Random matrix approach to quantum dissipation

Aurel Bulgac,[1,2,*] Giu Do Dang,[2,†] and Dimitri Kusnezov[3,‡]
[1]*Department of Physics, FM-15, University of Washington, Seattle, Washington 98195*[§]
[2]*Laboratoire de Physique Théorique et Hautes Energies, Université de Paris–Sud, Bâtiment 211, 91405 Orsay, France*
[3]*Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, Connecticut 06520-8120*
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The microscopic origin of dissipation in collective motion of a quantum many-body system is addressed in the framework of a parametric random matrix approach to the intrinsic dynamics. There are noticeable violations of the fluctuation-dissipation theorem and the energy diffusion has a markedly non-Gaussian character. Such features do not support the usual Fokker-Planck approach to dissipation in large amplitude motion of many fermion systems. [S1063-651X(96)05510-9]

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I. INTRODUCTION

While theory and experiment have gone a long way in the study of the collective nature of large amplitude nuclear motion, the theoretical understanding of the coupling between the collective and the intrinsic degrees of freedom, in particular, the understanding of the character and mechanism of the energy exchange between them, is still in its infancy. Most of the approaches are more or less phenomenological in nature. Hill and Wheeler [1] suggested more than forty years ago that Landau-Zener transitions are at the origin of nuclear dissipation. Implicit in this interpretation is the presumption that ''irreversibility'' is of quantum origin. Other quantum approaches have also been employed [2], such as in the linear response model [3], the hopping model [4], path integral method [5,6], and others. On the other hand, many formulations are basically classical in spirit as, for example, the so-called ''wall formula'' [7]. These include the more pragmatic phenomenological models, such as the Langevin and Fokker-Planck equations [8], Maxwell's model for friction with memory effects [9], and to a certain extent kinetic approaches, e.g., two-body dissipation mechanisms [7]. More recently, in an analysis of a generic problem of coupled slow and fast degrees of freedom, Berry and Robbins [10] obtained friction for the slow subsystem when treating the entire system classically, the key requirement being that both these subsystems had a continuous spectrum.

It is clear that the present status of our understanding of the quantum or classical nature of dissipation as well as its microscopic origin in many-body quantum systems is far from complete. In this article, we would like to approach the problem of dissipation in many-body systems using parametric random matrix theory techniques. This is based on the fact that, from the study of experimental nuclear properties from the low energy regime to compound nuclear states, it has become clear that the fluctuation properties of the complex many body system agree well with predictions from random matrix theory [11]. This has been extensively tested in recent years on the experimental side as well as in many nuclear models. With this in mind, we will construct a Hamiltonian which is consistent with the observed fluctuation properties in nuclei, using a parameter dependent random matrix description for the intrinsic nuclear states. This will allow us to explore the problems of diffusion and dissipation in collective motion.

The basis of our approach is explained in Sec. II, where we formulate the problem of collective motion using a path integral approach. An effective collective dynamics can be obtained by averaging over the fast degrees of freedom. The main purpose of this section is to discuss the central element of the path integral formalism, the influence functional. We limit the discussion to the behavior of the coupling between slow and fast modes and on how dissipation emerges. This is done in Sec. III for the case of a finite Hilbert space for the fast dynamics. In Sec. IV, we extend the results to the case of an infinite number of levels for the intrinsic subsystem, where analytical results are obtained and discussed in the adiabatic and diabatic limits. Finally, numerical results are given, in particular, for the evolution of the diffusion constant from the adiabatic to the diabatic limits.

II. INFLUENCE FUNCTIONAL APPROACH TO COLLECTIVE MOTION

The physical systems we explore are many body systems which exhibit excitations on two distinct time scales, described by collective (slow, which we shall denote with P,X) and intrinsic (fast) degrees of freedom (denoted by p,x). We assume that the Hamiltonian of such a system can be expressed in the following form:

$$H(X,P,x,p)=H_0(X,P)+H_1(X,x,p). \tag{1}$$

The dynamical couplings between the slow and fast modes, characterized by H_1, can be viewed as a heat bath coupled to the collective Hamiltonian H_0. In order to derive an effective

*Electronic address: bulgac@phys.washington.edu
†Electronic address: dodang@psisun.u-psud.fr
‡Electronic address: dimitri@nst4.physics.yale.edu
§Permanent address.

collective dynamics, we need to integrate out the effects described by H_1. The path integral formalism introduced by Feynman and Vernon [5,6] is particularly well suited for this purpose. The transition amplitude between direct product states $\Psi_i(X,x)=\phi_n(X)\chi_i(x)$ and $\Psi_f(X,x)=\phi_m(X)\chi_f(x)$ has the form (extension to more general wave functions is obvious)

$$\begin{aligned}A_{mf,ni}&=\langle\phi_m\chi_f|\exp\left[-\frac{i(t_1-t_0)H}{\hbar}\right]|\phi_n\chi_i\rangle\\&=\int dX_1dX_0\phi_m^*(X_1)\phi_n(X_0)\\&\quad\times\int\mathcal{D}X(t)\exp\left[\frac{i}{\hbar}S_0(X(t))\right]\langle\chi_f|\\&\quad\times T\exp\left(-\frac{i}{\hbar}\int_{t_0}^{t_1}dtH_1(X(t))\right)|\chi_i\rangle\\&=\int dX_1dX_0\phi_m^*(X_1)\phi_n(X_0)\int\mathcal{D}X(t)\\&\quad\times\exp\left[\frac{i}{\hbar}S_0(X(t))\right]K_{if}(X(t)).\end{aligned}\tag{2}$$

In the amplitude $A_{mf,ni}$, $X(t)$ represents a path starting at $X_0=X(t_0)$ at $t=t_0$ and ending at $X_1=X(t_1)$ at $t=t_1$. $S_0(X(t))$ is the classical action corresponding to the slow subsystem, described by the Hamiltonian $H_0(X)$, and T is a time ordering operator. (In most of the formulas we shall not display the dependence of various quantities on the slow variables x,p.) The quantity $K_{if}(X(t))$ is the matrix element of the time evolution operator of the fast subsystem for a given slow path $X(t)$, and describes the dynamical interactions between the two subsystems.

