PHYSICAL REVIEW E VOLUME 54, NUMBER 2 AUGUST 1996

Dynamical evolution of squeezing and antibunching effects in a quantum chaotic system: The three-level Lipkin model

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(Received 7 November 1995)

Time evolution of squeezing and antibunching effects in a nuclear model known as the three-level Lipkin model is analyzed. The two effects occur in the regime of classically regular motion, but disappear in the regime of classically chaotic motion, which is similar to that obtained in a spin-boson model (another quantum chaotic system). [S1063-651X(96)06207-1]

PACS number(s): 05.45.+b, 03.65.−w, 42.50.Dv

I. INTRODUCTION

The recently gained increase of understanding of classical Hamiltonian systems [1], which are nonintegrable and therefore display chaotic dynamical behavior, has led to the natural question: what is the universality of the quantum properties of such systems? During the past 15 years or so, studies have focused largely on the statistical properties of energy levels [2–5] and wave functions [2,6,7] of such systems. It has been shown that for systems whose classical dynamics is chaotic, the statistical fluctuations of energy spectra and wave functions are generally well described by the random matrix theory. Also, it was discovered that when such systems are allowed to depend on a parameter, the correlation between spectra belonging to different values of the parameter become universal upon an appropriate scaling of the parameter (see, for example, Ref. [5]). Interestingly, Mucciolo *et al.* [6] have recently proposed a universal scaling for all three Dyson ensembles, and Alhassid and Attias [7] have even established the universality of parametric correlation of eigenfunctions in chaotic and weakly disorder systems. Apart from the above studies, increased attention has also been paid to the universality of curvature distribution [8,9] and avoided crossing distribution [10,11]. In short, the available studies have indicated that systems which are classically chaotic, show a wide degree of universality in their quantum properties. Then, the main purpose of this paper is to answer the question, what are the other quantum properties of classically chaotic systems?

In a spin-boson model, which has been taken as "a physically quantum chaotic system" [12], it was found that the squeezing and antibunching effects disappear in the regime of classically chaotic motions [13]. In this paper we shall report that such quantum features are also exposed in a nuclear model when its classical counterpart is fully chaotic.

The nuclear model, known as the three-level Lipkin model [14], has been studied in the field of quantum chaos during recent years [15,16]. The Hamiltonian corresponding to the boson representation has the form

$$H=H^{(0)}+\lambda V, \tag{1}$$

$$H^{(0)}=\epsilon_1 b_1^\dagger b_1+\epsilon_2 b_2^\dagger b_2 - k_2\left(b_2^\dagger \sqrt{\Omega-\sum_i b_i^\dagger b_i}\, b_2^\dagger \sqrt{\Omega-\sum_i b_i^\dagger b_i}+\text{H.c.}\right), \tag{2}$$

$$\begin{aligned} V=&-k_1\left(b_1^\dagger \sqrt{\Omega-\sum_i b_i^\dagger b_i}\, b_1^\dagger \sqrt{\Omega-\sum_i b_i^\dagger b_i}+\text{H.c.}\right)\\ &+\mu_1\left(b_2^\dagger b_1 b_2^\dagger \sqrt{\Omega-\sum_i b_i^\dagger b_i}+\text{H.c.}\right)\\ &+\mu_2\left(b_1^\dagger b_2 b_1^\dagger \sqrt{\Omega-\sum_i b_i^\dagger b_i}+\text{H.c.}\right), \end{aligned} \tag{3}$$

where $i=1,2$. Letting particle number Ω become infinitely large and keeping the following parameters: $\epsilon_1'=\epsilon_1\Omega$, $\epsilon_2'=\epsilon_2\Omega$, $k_1'=k_1\Omega^2$, $k_2'=k_2\Omega^2$, $\mu_1'=\mu_1\Omega^2$, and $\mu_2'=\mu_2\Omega^2$ as constants, the classical counterpart of the above model is obtained [15]:

$$H_c=H_c^{(0)}+\lambda V_c, \tag{4}$$

$$H_c^{(0)}=\frac{\epsilon_1'}{2}(p_1^2+q_1^2)+\frac{\epsilon_2'}{2}(p_2^2+q_2^2) - k_2'(p_2^2-q_2^2)\left(1-\frac{p_1^2+q_1^2+p_2^2+q_2^2}{2}\right), \tag{5}$$

$$\begin{aligned} V_c=&-k_1'(p_1^2-q_1^2)\left(1-\frac{p_1^2+q_1^2+p_2^2+q_2^2}{2}\right)\\ &+\frac{\mu_1'}{\sqrt{2}}[(p_2^2-q_2^2)p_1+2p_2q_2q_1]\sqrt{1-\frac{p_1^2+q_1^2+p_2^2+q_2^2}{2}}\\ &+\frac{\mu_2'}{\sqrt{2}}[(p_1^2-q_1^2)p_2+2p_1q_1q_2]\sqrt{1-\frac{p_1^2+q_1^2+p_2^2+q_2^2}{2}}, \end{aligned} \tag{6}$$

