\frac{\partial(\omega\epsilon_{ij})}{\partial\omega}\frac{\partial}{\partial t}(E_i^*E_j)+\frac{1}{4}\frac{\partial(\omega\mu_{ij})}{\partial\omega}\frac{\partial}{\partial t}(H_i^*H_j)\\&+\frac{1}{2}\frac{\partial(\omega\xi_{ij}^{h})}{\partial\omega}\left(E_{m_i}^*\frac{\partial H_{m_j}}{\partial t}+H_{m_i}^*\frac{\partial E_{m_j}}{\partial t}\right)^{h}\\&+\frac{1}{2}\frac{\partial(\omega\xi_{ij}^{ah})}{\partial\omega}\left(E_{m_i}^*\frac{\partial H_{m_j}}{\partial t}-H_{m_i}^*\frac{\partial E_{m_j}}{\partial t}\right)^{ah},\end{aligned} \tag{28}$$

$$\begin{aligned}\overline{R(\vec{k},\vec{r})}=&-\left[\frac{1}{4}\omega\vec{\nabla}\cdot\left(\frac{\partial\epsilon_{ij}}{\partial\vec{k}}E_i^*E_j\right)+\frac{1}{4}\omega\vec{\nabla}\cdot\left(\frac{\partial\mu_{ij}}{\partial\vec{k}}H_i^*H_j\right)\right.\\&+\frac{1}{2}\omega\frac{\partial\xi_{ij}^{h}}{\partial\vec{k}}\left(E_{m_i}^*\frac{\partial H_{m_j}}{\partial\vec{r}}+H_{m_i}^*\frac{\partial E_{m_j}}{\partial\vec{r}}\right)^{h}\\&\left.+\frac{1}{2}\omega\frac{\partial\xi_{ij}^{ah}}{\partial\vec{k}}\left(E_{m_i}^*\frac{\partial H_{m_j}}{\partial\vec{r}}-H_{m_i}^*\frac{\partial E_{m_j}}{\partial\vec{r}}\right)^{ah}\right].\end{aligned} \tag{29}$$

IV. ENERGY BALANCE EQUATION

For quasimonochromatic fields in an anisotropic medium with temporal and spatial dispersion, Poynting's theorem has a form of the continuity equation

$$-\vec{\nabla}\cdot[\overline{\vec{S}}+\overline{\vec{A}}]=\frac{\partial\overline{W}}{\partial t}+\overline{\mathcal{P}}, \tag{30}$$

and may be characterized as the energy balance equation [1,2,11].

For weakly absorbing anisotropic media, one can interpret the terms in Eq. (30): $\overline{\vec{S}}$ as average Poynting's vector, $\overline{\vec{A}}$ as the average power flow density caused by spatial dispersion, W as the average density of the energy, and $\mathcal{P}$ as the average density of the dissipation losses. It is not possible (even for media without temporal and spatial dispersion) to interpret physically the terms in Eq. (30) when significant absorption takes place. On the other hand, to interpret the energy balance equation, the dissipation itself may be not so weak, but this dissipation has to be strongly reduced by choosing the corresponding frequency region [11] or by using the effect of induced transparency [17].

In this paper, we will show that for a complex medium with weak absorption, the terms in Poynting's equation may also be physically interpreted. For some particular cases of the field configuration and material structure we will have the energy conservation law in a form of continuity equation (30).

We can represent Poynting's equation (20) in the form

$$-\vec{\nabla}\cdot(\overline{\vec{S}}+\overline{\vec{A}_{DA}})=\frac{\partial\overline{W_{DA}}}{\partial t}+\overline{Q_{ME}}(\omega,t)+\overline{R_{ME}}(\vec{k},\vec{r})+\overline{\mathcal{P}}, \tag{31}$$

where

$$\overline{W_{DA}}=\frac{1}{4}\left[\frac{\partial(\omega\epsilon_{ij}^{h})}{\partial\omega}E_i^*E_j+\frac{\partial(\omega\mu_{ij}^{h})}{\partial\omega}H_i^*H_j\right] \tag{32}$$

is the density of the quasimonochromatic electromagnetic field in double anisotropic media characterized by the tensors $\overleftrightarrow{\epsilon}$ and $\overleftrightarrow{\mu}$,

$$\overline{\vec{A}_{DA}}=-\frac{1}{4}\omega\left[\frac{\partial\epsilon_{ij}^{h}}{\partial\vec{k}}E_i^*E_j+\frac{\partial\mu_{ij}^{h}}{\partial\vec{k}}H_i^*H_j\right] \tag{33}$$

is the density of the energy flow due to spatial dispersion effects in double anisotropic media. The terms $Q_{ME}(\omega,t)$ and $R_{ME}(\vec{k},\vec{r})$ in Eq. (31) are caused by magnetoelectric effects in bianisotropic media. These terms are defined by the last four terms in expressions Eq. (22) and Eq. (23), respectively.