From the amplitude, one computes the transition probability

$$\begin{aligned}|A_{mf,ni}|^2&=\int dX_1dX_0dX_1'dX_0'\phi_m^*(X_1)\phi_m(X_1')\phi_n(X_0)\\&\quad\times\phi_n^*(X_0')\int\mathcal{D}X(t)\mathcal{D}X'(t)\\&\quad\times\exp\left[\frac{i}{\hbar}(S_0(X(t))-S_0(X'(t))\right]\\&\quad\times K_{if}(X(t))K_{if}^*(X'(t))\\&=\int dX_1dX_0dX_1'dX_0'\phi_m^*(X_1)\phi_m(X_1')\phi_n(X_0)\\&\quad\times\phi_n^*(X_0')\int\mathcal{D}X(t)\mathcal{D}X'(t)\\&\quad\times\exp\left[\frac{i}{\hbar}(S_0(X(t))-S_0(X'(t))\right]\mathcal{L}_{if}(X,X')\\&=\int dX_1dX_1'\phi_m^*(X_1)\phi_m(X_1')\rho(X_1,X_1',t),\end{aligned}\tag{3}$$

where the quantity $\mathcal{L}_{if}(X,X')$ is called the influence functional and $\rho(X_1,X_1',t)$ is the density matrix of the slow subsystem at time t. The path $X'(t)$ satisfies similar boundary conditions as the path $X(t)$, namely, $X_0'=X'(t_0)$ at $t=t_0$ and ending at $X_1'=X'(t_1)$ at $t=t_1$. We will see in Sec. IV that the influence functional, under some rather general assumptions, and after performing an appropriate statistical averaging, has the form

$$\overline{\mathcal{L}_{if}(X,X')}=\exp\left(-\frac{i}{\hbar}\int_{t_0}^{t_1}dt\,V_{if}(X(t),X'(t))\right),\tag{4}$$

where $V_{if}(X(t),X'(t))$ is a function of the ''right'' $X(t)$ and ''left'' $X'(t)$ paths and the overline stands for ensemble statistical averaging, which we shall discuss below. This can be simplified further by summing over final and averaging over initial states.

The dynamics of the density matrix of the slow subsystem, $\rho(X,X',t)$, can be found from inspection of Eqs. (3), (4)

$$\begin{aligned}i\hbar\partial_t\rho(X,X',t)&=[H_0(X)-H_0(X')+V_{if}(X,X')]\rho(X,X',t)\\&=\mathcal{H}(X,X')\rho(X,X',t).\end{aligned}\tag{5}$$

The dynamical evolution of the slow subsystem, after having integrated out the fast subsystem, is described by an effective Schrödinger equation with twice as many degrees of freedom. The only thing that is left from the fast subsystem is an effective potential $V_{if}(X,X')$ acting in this double configuration space.

The description of the slow dynamics through the effective Hamiltonian $\mathcal{H}(X,X')$ has a number of advantages over other traditional approaches. For instance, in contrast to the optical model, unitarity is never violated, so that there is no loss of probability (see Ref. [12]). This is because the optical model is derived for the amplitudes $A_{mf,ni}$ (which are averaged over the internal motion) rather than the transition probabilities. Even though there is a constant energy exchange between the slow and fast subsystems, which can be interpreted as dissipation, the total energy is nevertheless conserved. The same is also true in principle for any other integrals of motion, such as total linear momentum, total angular momentum, parity, and so forth.

Feynman and Vernon [5] have discussed several possible cases, when the influence functional can be explicitly evaluated. The case of an infinite ensemble of harmonic oscillators, which play the role of the fast subsystem, has been extensively used in condensed matter physics to study the influence of dissipation on tunneling phenomena [13]. While an infinite ensemble of harmonic oscillators is appropriate to model phonons, it is not suited for the many fermion systems we have in mind. In the situations studied in Ref. [13] the phonon energies extend down to zero and thus there is no formal separation of time scales, between fast and slow degrees of freedom. There is another more important reason why a Caldeira and Leggett approach [13] is not appropriate for our purposes here. There, the character of the dissipation is assumed to be in the ohmic regime. The dynamics of the fast degrees of freedom is postulated in such a way as to lead, within the adopted framework, to the expected dissipa-

tive character. In our case, we know a great deal about the fast dynamics, but we do not know the character of the dissipation mechanism, which is the problem we want to study.

The density matrix approach can also be formulated directly from the Schrödinger equation

$$i\hbar\partial_t\Psi(X,x,t)=[H_0(X,P)+H_1(X,x,p)]\Psi(X,x,t), \quad (6)$$

where $\Psi(X,x,t)$ is the wave function of the entire system. By tracing over the fast variables x, one defines the density matrix for the slow subsystem in the usual manner

$$\rho(X,X',t)=\int dx\,\Psi^*(X,x,t)\Psi(X',x,t). \quad (7)$$

The resulting dynamics is now

$$\begin{aligned} i\hbar\partial_t\rho(X,X',t)=&[H_0(X)-H_0(X')]\rho(X,X',t)\\ &+\int dx\,\Psi^*(X',x,t)\\ &\times[H_1(X,x)-H_1(X',x)]\Psi(X,x,t). \end{aligned} \quad (8)$$

The main difficulty with this approach lies in the evaluation of the last term, which requires the knowledge of the full wave function of the many body system. Within the framework we develop in this article, we will show that the effective potential $V(X,X')$ in the alternate formulation of Eq. (5), has a surprisingly simple form after performing the statistical average over different realizations of the fast subsystem. The path integral formulation of the problem allows us to devise relatively simple ways to evaluate the influence functional as well as this effective potential. The statistical averaging procedure is equivalent to averaging over the fast motion or ergodicity of the fast motion, which is the formal basis of the applicability of the random matrix theory to many-body systems.

It should be noticed, from the definition of $\mathcal{H}$, that

$$\mathcal{H}(X,X')=-\mathcal{H}^*(X',X), \quad (9)$$

so that not all solutions of Eq. (5) have a direct physical interpretation as density matrices for the slow subsystem. Only those solutions which satisfy the requirement that

$$\rho(X,X',t)=\rho^*(X',X,t) \quad (10)$$

describe an actual time evolution.

III. DRIVEN SYSTEM WITH CONSTANT COLLECTIVE VELOCITY

In order to derive an expression for the influence functional and to assess how the energy is transferred from the slow (X) to the fast (x) modes, we will first study the case where the slow modes evolve with a constant velocity V_0 according to $X(t)=V_0t$. This corresponds to neglecting the reaction of the fast system on the slow motion. With this assumption, we can solve for the quantum dynamics of the fast subsystem, and obtain analytic results for situations analogous to the conventional adiabatic and diabatic limits.

Removing the part of the total Hamiltonian corresponding to this constant motion, what remains is the Hamiltonian for the fast subsystem. This can be defined through its matrix elements in a fixed, N-dimensional basis as

$$[H(X)]_{ij}=[H_0]_{ij}+[H_1(X)]_{ij}. \quad (11)$$

H_0 is taken to be diagonal and defines the average density of states, with $\langle k|H_0|l\rangle=[H_0]_{kl}=\varepsilon_k\delta_{kl}$. Although H_0 does not depend parametrically on X, it will become evident from our results that the introduction of a coordinate dependence in this part of the Hamiltonian is straightforward.

In the basis of the eigenstates of H_0, we define $H_1(X)$ as a parameter dependent, $N\times N$ real Gaussian random matrix, which is completely specified by its first two moments

$$\begin{aligned} &\overline{[H_1(X)]_{kl}}=0,\\ &\overline{[H_1(X)]_{ij}[H_1(Y)]_{kl}}=[\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}]\mathcal{G}_{ij}(X-Y). \end{aligned} \quad (12)$$

We assume that $H(X)$ has time reversal symmetry, so that the averages are performed with respect the Gaussian orthogonal ensemble (GOE). If we break time reversal invariance, $H(X)$ becomes a complex Hermitian matrix, and one must consider the Gaussian unitary ensemble (GUE). As noted in Ref. [12], it is simple to extend our formalism results to the GUE. $\mathcal{G}_{ij}(X-Y)$ is a ''bell shaped'' correlation function with a characteristic width X_0. The dependence on i,j allows for the description of banded matrices, where an effective number of states $N_0\leqslant N$ are coupled by $H_1(X)$ [12]. The coordinate dependence of this parameterization implies that correlations in the system corresponding to different ''shapes'' X are effective only within a ''distance'' $O(X_0)$. In a finite system such as an atomic nucleus, if the slow coordinate is deformation, then X_0 can be, for example, the deformation necessary to change completely the character of the wave function of the intrinsic degrees of freedom. In the case of the quadrupole deformation, $X_0\approx\Delta\beta\approx 5/A$ [14] (here β is a dimensionless measure of the asphericity of a nucleus and A is the number of nucleons), which is indeed a very small change of the deformation. The average level density of H for each fixed shape X is determined mainly by H_0, while its spectral fluctuation properties are determined by $H_1(X)$.