after transforming the boson creation and annihilation operators $b_j^\dagger$, b_j to q_j and p_j with $b_j^\dagger=\sqrt{\Omega/2}(q_j-ip_j)$, $b_j=\sqrt{\Omega/2}(q_j+ip_j)$ $(j=1,2)$. Taking $\epsilon_1'=30$, $\epsilon_2'=48$, $k_1'=117$, $k_2'=189$, $\mu_1'=207$, and $\mu_2'=164.7$, it can be regular or chaotic according to the choice of the parameter λ [15]: (a) the system corresponding to $\lambda=0$ is regular; (b) the system is strongly chaotic when λ reaches 1.

Now we study the antibunching and squeezing effects in this classically chaotic quantum system. Throughout this paper, we take $\Omega=36$ [note: the Hilbert space is $M=(\Omega+1)(\Omega+2)/2=703$].

II. ANTIBUNCHING EFFECT

In quantum optics and laser physics, high-order correlation of the radiation field should be investigated in order to obtain further information and insight into the characteristics of resonance fluorescence. Experimentally, this is done using two detectors to measure the joint probability of detecting a photon at time t and a subsequent one at time $t+\tau$. A usually measured quantity is the second-order correlation function

$$g^{(2)}=\frac{\langle a^\dagger a^\dagger aa\rangle}{\langle a^\dagger a\rangle^2}, \tag{7}$$

which is proportional to the joint probability. $a^\dagger$ and a are the creation and annihilation operators of the radiation field, respectively. Then, the following phenomena are defined.

(i) For $g^{(2)}>1$, the probability of detecting a photon at $t+\tau$ is increased after detecting a photon at t, and photons tend to arrive in bunches rather than strictly at random. This phenomenon is called the positive correlation or bunching effect.

(ii) For $g^{(2)}<1$, the probability of detecting a photon at $t+\tau$ is decreased after detecting a photon at t, and photons tend to repel each other. This phenomenon is called the negative correlation or antibunching effect.

(iii) For $g^{(2)}=1$, the probability of detecting a photon at $t+\tau$ does not change after detecting a photon at t, and photons arrive at random. This phenomenon is called the noncorrelation effect.

More details about them are given in Ref. [17].

Here we introduce a similar quantity $G_i^{(2)}$,

$$G_i^{(2)}=\frac{\langle b_i^\dagger b_i^\dagger b_i b_i\rangle}{\langle b_i^\dagger b_i\rangle^2}, \tag{8}$$

in the three-level Lipkin model. For convenience in numerical calculation, we define the function

$$C_i=\langle b_i^\dagger b_i^\dagger b_i b_i\rangle-\langle b_i^\dagger b_i\rangle^2. \tag{9}$$

Then, we also call it the bunching effect, the noncorrelation effect, and the antibunching effect for $G_i^{(2)}>1$ $(C_i>0)$, $G_i^{(2)}=1$ $(C_i=0)$, and $G_i^{(2)}<1$ $(C_i<0)$, respectively. Now we take the quantity $C_1=\langle b_1^\dagger b_1^\dagger b_1 b_1\rangle-\langle b_1^\dagger b_1\rangle^2$ as an example.

First, we consider the case where the system is restricted to the following initial condition: one of the eigenstates of the Hamiltonian (1). It is clear that C_1 does not change with the time. The numerical results are presented in Fig. 1. Figure 1(a) tells us that for $\lambda=0$, the classically regular case, the antibunching effect occurs at almost all the initial states (except some states, at which $C_1=0$ and the noncorrelation effect appears). Moreover, with the increase of the parameter λ, as shown in Fig. 1(b), the number of initial states which can exhibit the antibunching effect is decreased greatly. Furthermore, when $\lambda=1$, as shown in Fig. 1(c), we find that almost all the initial states produce the bunching effect rather than the antibunching effect.