Let us introduce the following quantity:

$$\begin{aligned}W_{ME}&=\frac{1}{4}\left[\frac{\partial(\omega\xi_{ij})}{\partial\omega}E_i^*H_j+\frac{\partial(\omega\zeta_{ij})}{\partial\omega}H_i^*E_j\right]\\&=\frac{1}{4}\left[\frac{\partial(\omega\xi_{ij})}{\partial\omega}E_{m_i}^*H_{m_j}+\frac{\partial(\omega\zeta_{ij})}{\partial\omega}H_{m_i}^*E_{m_j}\right].\end{aligned} \tag{34}$$

For

$$E^*_{m_i}\frac{\partial H_{m_j}}{\partial t}=H_{m_j}\frac{\partial E^*_{m_i}}{\partial t}. \tag{35}$$

One obtains from Eq. (34)

$$\frac{\partial W_{ME}}{\partial t}=\frac{1}{2}\left[\frac{\partial(\omega\xi_{ij})}{\partial\omega}E^*_{m_i}\frac{\partial H_{m_j}}{\partial t}+\frac{\partial(\omega\zeta_{ij})}{\partial\omega}H^*_{m_i}\frac{\partial E_{m_j}}{\partial t}\right]. \tag{36}$$

For a time average quantity, we have

$$\frac{\partial\overline{W_{ME}}}{\partial t}=\frac{1}{2}\left[\frac{\partial(\omega\xi^h_{ij})}{\partial\omega}\left(E^*_{m_i}\frac{\partial H_{m_j}}{\partial t}\right)^h+\frac{\partial(\omega\xi^{ah}_{ij})}{\partial\omega}\times\left(E^*_{m_i}\frac{\partial H_{m_j}}{\partial t}\right)^{ah}+\frac{\partial(\omega\zeta^h_{ij})}{\partial\omega}\left(H^*_{m_i}\frac{\partial E_{m_j}}{\partial t}\right)^h+\frac{\partial(\omega\zeta^{ah}_{ij})}{\partial\omega}\left(H^*_{m_i}\frac{\partial E_{m_j}}{\partial t}\right)^{ah}\right]. \tag{37}$$

This expression corresponds to the last four terms in Eq. (22) and, therefore, we can write

$$\overline{Q_{ME}}(\omega,t)=\frac{\partial\overline{W_{ME}}}{\partial t}. \tag{38}$$

An expression for $\overline{W_{ME}}$ one obtains on the basis of Eqs. (34) and (B5):

$$\overline{W_{ME}}=\frac{1}{4}\left\{\frac{\partial[\omega(\zeta^h_{ij}+\xi^h_{ij})]}{\partial\omega}(H^*_iE_j)^h+\frac{\partial[\omega(\zeta^{ah}_{ij}-\xi^{ah}_{ij})]}{\partial\omega}(H^*_iE_j)^{ah}\right\}. \tag{39}$$

Now we introduce the quantity

$$A_{ME}=-\frac{1}{4}\omega\left[\frac{\partial\xi_{ij}}{\partial\vec{k}}E^*_iH_j+\frac{\partial\zeta_{ij}}{\partial\vec{k}}H^*_iE_j\right]=-\frac{1}{4}\omega\left[\frac{\partial\xi_{ij}}{\partial\vec{k}}E^*_{m_i}H_{m_j}+\frac{\partial\zeta_{ij}}{\partial\vec{k}}H^*_{m_i}E_{m_j}\right]. \tag{40}$$

For

$$E^*_{m_i}\frac{\partial H_{m_j}}{\partial\vec{r}}=H_{m_j}\frac{\partial E^*_{m_i}}{\partial\vec{r}} \tag{41}$$

we have from Eq. (40)

$$\frac{\partial\overline{A_{ME}}}{\partial\vec{r}}=-\frac{1}{2}\omega\left[\frac{\partial\xi^h_{ij}}{\partial\vec{k}}\left(E^*_{m_i}\frac{\partial H_{m_j}}{\partial\vec{r}}\right)^h+\frac{\partial\xi^{ah}_{ij}}{\partial\vec{k}}\left(E^*_{m_i}\frac{\partial H_{m_j}}{\partial\vec{r}}\right)^{ah}+\frac{\partial\zeta^h_{ij}}{\partial\vec{k}}\left(H^*_{m_i}\frac{\partial E_{m_j}}{\partial\vec{r}}\right)^h+\frac{\partial\zeta^{ah}_{ij}}{\partial\vec{k}}\left(H^*_{m_i}\frac{\partial E_{m_j}}{\partial\vec{r}}\right)^{ah}\right]. \tag{42}$$