Even though our formalism is not restricted by the form of $\mathcal{G}$ it is convenient to use an explicit parameterization, which incorporates the density of states and the bandwidth of the statistical fluctuations explicitly [6]

$$\mathcal{G}_{ij}(X)=\frac{W_0}{\sqrt{\rho(\varepsilon_i)\rho(\varepsilon_j)}}\exp\left[-\frac{(\varepsilon_i-\varepsilon_j)^2}{2\kappa_0^2}\right]G\left(\frac{X}{X_0}\right). \quad (13)$$

We will treat the function $G(x)$ as a phenomenological quantity. Here $G(x)=G(-x)=G^*(x)\leqslant 1$, $G(0)=1$, and W_0, κ_0 [$N_0\approx\kappa_0\rho(\varepsilon)$] and X_0 are characteristic of the given system. In terms of these quantities, we will express the velocity V_0 as a dimensionless quantity, with the implicit dimensions of X_0/T_0, where T_0, the characteristic time scale, is taken to be unity. To use again a nuclear example, an appropriate value for κ_0 is of the order of 10 MeV [6,15].

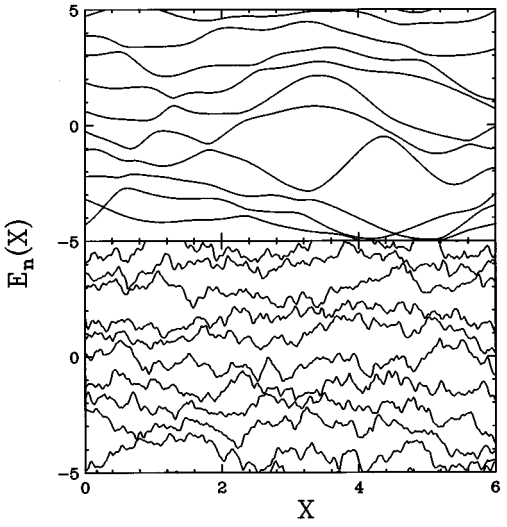

FIG. 1. Instantaneous eigenvalue spectrum $E_n(X)$ as a function of the ''shape'' (X), for a Hamiltonian of the ensemble defined by Eqs. (11)–(13), with $[H_0]_{jk}=k\delta_{jk}$, using the correlator $G(x)=\exp(-x^2/2)$ (top) and $G(x)=\exp(-|x|)$ (bottom).

For any fixed X, the spreading width $\Gamma^{\downarrow}$ is essentially energy independent (more exactly, there is no exponential energy dependence, in spite of the exponential increase of the average level density), in agreement with experimental findings [8]. Moreover, the distribution of the matrix elements of $H_1(X)$ has a shape very similar [16] to that extracted from either nuclear and atomic shell model Hamiltonians or other models for many-body systems [15,17].

The distinction between various choices for $G(x)$ is important. If we consider the leading order behavior of $G(x)\simeq 1-c|x|^{\alpha}+\cdots$, then $\alpha=2$ results in the instantaneous eigenvalues of $H(X)$ evolving smoothly in X, while $0\leqslant\alpha<2$ correspond to the instantaneous eigenvalues diffusing stochastically in X. Other values of α are not possible within this type of random matrix theory [18]. This is illustrated in Fig. 1 where we take $[H_0]_{kl}=k\delta_{kl}$, resulting in a constant average level density for the instantaneous spectra, $E_n(X)$, of $H(X)$. In Fig. 1 (top), we use a Gaussian correlator $G(x)=\exp(-x^2/2)$, and in Fig. 1 (bottom), an exponential $G(x)=\exp(-|x|)$. Notice that the conventional adiabatic limit does not exist for the exponential, as the individual energy levels undergo Brownian motion on short distance scales. (Such nonsmooth paths are encountered in the quantum treatment of the slow dynamics in the path integral formulation. In that case one has to perform a summation over nondifferentiable paths $X(t)$, due to quantum fluctuations.) The spectra in Fig. 1 are intended only to give an intuitive picture about the character of the Hamiltonian. In our approach we never diagonalize the instantaneous Hamiltonian $H_1(X)$, but solve the corresponding time dependent Schrödinger equation. The approximation we consider is the leading order in $1/N_0$ [12]. In this limit of large N_0, our approach can deal with any type of correlation function $G(x)$ and a well defined adiabatic limit exists even in the case of exponential correlations.

The time evolution of the fast subsystem is found by solving the time dependent Schrödinger equation

$$\psi(t)=T\exp\left[-\frac{i}{\hbar}\int_0^t ds\,H(X(s))\right]\psi(0)=\mathcal{U}(t)\psi(0), \tag{14}$$

where T is the time ordering operator, and $\mathcal{U}(t)$ the propagator. (We assume that the initial state $\psi(0)$ is uncorrelated with the Hamiltonian $H_1(X(t))$ at later times; correlated initial conditions have been discussed elsewhere [12].) Using Eqs. (12) and resumming all leading order diagrams in the perturbation expansion of $\mathcal{U}(t)$ in the limit $N_0\gg 1$, one can show that the average propagator $U(t)=\overline{\mathcal{U}(t)}$ is diagonal in the representation we have chosen and its diagonal matrix elements satisfy the following system of coupled integral equations [12]:

$$\begin{aligned}U_k(t)=&\exp(-i\varepsilon_k t/\hbar)-\frac{1}{\hbar^2}\int_0^t ds_1\int_0^{s_1} ds_2\\&\times\exp(-i\varepsilon_k s_2/\hbar)U_k(t-s_1)\\&\times\sum_{n=1}^{N}\mathcal{G}_{kn}(X(s_1)-X(s_2))U_n(s_1-s_2).\end{aligned} \tag{15}$$

In order to compute averages of observables, we need to introduce the set of generalized occupation number probabilities

$$\begin{aligned}\mathcal{N}_k(t_1,t_2)&=\overline{\langle\psi(t_1)|k\rangle\langle k|\psi(t_2)\rangle}\\&=\sum_l\overline{\langle l|\mathcal{U}^{\dagger}(t_1)|k\rangle\langle k|\mathcal{U}(t_2)|l\rangle}n_l(0),\end{aligned} \tag{16}$$

where $n_l(t)\equiv\mathcal{N}_l(t,t)$ is the occupation probability of the state $|l\rangle$. $\mathcal{N}_k(t_1,t_2)$ satisfy the following set of integral equations:

$$\begin{aligned}\mathcal{N}_k(t_1,t_2)=&U_k^*(t_1)U_k(t_2)n_k(0)\\&+\frac{1}{\hbar^2}\int_0^{t_1}ds_1\int_0^{t_2}ds_2\sum_l\mathcal{N}_l(s_1,s_2)\\&\times\mathcal{G}_{lk}(s_1-s_2)U_k^*(t_1-s_1)U_k(t_2-s_2).\end{aligned} \tag{17}$$

These equations specify the time evolution of the system, and below we shall study the numerical solutions of Eqs. (15) and (17) and the velocity dependence of the diffusion constant, and in the next section, the extension of the formalism to the regime $1\ll N_0<N\to\infty$, where we find analytic limits and a great simplification of the formalism.