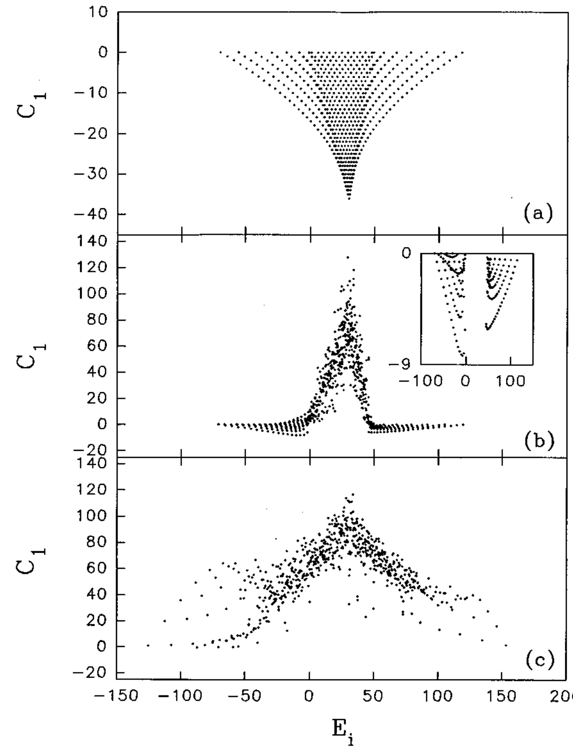

FIG. 1. C_1 versus the energy eigenvalue E_i of the Hamiltonian when the system is initially prepared in the eigenstate ϕ_i of the Hamiltonian (every point represents the result at one initial condition). (a) The regular case, $\lambda=0$; (b) the intermediate case, $\lambda=0.2$; (c) the strongly chaotic case, $\lambda=1$.

Then, in Fig. 2 we show the time evolution of C_1 when the system is initially in a coherent state [16]

$$|\Phi_c\rangle=\exp\left[\sum_i\left(Z_i b_i^\dagger\sqrt{\Omega-\sum_i b_i^\dagger b_i}\right)\right]|0,0\rangle$$

$$(i=1,2), \tag{10}$$

where $Z_i=Z_i'/\Omega$, $Z_1'=20.9$, and $Z_2'=24.7$. It is seen from Fig. 2(a) that the antibunching effect occurs at all the times in the classically regular case $(\lambda=0)$. However, the antibunching effect disappears gradually with the increase of λ. In particular, we find that in the strongly chaotic case, as shown in Fig. 2(c), the antibunching effect disappears almost entirely [except at the early time, see inset in Fig. 2(c)].

It has been shown that a regular pattern can be viewed as a quantum effect of classically Kol'mogorov-Arnol'd-Moser (KAM) tori [3,18]. This conclusion is also reflected from

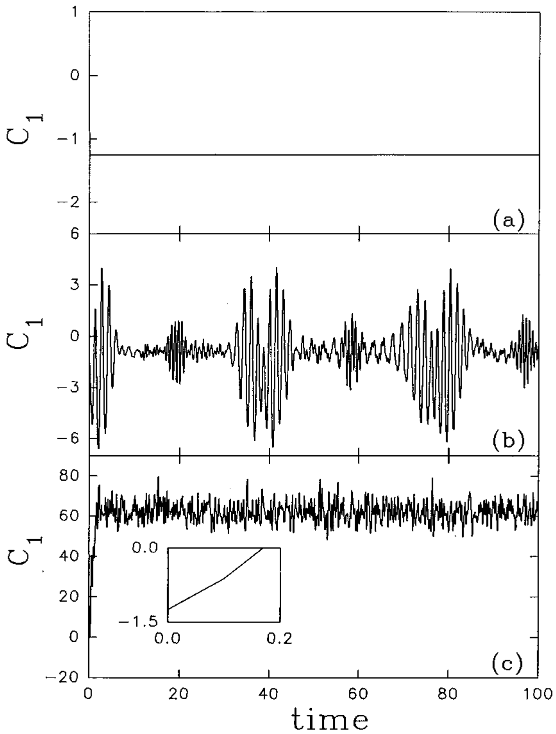

FIG. 2. The time evolution of C_1 when the system is initially prepared in a coherent state. (a) The regular case, $\lambda=0$; (b) the intermediate case, $\lambda=0.2$; (c) the strongly chaotic case, $\lambda=1$.