For the last four terms in Eq. (23) one can write

$$\overline{R_{ME}}(\vec{k},\vec{r})=\frac{\partial\overline{A_{ME}}}{\partial\vec{r}}. \tag{43}$$

For a spatially homogeneous medium, one obtains

$$\overline{R_{ME}}(\vec{k},\vec{r})=\vec{\nabla}\cdot\overline{\vec{A}_{ME}}, \tag{44}$$

where

$$\overline{\vec{A}_{ME}}=-\frac{1}{4}\left[\frac{\partial(\zeta^h_{ij}+\xi^h_{ij})}{\partial\vec{k}}(H^*_iE_j)^h+\frac{\partial(\zeta^{ah}_{ij}-\xi^{ah}_{ij})}{\partial\vec{k}}(H^*_iE_j)^{ah}\right]. \tag{45}$$

Equations (35) and (41) describe a certain structure of the slowly varying complex envelopes. Such an arbitrary construction of a field structure is possible since the fields $\vec{E}_m$ and $\vec{H}_m$ may be introduced independently.

If representations (38) and (44) are possible, we have continuity-equation form Eq. (30) of Eq. (20), where

$$\overline{W}=\overline{W_{DA}}+\overline{W_{ME}} \tag{46}$$

and

$$\overline{\vec{A}}=\overline{\vec{A}_{DA}}+\overline{\vec{A}_{ME}}. \tag{47}$$

We have obtained the continuity-equation form of Poynting's theorem for a certain structure of the quasimonochromatic field but for an arbitrary form of the constitutive tensors. One can see that expression (46) coincides with the expression for the average stored density in [5]. We have shown, however, that in bianisotropic media the notion of the average stored energy density takes place only for a certain structure of the field.

V. VELOCITY OF ENERGY TRANSPORT AND GROUP VELOCITY

For a lossless case characterized by relations (26) or (27) and for the field structure described by relations (35) and (41) one can introduce a notion of the energy transport velocity

$$\vec{V}e=\frac{\overline{W}}{\overline{\vec{S}}+\overline{\vec{A}}}, \tag{48}$$

where $\overline{\vec{S}}$ is average Poynting's vector [see Eq. (21)], $\overline{W}$ and $\overline{\vec{A}}$ are, respectively, an average density of the energy [see Eq. (46)] and an average power flow caused by spatial dispersion [see Eq. (47)]. One can also use expression (48) for a case of electromagnetic wave propagation with a weak absorption.

A group velocity is defined as the velocity with which the whole wave packet moves without a distortion. For the quasimonochromatic field, the dispersion equation $\omega=\omega(\vec{k})$

may be represented by the first two terms of the Taylor series and one can see that the peak of the wave packet is propagated with the group velocity [11,21]

$$\vec{V}g=\frac{\partial\omega}{\partial\vec{k}}. \tag{49}$$

This definition of the group velocity is correct for a lossless case. In the case of losses, one has to use special analysis to determine the group velocity [11].

We will now consider the expression for the group velocity in a lossless case (the vector $\vec{k}$ is real) of a complex medium. In Maxwell's equations, the fields are represented as

$$\vec{E}=\vec{E}_0(\omega,\vec{k})e^{i(\omega t-\vec{k}\cdot\vec{r})}. \tag{50}$$

We have analogous representations for $\vec{H}$, $\vec{B}$, and $\vec{D}$. One can obtain from Maxwell's equations:

$$\omega(\vec{B}_0\cdot\vec{H}_0^*+\vec{E}_0^*\cdot\vec{D}_0)=\vec{k}\cdot(\vec{E}_0\times\vec{H}_0^*+\vec{E}_0^*\times\vec{H}_0). \tag{51}$$