Constant level density

Consider the situation of a constant average level density ($[H_0]_{kl}=k\delta_{kl}$, $-N/2\leqslant k$, $l\leqslant N/2$), as in the case of a two-dimensional stadium billiard. Equations (15)–(17) have been solved numerically for $N=101$, a bandwidth $N_0=21$, a Gaussian correlation $G(x)=\exp(-x^2/2)$ with $X_0=1$, and initial conditions $n_k(0)=\delta_{k,0}$. (Hereafter, in all numerical results we show we take $\hbar=1$ and express V_0 in units of X_0/T_0.) The resulting occupation numbers $n_k(t)$ are shown

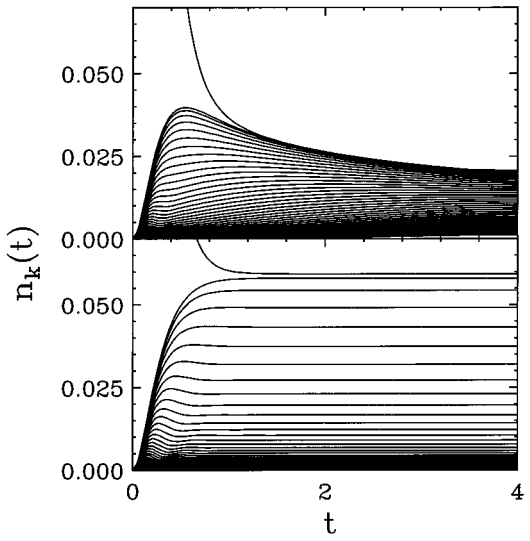

FIG. 2. The time dependence of the occupation probabilities $n_k(t)$, for $k=0,\ldots,50$ [in this case $n_k(t)=n_{-k}(t)$], where k counts from top to bottom in the figure, for the case of fast, $V_0=4$ (top), and slow, $V_0=1/16$ (bottom), driving velocities. The units of lengths are defined as $X_0=1$ and that of time follows from $\hbar=1$ and $\rho_0=1$.

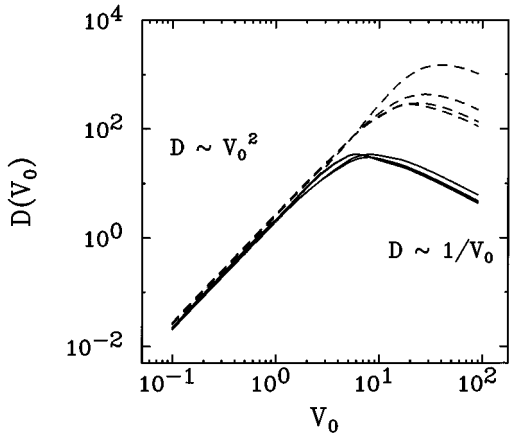

FIG. 3. Velocity dependence of the diffusion constant $D(V_0)$ from the adiabatic to the diabatic limits. We use $X_0=1$ (which thus sets the unit of length), $W_0\rho_0=1$ (ρ_0 thus defines the energy units), $\kappa_0\rho_0=5$ (the lowest four curves) and $\kappa_0\rho_0=15$ (the highest four curves). The curves correspond upwardly to $\beta/\rho_0=0.0$, 0.05, 0.1, and 0.2, respectively.

in Fig. 2 for the cases of fast ($V_0=4$, top) and slow ($V_0=1/16$, bottom) driving velocities. As the results are symmetric with respect to the index k, $n_k(t)=n_{-k}(t)$, only levels with $k=0-50$ are shown, counting from the top of the figure. One might expect that even a small driving velocity would result in a complicated time evolution, as the Hamiltonian is time dependent and has many small gaps in the instantaneous spectrum, where Landau-Zener transitions might be expected to occur and thus ''irreversibility'' is induced [1]. Actually, as we have discussed at length in Ref. [12], this mechanism, which has been advocated in many previous treatments [2], is valid only for isolated level crossings and thus is unrealistic when there are many nonisolated ones as shown in Fig. 1.

In Fig. 2, one can distinguish two time scales: a relatively rapid initial transient behavior, followed by a slower evolution. While the initial transient is almost identical in both cases of fast and slow motion, governed by the same spreading width $\Gamma^{\downarrow}$, the long time behavior is strikingly different. For small driving velocities, the time evolution apparently rapidly equilibrates, and can be understood in terms of the $V_0\to 0$ limit, corresponding to constant random matrix theory. For large velocities there is a steady evolution to a different probability distribution. The initial transient behavior arises only because our initial occupation probabilities $n_k(0)$ do not correspond to an instantaneous eigenstate of $H(0)$. The subsequent long time behavior is due to the explicit time dependence of the Hamiltonian $H(t)$ and would be absent for a time independent one.

The diffusion process associated with these time evolutions can be characterized by the energy variance $\Delta_E(t)$ and the energy diffusion constant $D(V_0)$, defined by

$$\Delta_E^2(t)=\overline{\langle\psi(t)|[H(t)-E(t)]^2|\psi(t)\rangle}\underset{t\to\infty}{\sim}\text{const}+2D(V_0)t, \tag{18}$$

where $E(t)=\overline{\langle\psi(t)|H(t)|\psi(t)\rangle}$. It is important to note that the energy variance is time dependent only for a time dependent Hamiltonian. For example, if we were to consider the case $V_0\equiv 0$ one would still have the initial transient time evolution of the occupation number probabilities we have mentioned above. However, there will be no time dependence of the energy variance in such a case. This is a trivial statement for the time evolution of a given time independent Hamiltonian. The fact that this feature is preserved after performing the statistical average in the $1/N$-leading approximation we use throughout this work, is one more consistency check, which can be proven rigorously.

In Fig. 3, $D(V_0)$ can be seen to exhibit a quadratic velocity dependence in the adiabatic limit, in contradiction to numerous previous claims [2]. In the case we consider here of a symmetrical initial distribution $n_k(0)$ and constant level density, the average energy $E(t)$ is time independent, hence the reaction force on the slow system, in particular, the friction force, exactly vanishes. This is consistent in a somewhat trivial way with a fluctuation-dissipation theorem in the following sense. Expressed as $\gamma=\beta D$, where γ is the ''friction'' coefficient and $\beta=1/T=d\ln\rho(\varepsilon)/d\varepsilon\equiv 0$ is the inverse thermodynamic temperature, we have the expected result $dE(t)/dt=\gamma\equiv 0$. (Note that the rate of energy loss for the slow subsystem is $-dE(t)/dt=-\gamma$.) When H is time independent, both $\Delta_E(t)$ and $E(t)$ are also time independent. The absence of ''friction'' in this case is relatively easy to understand. The initial occupied state was chosen in the middle of the spectrum, in order to minimize the spectrum edge effects. Since the average level density is constant, there are an equal number of levels above and below the initially occupied state. This in conjunction with the fact that the Hamiltonian is symmetric leads to this trivial behavior. As soon as we consider realistic systems with increasing level densities, the fast subsystem will heat up.

IV. CHARACTERISTIC FUNCTIONAL APPROACH

While the coupled dynamics of the occupation number probabilities becomes an increasingly difficult computational

problem for large N, it is possible to take an additional limit $N\to\infty$ and obtain some major simplifications of the formalism. For this purpose, let us introduce the characteristic functional

$$\mathcal{N}(t_1,t_2,\tau)=\overline{\langle\psi(t_1)|\exp\left[\frac{iH_0(\tau-t_1+t_2)}{\hbar}\right]|\psi(t_2)\rangle}$$
$$=\int d\varepsilon_k\rho(\varepsilon_k)\mathcal{N}_k(t_1,t_2)\exp\left[\frac{i\varepsilon_k(\tau-t_1+t_2)}{\hbar}\right]. \tag{19}$$

This is essentially the Fourier transform of the generalized occupation numbers (16) and (17) with respect to the index k

$$\mathcal{N}_k(t_1,t_2)=\frac{1}{2\pi\hbar\rho(\varepsilon_k)}\int d\tau\mathcal{N}(t_1,t_2,\tau)$$
$$\times\exp\left[-\frac{i\varepsilon_k(\tau-t_1+t_2)}{\hbar}\right]. \tag{20}$$

The equal time functional $\mathcal{N}(t,t,\tau)$ is correspondingly the Fourier transform of the occupation number probabilities $n_k(t)$. The distribution of the occupation number probabilities can be defined in terms of a cumulant expansion

$$\mathcal{N}(t,t,\tau)=\exp\left[\sum_n\overline{\langle\langle\psi(t)|H_0^n|\psi(t)\rangle\rangle}\frac{(i\tau)^n}{\hbar^n n!}\right], \tag{21}$$

where $\overline{\langle\langle\psi(t)|H_0^n|\psi(t)\rangle\rangle}$ are the cumulants. If we assume an initial condition consisting of the occupation of a single state $n_0(0)=1$ and $n_j(0)=n_{-j}(0)=0$ for $j\neq0$, then we find that for a system with a realistic level density of the form $\rho(\varepsilon)=\rho_0\exp(\beta\varepsilon)$ the propagator $\sigma(t)=\exp(i\varepsilon_k t/\hbar)U_k(t)$ [note $\sigma(t)$ is state independent] and $\mathcal{N}(t_1,t_2,\tau)$ satisfy the evolution equations

$$\sigma(t)=1-\frac{2\pi W_0}{\hbar}\int_0^t ds_1\int_0^{s_1}ds_2\sigma(s_1-s_2)\sigma(s_2)$$
$$\times P(s_1-s_2)G\left(\frac{(s_1-s_2)V_0}{X_0}\right), \tag{22}$$

$$\mathcal{N}(t_1,t_2,\tau)=\sigma^*(t_1)\sigma(t_2)+\frac{2\pi W_0}{\hbar}\int_0^{t_1}ds_1\int_0^{t_2}ds_2$$
$$\times\mathcal{N}(s_1,s_2,\tau)P(s_1-s_2-\tau)G\left(\frac{(s_1-s_2)V_0}{X_0}\right)$$
$$\times\sigma^*(t_1-s_1)\sigma(t_2-s_2), \tag{23}$$

with the correlator $\mathcal{G}_{ij}(X)$ of the form of Eq. (13) and $P(s)$ is given by

$$P(s)=P^*(-s)=\frac{\kappa_0}{\sqrt{2\pi}\hbar}\exp\left[-\frac{\kappa_0^2}{2\hbar^2}\left(s+i\frac{\hbar\beta}{2}\right)^2\right]. \tag{24}$$

The case $\beta=0$ corresponds to the situation we have analysed in the preceding section, of constant average level density, while the case of finite β approximates a many fermion system.

The great advantage of this form of the evolution equations is that one has to solve only one equation at a time for each value of the parameter τ, instead of N-coupled equations [compare Eq. (23) with Eqs. (17)], which is a significant simplification. Moreover, various analytic solutions can be obtained, as we exemplify below, by analyzing the adiabatic and the diabatic evolutions of the occupation numbers.

A. Adiabatic limit

It is useful to introduce two time scales: (i) the characteristic time scale for the slow motion $\tau_{\rm slow}=X_0/V_0$ and (ii) the characteristic time scale for the fast degrees of freedom, $\tau_{\rm fast}=\hbar/\kappa_0$. The *adiabatic limit* corresponds to $\tau_{\rm slow}=X_0/V_0\gg\tau_{\rm fast}=\hbar/\kappa_0$. If the condition $\kappa_0\beta\ll1$ is also fulfilled (this condition is essentially equivalent to having $\beta=0$), one can replace $P(s)$ by an appropriately chosen Dirac δ function in the equations for the propagator $\sigma(t)$ and the generalized occupation number probabilities $\mathcal{N}(t_1,t_2,\tau)$. Then the equation for the propagator $\sigma(t)$ can be solved and the solution is

$$\sigma(t)=\exp\left(-\frac{\pi W_0 t}{\hbar}\right). \tag{25}$$

Even though this expression satisfies the initial condition $\sigma(0)=1$, it is not valid for very small times $t\simeq O(\tau_{\rm fast})$ [12]. During times of the order of $\tau_{\rm fast}$ the function $P(s)$ cannot be approximated by a Dirac δ function, these time intervals within this approximation being effectively compressed to zero. One can now establish that for $t>\tau>0$

$$\mathcal{N}(t,t,\tau)=\exp\left\{-\frac{2\pi W_0 t}{\hbar}\right\}+\frac{2\pi W_0}{\hbar}$$
$$\times\exp\left\{-\frac{\pi W_0\tau}{\hbar}\right\}G\left(\frac{V_0\tau}{X_0}\right)$$
$$\times\int_\tau^t ds\exp\left\{-\frac{2\pi W_0(t-s)}{\hbar}\right\}$$
$$\times\mathcal{N}(s,s-\tau,\tau), \tag{26}$$

$$\mathcal{N}(t,t-\tau,\tau)=\exp\left\{-\frac{2\pi W_0 t}{\hbar}+\frac{\pi W_0\tau}{\hbar}\right\}+\frac{2\pi W_0}{\hbar}G\left(\frac{V_0\tau}{X_0}\right)$$
$$\times\int_\tau^t ds\exp\left\{-\frac{2\pi W_0(t-s)}{\hbar}\right\}\mathcal{N}(s,s-\tau,\tau). \tag{27}$$