The temporal and space convolution form of the constitutive relations in complex media makes it possible to represent relations (3) as [11,13]

$$\begin{aligned}\vec{D}_0(\omega,\vec{k})&=\overleftrightarrow{\epsilon}(\omega,\vec{k})\vec{E}_0(\omega,\vec{k})+\overleftrightarrow{\xi}(\omega,\vec{k})\vec{H}_0(\omega,\vec{k}),\\ \vec{B}_0(\omega,\vec{k})&=\overleftrightarrow{\zeta}(\omega,\vec{k})\vec{E}_0(\omega,\vec{k})+\overleftrightarrow{\mu}(\omega,\vec{k})\vec{H}_0(\omega,\vec{k}).\end{aligned} \tag{52}$$

Let us totally differentiate Eq. (51) by $\vec{k}$. After some transformations analogous the procedure used in [11] and [21] for an anisotropic medium and taking into account Eq. (52), one obtains

$$\begin{aligned}&\frac{\partial\omega}{\partial\vec{k}}\left[\frac{\partial(\omega\overleftrightarrow{\epsilon})}{\partial\omega}\vec{E}_0\vec{E}_0^*+\frac{\partial(\omega\overleftrightarrow{\mu})}{\partial\omega}\vec{H}_0\vec{H}_0^*+\frac{\partial(\omega\overleftrightarrow{\xi})}{\partial\omega}\vec{H}_0\vec{E}_0^*\right.\\
&\quad\left.+\frac{\partial(\omega\overleftrightarrow{\zeta})}{\partial\omega}\vec{E}_0\vec{H}_0^*\right]+\omega\frac{\partial\overleftrightarrow{\epsilon}}{\omega\vec{k}}\vec{E}_0\vec{E}_0^*+\omega\frac{\partial\overleftrightarrow{\mu}}{\partial\vec{k}}\vec{H}_0\vec{H}_0^*\\
&\quad+\omega\frac{\partial\overleftrightarrow{\xi}}{\partial\vec{k}}\vec{H}_0\vec{E}_0^*+\omega\frac{\partial\overleftrightarrow{\zeta}}{\partial\vec{k}}\vec{E}_0\vec{H}_0^*+\omega\vec{E}_0^*\left(\overleftrightarrow{\epsilon}\frac{d\vec{E}_0}{d\vec{k}}+\overleftrightarrow{\xi}\frac{d\vec{H}_0}{d\vec{k}}\right)\\
&\quad+\omega\vec{H}_0^*\left(\overleftrightarrow{\zeta}\frac{d\vec{E}_0}{d\vec{k}}+\overleftrightarrow{\mu}\frac{d\vec{H}_0}{d\vec{k}}\right)-\omega\frac{d\vec{E}_0}{d\vec{k}}(\overleftrightarrow{\epsilon}^*\vec{E}_0^*+\overleftrightarrow{\xi}^*\vec{H}_0^*)\\
&\quad-\omega\frac{d\vec{H}_0}{d\vec{k}}(\overleftrightarrow{\zeta}^*\vec{E}_0^*+\overleftrightarrow{\mu}^*\vec{H}_0^*)=\vec{E}_0\times\vec{H}_0^*+\vec{E}_0^*\times\vec{H}_0.\end{aligned} \tag{53}$$

As a result of time averaging, for lossless case characterized by relations (26), we have an expression for $\vec{V}g$ which fully coincides with expression (48) for $\vec{V}e$.

One can see, however, that we have obtained the expression (48) for $\vec{V}e$ for a certain structure of complex envelopes which is described by the relations (35) and (41). On the other hand, to obtain an expression for $\vec{V}g$, we did not use any additional conditions for the field structure.

VI. DISCUSSION

Different forms of constitutive relations may give different macroscopic treatment of energy relations for quasimonochromatic fields. Constitutive relations (2) and (3) may be useful in microwave frequencies. At the optical region, the separation of electric polarization and magnetization currents may not be uniquely defined [1,2,9,11]. In this case, it is convenient to combine all induced effects in the bianisotropic material in a renormalized electric dipole moment. The bianisotropic medium is characterized now by a tensor of the permittivity with parameters dependable from a frequency ω and a wave vector $\vec{k}$ [13]. This kind of the constitutive relations gives the energy balance equation formally coincided with the energetic relations for anisotropic optical crystals [11].

Our analysis of complex media based on the constitutive relations (3) has demonstrated that a transparency of a complex medium for the propagation of monochromatic electromagnetic waves, as well as an energy transport for quasimonochromatic waves, may be determined not only by parameters of a medium but also by the structure of the field. We have to use combined consideration of the field structure and the properties of a medium.