The second of these equations can be reduced to a linear homogeneous equation of first degree for $\mathcal{N}(t,t-\tau,\tau)$, by taking the time derivative, and can be easily solved. The generalized occupation number probabilities $\mathcal{N}(t,t,\tau)$ can then be directly computed by integration, after which one obtains (for $t\geq|\tau|$)

$$\mathcal{N}(t,t,\tau)=\exp\left\{-\frac{2\pi W_0}{\hbar}\left[1-G\left(\frac{\tau V_0}{X_0}\right)\right](t-|\tau|)-\frac{2\pi W_0|\tau|}{\hbar}\right\}. \tag{28}$$

In the adiabatic limit $G(\tau V_0/X_0)\to 1$ and the first term in the exponential vanishes. The occupation numbers $n_k(t)$ after the initial transient time interval seen in Figs. 2 are then found to be

$$n_k=\frac{W_0}{\varepsilon_k^2+\pi^2 W_0^2}. \tag{29}$$

This Lorentzian shape is identical with the constant random matrix theory result, see Ref. [12], with the identification $\Gamma^{\downarrow}=2\pi W_0$. In the adiabatic limit, during a time $t\approx\tau_{\rm fast}$ the slow variables hardly change and thus the dynamics of the fast system is almost identical to the dynamics governed by a constant random Hamiltonian. Our initial state in the middle of the spectrum, chosen as $n_0(t=0)=1$, is spread over an energy interval $\approx\Gamma^{\downarrow}=2\pi W_0$ and the distribution has a Lorentzian shape. If the Hamiltonian is time independent, after this time there would be essentially no further evolution of the average occupation number probabilities. The subsequent dynamical evolution of the fast system occurs only because the Hamiltonian $H(X(t))$ is time dependent, and only the subsequent time evolution of the system leads to dissipation and entropy production in the long time limit.

One can now explicitly evaluate the cumulants $\overline{\langle\langle\psi(t)|H_0^n|\psi(t)\rangle\rangle}$. All odd moments of H_0 vanish identically [since $G(x)=G(-x)$ and thus there are only even powers of τ in the expansion (21)]. The reason for this is our assumption that $\kappa\beta\to 0$, which we shall lift shortly below. In the limit $t\to\infty$, all even cumulants of H_0 increase linearly in time. If $G(x)=\exp(-x^2/2)$ (we shall use this form hereafter for illustrative purposes) then in the limit $t\to\infty$

$$\overline{\langle\langle\psi(t)|H_0^{2n}|\psi(t)\rangle\rangle}=\frac{2\pi W_0 t}{\hbar}\left(\frac{\hbar V_0}{X_0}\right)^{2n}\frac{(2n)!}{2^n n!},$$

$$D(V_0)=\frac{\pi\hbar W_0 V_0^2}{X_0^2}=\left[\frac{\hbar\Gamma^{\downarrow}}{2X_0^2}\right]V_0^2=\left[\frac{\hbar\Gamma^{\downarrow}}{2\tau_{\rm slow}^2}\right] \tag{30}$$

$(\Gamma^{\downarrow}=2\pi W_0)$ resulting in a non-Gaussian distribution. A Gaussian process would have only the first two cumulants nonvanishing. The diffusion constant is extracted from the time dependence of the second cumulant according to Eq. (18).

These solutions for the propagator and the generalized occupation number probabilities are remarkable in several respects. We have obtained them for an arbitrary $G(x)$ in an analytical form. Since the dynamical evolution of the fast system is much faster than the slow motion, the propagator in this approximation in insensitive to the form of $G(x)$. Hence, the propagator will have the same expression even when the time dependence of the slow degrees of freedom is arbitrary, not necessarily $X(t)=V_0 t$, as we have assumed so far. In particular, noting that the influence functional is given by

$$\mathcal{L}(t)\equiv\mathcal{N}(X(t),X'(t),0), \tag{31}$$

we see that it satisfies a relatively simple evolution equation

$$\mathcal{L}(t)=\exp\left\{-\frac{2\pi W_0 t}{\hbar}\right\}+\frac{2\pi W_0}{\hbar}\times\int_0^t ds\exp\left\{-\frac{2\pi W_0(t-s)}{\hbar}\right\}G\left(\frac{X(s)-X'(s)}{X_0}\right)\mathcal{L}(s). \tag{32}$$

Note also that the ''left'' $X(t)$ and ''right'' $X'(t)$ trajectories are different. For $X(t)\equiv X'(t)$, this influence functional satisfies the identity $\mathcal{L}(t)\equiv 1$ as required. Solving the integral equation leads to

$$\mathcal{L}(t)=\exp\left\{-\frac{2\pi W_0}{\hbar}\int_0^t\left[1-G\left(\frac{X(s)-X'(s)}{X_0}\right)\right]ds\right\}. \tag{33}$$

It should be clear that in this case one can use a correlator $\mathcal{G}_{ij}(X,Y)$ of a general form (therefore relaxing the ''translational invariance'' we have assumed to this point).

Notice that, as the result of the symmetric initial distribution and $\beta=0$, there is no friction (imaginary components) in the above influence functional. To get friction, consider the next order corrections to the adiabatic limit $\kappa\beta\ll 1$. In this limit, we replace the function $P(s)$ in Eqs. (22) and (23) with

$$P(s)\to\delta\left(s+\frac{i\beta\hbar}{2}\right). \tag{34}$$

The evolution equation for the propagator, and therefore its solution, remains unchanged up to first order corrections in β. The equation for the generalized occupation number probabilities, however, now reads

$$\mathcal{N}(t_1,t_2,\tau)=\exp\left[-\frac{\pi W_0(t_1+t_2)}{\hbar}\right]+\frac{2\pi W_0}{\hbar}\int_0^{t_1}ds_1\int_0^{t_2}ds_2\mathcal{N}(s_1,s_2,\tau)\times\delta\left(s_1-s_2-\tau+\frac{i\beta\hbar}{2}\right)G\left(\frac{X(s_1)-X'(s_2)}{X_0}\right)\times\exp\left[-\frac{\pi W_0(t_1-s_1+t_2-s_2)}{\hbar}\right]. \tag{35}$$

By analytically continuing $G[(X(s_1)-X'(s_2))/X_0]$, and extending the integration contour into complex time plane, this equation can be solved following the same steps outlined above. When $X(t)=X'(t)$ one thus obtains that

$$\mathcal{N}(t,t,\tau)=\exp\left\{\frac{\pi W_0}{\hbar}\int_0^t ds\frac{\dot{X}^2(s)}{X_0^2}\left(i\tau+\frac{\hbar\beta}{2}\right)^2\right\}. \tag{36}$$

This solution is valid only if $t\gg\tau$ and only in the first order in β and second order in τ. It follows then that:

$$\langle\langle\psi(t)|H_0|\psi(t)\rangle\rangle=\frac{1}{2}\beta\langle\langle\psi(t)|H_0^2|\psi(t)\rangle\rangle$$
$$=\frac{\beta\pi\hbar W_0}{X_0^2}\int_0^t ds\dot{X}(s)^2 \tag{37}$$

which is an integral from of the fluctuation-dissipation theorem. However, since the higher cumulants are nonvanishing, this dynamics is certainly non-Gaussian. Consider again the case of constant velocity, $X(t)=V_0t$. After similar manipulations of Eqs. (22) and (23) one obtains for $\beta>0$ and $t\to\infty$ that

$$\mathcal{N}(t,t,\tau)=\exp\left\{\frac{2\pi W_0}{\hbar}\left[G\left(\frac{\tau V_0}{X_0}-\frac{i\hbar\beta V_0}{2X_0}\right)\right.\right.$$
$$\left.\left.-G\left(-\frac{i\hbar\beta V_0}{2X_0}\right)\right]t\right\}. \tag{38}$$

Once again, this formula is correct up to first order in β only. Since for $\beta>0$ the average level density is increasing with energy, there are on the average more transitions upward in energy than downward and hence the fast subsystem is heated. For this reason the odd cumulants $\langle\langle\psi(t)|H_0^n|\psi(t)\rangle\rangle$ are nonvanishing, and ''friction'' is present since $\langle\langle\psi(t)|H_0|\psi(t)\rangle\rangle>0$. From Eq. (38) it follows that the odd cumulants are given by the following expressions:

$$\langle\langle\psi(t)|H_0^{2n-1}|\psi(t)\rangle\rangle=\frac{\beta}{2}\langle\langle\psi(t)|H_0^{2n}|\psi(t)\rangle\rangle. \tag{39}$$

The case $n=1$ corresponds to the Einstein fluctuation-dissipation theorem.

B. Diabatic limit

Another simple solution can be obtained in the *diabatic limit*, when $\tau_{\rm slow}=X_0/V_0\ll\tau_{\rm fast}=\hbar/\kappa_0$. Then it is reasonable to replace the correlator $G(s)$ by an appropriately chosen Dirac δ function, namely,

$$G\left(\frac{(s_1-s_2)V_0}{X_0}\right)\to\delta(s_1-s_2)\frac{V_0}{X_0}\int_{-\infty}^{\infty}ds\,G(s). \tag{40}$$

One can proceed now in a similar fashion as in the adiabatic limit and determine the propagator $\sigma(t)$ and subsequently the generalized occupation probabilities. After some lengthy algebra, we find

$$\mathcal{N}(t,t,\tau)=\exp\left\{-\frac{2\pi X_0W_0\kappa_0}{\hbar^2V_0}\left[\exp\left(\frac{\kappa_0^2\beta^2}{8}\right)\right.\right.$$
$$\left.\left.-\exp\left(\frac{\kappa_0^2}{2}\left(\frac{\beta}{2}+\frac{i\tau}{\hbar}\right)^2\right)\right]t\right\}. \tag{41}$$

This is similar to the functional form found in the adiabatic limit. Again, all the cumulants of H_0 increase linearly in time

$$\overline{\langle\langle\psi(t)|H_0^n|\psi(t)\rangle\rangle}=\left[\frac{2\pi X_0W_0\kappa_0}{\hbar^2V_0}\exp\left(\frac{\beta^2\kappa_0^2}{8}\right)\left(\frac{i\kappa_0}{\sqrt{2}}\right)^n\right.$$
$$\left.\times H_n\left(-\frac{i\kappa_0\beta}{2\sqrt{2}}\right)\right]t, \tag{42}$$

where $H_n(x)$ are Hermite polynomials, resulting in a non-Gaussian diffusion of the occupation numbers. From Eq. (18) and the second cumulant we find in this limit a completely different velocity dependence

$$D(V_0)=\left[\frac{\Gamma^{\downarrow}X_0\kappa_0^3(\beta^2\kappa_0^2+4)}{8\hbar^2}\exp\left(\frac{\beta^2\kappa_0^2}{8}\right)\right]\frac{1}{V_0}. \tag{43}$$

It is worth noting that in Eq. (18) we use the full Hamiltonian for the fast system, namely, $H(t)=H_0+H_1(t)$, in order to extract the diffusion constant $D(V_0)$, while in this section we only use moments of H_0 for the same purpose. In the $t\to\infty$ limit these two ways of determining $D(V_0)$ lead to identical results. The fluctuation-dissipation theorem, obtained from the first and second cumulants is

$$\beta D=\gamma\left(1+\frac{\beta^2\kappa_0^2}{4}\right), \tag{44}$$

showing that $\beta D=\gamma$ is not generally satisfied.

C. Numerical results for arbitrary velocities

For arbitrary velocities we have to resort to a numerical solution of Eqs. (22) and (23). As we have discussed above, these equations correspond to a Hamiltonian with an infinite number of levels and we have solved them for a variety of parameter values and several typical results are presented here. In Fig. 3 we show the behavior of the diffusion constant, $D(V_0)$, from the adiabatic [Eq. (30)] to the diabatic [Eq. (43)] limit for some values of the parameter β. In all cases, $D(V_0)$ evolves from quadratic (in the adiabatic limit) to inverse velocity dependence (in the diabatic limit). At high velocities, the system becomes increasingly less opaque, as reflected in the decreasing diffusion constant. A similar behavior is observed for the first cumulant, i.e., the average rate of heating, which as a function of the velocity V_0 has a similar aspect with $D(V_0)$. This is reminiscent of the motional narrowing phenomenon in NMR. At low enough velocities the fast system has almost sufficient time to ''accommodate'' to the new environment, while the ''shape'' X changes. In the opposite limit of high velocities, the ''shape'' X evolves so rapidly that the system can barely react to the changes. Consequently, the energy diffusion is maximal only for some intermediate velocity regime, when the ''slow'' motion is in ''resonance'' with the ''fast'' dynamics, namely, when $\tau_{\rm slow}=X_0/V_0$ is comparable to $\tau_{\rm fast}=\hbar/\kappa_0$. In Fig. 4, we plot the ratio of $\beta D/\gamma$, where D and γ are computed from the first and second order cumulants as a function of velocity V_0. When the ratio is unity, the Einstein form of the fluctuation-dissipation theorem is found. Noticeable differences occur at large velocities.

The behavior of $D(V_0)$ and the crossover from the adiabatic to the diabatic velocity dependence can be seen in a

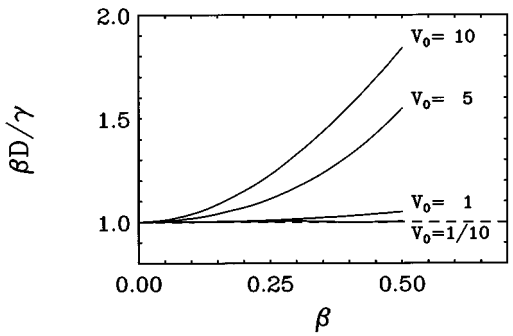

FIG. 4. Deviation from the fluctuation-dissipation theorem as a function of $\beta=1/T$. For low velocities, the theorem is largely satisfied. However, for large velocities it is violated even at moderate temperatures. Here we use $X_0=1$, $W_0\rho_0=1$, and $\kappa_0\rho_0=5$.

simple parametrization of the results in Fig. 3. By combining the results for the constant level density situation, we find that

$$D(V_0)=\frac{\Gamma^{\downarrow}\kappa_0^2}{2\hbar}\frac{(V_0/V_c)^2}{1+(V_0/V_c)^3} \tag{45}$$

where the critical velocity V_c is defined as

$$V_c=\frac{X_0\kappa_0}{\hbar}. \tag{46}$$

A similar formula is easily obtained for the case of nonconstant level density ($\beta\neq 0$), in which case V_c acquires a β dependence as well. From this form of the diffusion constant, one can see that the crossover occurs for velocities near the critical velocity $V_0\sim V_c$ as argued above.