For bianisotropic media, Poynting's theorem has the continuity-equation form only for a certain structure of the quasimonochromatic field. The energy transport is possible if complex envelopes satisfy Eqs. (35) and (41). Only in this case one can introduce a notion of an average stored energy $\overline{W}$ and an additional average power flow $\overline{\vec{A}}$ caused by the effects of spatial dispersion.

For the field structure described by Eqs. (35) and (41), we can consider energy transport velocity $\vec{V}e$. On the other hand, one can formally introduce a notion of a group velocity $\vec{V}g$ as the velocity of moving wave packet without distortion. The group velocity correlates with the energy transport velocity only if relations (35) and (41) take place.

APPENDIX A: FOURIER TRANSFORMATION

Four integrals in constitutive relations (2) are similar to each other; therefore we will analyze one of them. We denote this integral as $F_i(\vec{r},t)$. For the time- invariant and spatially homogeneous medium and for the quasimonochromatic field, we have

$$\begin{aligned}F_i(\vec{r},t)&=\int_{-\infty}^{t}dt'\int d\vec{r}\,'\alpha_{ij}(\vec{r}\,'-\vec{r},t'-t)E_j(\vec{r}\,',t')\\
&=e^{i(\omega t-\vec{k}\cdot\vec{r})}\int_{-\infty}^{t}dt'\int d\vec{r}\,'\alpha_{ij}(\vec{r}-\vec{r}\,',t'-t)\\
&\quad\times E_{m_j}(\vec{r}\,',t')e^{i[\omega(t'-t)-\vec{k}\cdot(\vec{r}\,'-\vec{r})]}.\end{aligned} \tag{A1}$$

After substituting (7) into (A1), one obtains

$$F_i(\vec{r},t)=\left[\alpha_{ij}(\omega,\vec{k})E_{m_j}(\vec{r},t)+i\,\frac{\partial\alpha_{ij}(\omega,\vec{k})}{\partial\vec{k}}\,\frac{\partial E_{m_j}(\vec{r},t)}{\partial\vec{r}}-i\,\frac{\partial\alpha_{ij}(\omega,\vec{k})}{\partial\omega}\,\frac{\partial E_{m_j}(\vec{r},t)}{\partial t}\right]e^{i(\omega t-\vec{k}\cdot\vec{r})}, \qquad \text{(A2)}$$

where

$$\alpha_{ij}(\omega,\vec{k})=\int_{-\infty}^{t}dt'\int d\vec{r}'\,\alpha_{ij}(\vec{r}'-\vec{r},t'-t)\times e^{i[\omega(t'-t)-\vec{k}\cdot(\vec{r}'-\vec{r})]}, \qquad \text{(A3)}$$

$$\frac{\partial\alpha_{ij}(\omega,\vec{k})}{\partial\vec{k}}=-i\int_{-\infty}^{t}dt'\int d\vec{r}'\,\alpha_{ij}(\vec{r}'-\vec{r},t'-t)(\vec{r}'-\vec{r})\times e^{i[\omega(t'-t)-\vec{k}\cdot(r'-\vec{r})]}, \qquad \text{(A4)}$$

$$\frac{\partial\alpha_{ij}(\omega,\vec{k})}{\partial\omega}=i\int_{-\infty}^{t}dt'\int d\vec{r}'\,\alpha_{ij}(\vec{r}'-\vec{r},t'-t)(t'-t)\times e^{i[\omega(t'-t)-\vec{k}\cdot(\vec{r}'-\vec{r})]}. \qquad \text{(A5)}$$

APPENDIX B: DERIVATION OF EQ. (20)

A contraction of Hermitian and anti-Hermitian tensors of the second rank is an imaginary quantity. It is evident on the basis of the following relation:

$$(A_{ij}^{h})^{*}(B_{ij}^{ah})^{*}=-A_{ji}^{h}B_{ji}^{ah}=-A_{ij}^{h}B_{ij}^{ah}, \qquad \text{(B1)}$$

which is the definition of an imaginary quantity. Analogously, one can be convinced that $A_{ij}^{h}B_{ij}^{h}$ and $A_{ij}^{ah}B_{ij}^{ah}$ are real quantities.

To obtain Eq. (20), we have to consider separately every term in Eq. (19). Let us consider the first three terms in the right-hand side of Eq. (19). The term $\epsilon_{ij}E_{m_j}E_{m_i}^{*}=\epsilon_{ij}E_{m_i}^{*}E_{m_j}$ is a contraction of two tensors: the tensor of the permittivity $\overleftrightarrow{\epsilon}$ and the dyad $\vec{E}^{*}\vec{E}$. Since the dyad $\vec{E}^{*}\vec{E}$ is Hermitian, we have

$$\mathrm{Re}(i\omega\epsilon_{ij}E_{m_j}E_{m_i}^{*})=\omega\epsilon_{ij}^{ah}E_i^{*}E_j\,. \qquad \text{(B2)}$$

One can see that for quasimonochromatic fields, the dyads $E_{m_i}^{*}(\partial E_{m_j}/\partial t)$ and $E_{m_i}^{*}(\partial E_{m_j}/\partial\vec{r})$ are also Hermitian [20]. Therefore, we have

$$\mathrm{Re}\left[\frac{\partial(\omega\epsilon_{ij})}{\partial\omega}\,\frac{\partial E_{m_j}}{\partial t}\,E_{m_i}^{*}\right]=\frac{1}{2}\,\frac{\partial(\omega\epsilon_{ij}^{h})}{\partial\omega}\left(E_{m_i}^{*}\,\frac{\partial E_{m_j}}{\partial t}+E_{m_j}\,\frac{\partial E_{m_i}^{*}}{\partial t}\right)=\frac{1}{2}\,\frac{\partial(\omega\epsilon_{ij}^{h})}{\partial\omega}\,\frac{\partial}{\partial t}\,(E_i^{*}E_j) \qquad \text{(B3)}$$

and

$$\mathrm{Re}\left[\omega\,\frac{\partial\epsilon_{ij}}{\partial\vec{k}}\,\frac{\partial E_{m_j}}{\partial\vec{r}}\,E_{m_i}^{*}\right]=\frac{1}{2}\,\omega\,\frac{\partial\epsilon_{ij}^{h}}{\partial\vec{k}}\left(E_{m_i}^{*}\,\frac{\partial E_{m_j}}{\partial\vec{r}}+E_{m_j}\,\frac{\partial E_{m_i}^{*}}{\partial\vec{r}}\right)=\frac{1}{2}\,\omega\,\frac{\partial\epsilon_{ij}^{h}}{\partial\vec{k}}\,\frac{\partial}{\partial\vec{r}}\,(E_i^{*}E_j)=\frac{1}{2}\,\omega\vec{\nabla}\cdot\left(\frac{\partial e_{ij}^{h}}{\partial\vec{k}}\,E_i^{*}E_j\right). \qquad \text{(B4)}$$

To obtain Eq. (B4), we took into account that a medium is spatially homogeneous. We have analogous relations for the last three terms in the right-hand side of Eq. (19).

One can be convinced that for arbitrary field configuration

$$(E_{m_i}^{*}H_{m_j})^{h}+(E_{m_i}^{*}H_{m_j})^{ah}=(H_{m_i}^{*}E_{m_j})^{h}-(H_{m_i}^{*}E_{m_j})^{ah}. \qquad \text{(B5)}$$

On the basis of this relation, we have

$$\xi_{ij}H_{m_j}E_{m_i}^{*}+\zeta_{ij}E_{m_j}H_{m_i}^{*}=(\xi_{ij}^{h}+\zeta_{ij}^{h})(H_{m_i}^{*}E_{m_j})^{h}+(\zeta_{ij}^{ah}-\xi_{ij}^{ah})(H_{m_i}^{*}E_{m_j})^{ah}+(\zeta_{ij}^{h}-\xi_{ij}^{h})(H_{m_i}^{*}E_{m_j})^{ah}+(\xi_{ij}^{ah}+\zeta_{ij}^{ah})(H_{m_i}^{*}E_{m_j})^{h} \qquad \text{(B6)}$$

and, therefore,

$$\mathrm{Re}[i\omega(\xi_{ij}H_{m_j}E_{m_i}^{*}+\zeta_{ij}E_{m_j}H_{m_i}^{*})]=\omega[(\zeta_{ij}^{h}-\xi_{ij}^{h})(H_{m_i}^{*}E_{m_j})^{ah}+(\xi_{ij}^{ah}+\zeta_{ij}^{ah})(H_{m_i}^{*}E_{m_j})^{h}]=\omega[(\zeta_{ij}^{h}-\xi_{ij}^{h})(H_i^{*}E_j)^{ah}+(\xi_{ij}^{ah}+\zeta_{ij}^{ah})(H_i^{*}E_j)^{h}]. \qquad \text{(B7)}$$

For the other four terms in Eq. (19), we have the obvious transformations.

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