In Fig. 5 we show the first four cumulants

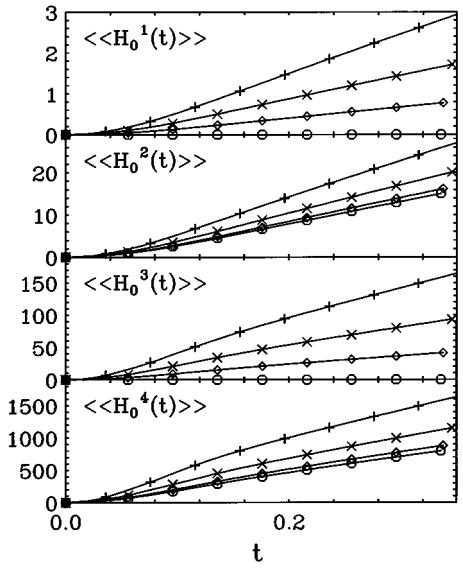

FIG. 5. The first four cumulants $\langle\langle\psi(t)|H_0^n|\psi(t)\rangle\rangle$ as a function of time for $X_0=1$, $W_0\rho_0=1$, $\kappa_0\rho_0=5$, and $V_0=10$. The solid lines are there as a guide. The curves correspond to $\beta/\rho_0=0$ (o), 0.1 ($\diamond$), 0.2($\times$), and 0.3($+$). For small values of β one finds a very good agreement with Eq. (39) as expected.

$\langle\langle\psi(t)|H_0^n|\psi(t)\rangle\rangle$ as functions of time for several average level densities $\rho(\varepsilon)=\rho_0\exp(\beta\varepsilon)$. The velocity V_0 and range of time have been chosen such that $V_0t>X_0$ and thus the asymptotic behavior already sets in. These values of the parameters are approximately midway between the adiabatic and diabatic limits. In spite of this, the relations between the cumulants, Eqs. (39), are satisfied to good accuracy. In particular, as one can also see in Fig. 4, the expected relation following from the Einstein fluctuation-dissipation theorem between the friction and diffusion coefficients is satisfied reasonably well for $\beta/\rho_0\leqslant 0.1$. As we have discussed above, the large values of the cumulants beyond the second one are indicative of long tails in the occupation number probability distributions. As in the adiabatic and diabatic cases, these cumulants increase approximately linearly with time.

V. CONCLUSION

We have presented numerical and analytical solutions of the time dependent evolution equations of a driven complex quantum system, such as a nucleus, when the level density is large. The parametric random matrix approach chosen here incorporates the essential attributes of the intrinsic dynamics: an exponentially increasing level density, GOE spectral fluctuations and loss of correlations during large amplitude collective motion. We have shown that the resulting energy diffusion process is non-Gaussian in character, that the energy distribution has long tails and determined the nontrivial velocity dependence of the energy diffusion constant, which vanishes for $V_0\rightarrow 0$ and $V_0\rightarrow\infty$.

The present approach treats the fast subsystem quantum mechanically and the slow subsystem classically, as has been done often in the past [2]. As we have discussed above however, the inclusion of quantum effects into the slow motion is now possible. The energy diffusion process is described in terms of intrinsic characteristics of the many-body system (thermodynamic temperature β, spreading width $\Gamma^{\downarrow}$, κ_0 and X_0) and V_0. It is notable that the average level spacing or average level density are absent in these results, contrary to what one might have expected. As it was noted in Ref. [12] also, the presence of the average level density or average level spacing in an expression for the diffusion constant is unreasonable, since this would lead to an exponential dependence of the diffusion and friction coefficients with the size of the system. The average level spacing decreases exponentially with the number of particles, since its natural logarithm is proportional to the negative of the entropy of the system, which is an extensive quantity.

It is not clear yet whether the intrinsic characteristics $\Gamma^{\downarrow}$, κ_0 and X_0 have a meaningful classical limit separately or only in a given combination, and this seemingly points to an apparent lack of a classical limit for the fast degrees of freedom ($\hbar\rightarrow 0$) of the solutions (30), (43). In Ref. [10], friction was obtained only in a classical treatment of both fast and slow system, while in Ref. [6], dissipation and friction appear only in an explicit quantum treatment (path integral) of the entire system and the presence of quantum fluctuations in the slow subsystem was essential. The wall formula [7] leads to a diffusion constant $D\propto V_0^2$ as we have obtained here for small V_0 [see Fig. 3 and Eq. (30)], but is essentially a classical result, which does not depend in any significant way on

$\hbar$, and apparently reflects a different underlying mechanism. It will be highly desirable to identify the classical limit of the present approach.

The most salient feature of the solutions (28) and (38) becomes evident when one considers the asymptotic behavior of the cumulants. Since cumulants of higher than second order are nonvanishing, Gaussian processes are not obtained in any of these limiting cases for the energy diffusion. As a result the distribution of occupation numbers has very long tails. In particular, for $V_0 \equiv 0$ the distribution corresponding to Eq. (28) has a Lorentzian shape, as might be expected from constant random matrix theory. These features imply that a Langevin or Fokker-Planck approach to energy dissipation is at least questionable. As we have discussed in Ref. [12] these results apply equally to the GUE case. One might be surprised by this result, since the frequency of small level separations for a GOE Hamiltonian is larger than for GUE one and thus one would expect that in the adiabatic limit there should be more transitions for GOE than for GUE case as would be the case if the energy level separation distribution and the Landau-Zener mechanism were the dominant factors. However, the Landau-Zener formula has a limited applicability as one can easily ascertain from the following simple example. Imagine a spin 1/2 in a magnetic field, which changes its direction in some arbitrary manner. Since the magnitude of the magnetic field is constant the splitting between the two instantaneous levels is time independent. Now change the direction of the field very slowly and very rapidly and consider that initially the spin was in the lower level. According to the Landau-Zener formula there will be no transitions in either case. It should be obvious however that there will be transitions from the lower to the upper level and the intensity of these transitions depends on the rate of change of the direction of the magnetic field.

One interesting aspect of our analysis is the form of the influence functional. From Eq. (8) we do not expect the equation for the reduced density matrix to be local in time. Rather we expect

$$i\hbar\partial_t \rho(X,X',t) = [H_0(X) - H_0(X')]\rho(X,X',t) + \int_{t_0}^{t} dt' V(X,X',t,t')\rho(X,X',t'). \tag{47}$$

Even though memory effects are explicitly present in the evolution equations for the fast subsystem, they vanish in the equation for the slow subsystem, as seen in the explicit solutions presented here [e.g., Eq. (33)]. This is true in the adiabatic limit, which is perhaps of most interest. In this respect, our approach is qualitatively different from the earlier and more familiar approach in condensed matter physics of Caldeira and Leggett [13], based on the form for the influence functional for an infinite number of oscillators [5].

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