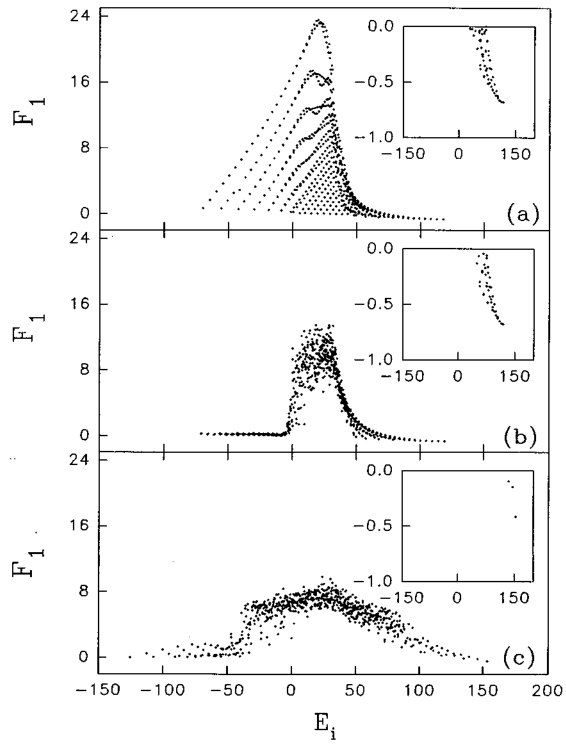

FIG. 3. F_1 versus the energy eigenvalue E_i when the system is initially prepared in the eigenstate ϕ_i of the Hamiltonian. (a) The regular case, $\lambda=0$; (b) the intermediate case, $\lambda=0.2$; (c) the strongly chaotic case, $\lambda=1$.

Fig. 1. In Fig. 1(a), since $[H^{(0)},b_1^\dagger b_1]=0$, the system $H^{(0)}$ has two independent conservative quantities, the energy and C_1. Then, as predicted in Refs. [3,18], they form a regular pattern. However, when λ grows from 0, the regular pattern shown in Fig. 1(a) is distorted. Figure 1(b) shows the intermediate case ($\lambda=0.2$): the points are distributed randomly in the middle region, while at two bottom sides a regular pattern is still observed. Furthermore, Fig. 1(c) shows that the irregular region expands to the whole area when λ is close to 1.

III. SQUEEZING EFFECT

Creating quantum states known as squeezed states, which fulfill the uncertainty relation and give a reduced uncertainty in the measurement of a particular observable at the expense of increased uncertainty in the measurement of a second noncommuting observable, has been an interesting topic for the past 15 years or so. Interest in such states is stimulated by its potential application [19] in gravity wave detection, high-resolution spectroscopy, quantum nondemolition experiments, quantum communications, and low-light-level microscopy. In order to reduce the effect of quantum fluctuations, many techniques [20] have been suggested.

Recently, Zyczkowski [21] has analyzed the time evolution of squeezed states in a quantum kicked rotator model, which has been taken as a quantum chaotic system. He found that squeezing influences the shape of quantum revivals obtained in the regime of classically regular motion, but does not facilitate the diffusion in angular momentum in the regime of classically chaotic motion.

In this paper we investigate the squeezing properties in the three-level Lipkin model (another quantum chaotic system). For this, we define two Hermitian quadrature operators

$$Q_j=\frac{1}{2}(b_j+b_j^\dagger), \quad P_j=\frac{1}{2i}(b_j-b_j^\dagger). \tag{11}$$

The above operators obey the commutation relation

$$[Q_j,P_j]=i\frac{1}{2}. \tag{12}$$

Correspondingly, the Heisenberg uncertainty relation is

$$(\Delta Q_j)^2(\Delta P_j)^2\geqslant\frac{1}{16}. \tag{13}$$

It is convenient to define the functions

$$F_1=(\Delta Q_j)^2-\frac{1}{4}, \quad F_2=(\Delta P_j)^2-\frac{1}{4}, \tag{14}$$

Then, the quantum fluctuations in Q_j (or P_j) are squeezed if $F_1<0$ (or $F_2<0$). In what follows, we take the quantum fluctuations in $Q_1=\frac{1}{2}(b_1^\dagger+b_1)$ as an example.

Figure 3 presents the numerical results when the system is initially prepared in one of the eigenstates of the Hamiltonian (1). It can be seen from Fig. 3(a) that for $\lambda=0$, the regular case, the quantum fluctuations in Q_1 can be squeezed at some initial states. Then, when $\lambda=0.2$, the intermediate case, we find from Fig. 3(b) that the number of the initial states, which can produce reduced quantum fluctuations, is decreased. Especially, when $\lambda=1$, the classically chaotic case, as shown in Fig. 3(c), only three states can produce the squeezing effect.

Finally, we have examined the case where the system is initially prepared in a coherent state given by Eqs. (10). The numerical results have shown that the squeezing effect occurs in the regime of classically regular motion, but disappears in the regime of classically chaotic motion. It should be mentioned here that in the classically chaotic case, we have also examined the quantum fluctuations in $P_1 = 1/2i\,(b_1 - b_1^\dagger)$, but they cannot be squeezed.

IV. CONCLUSIONS

In this paper we have studied the time evolution of the squeezing and antibunching effects in a three-level Lipkin model (a quantum chaotic system). The above numerical experiments have shown that the two effects occur in the regime of classically regular motion, but disappear in the regime of classically chaotic motion. This conclusion is similar to that [13] obtained in a spin-boson model (another quantum chaotic system). Is it universal in all quantum chaotic systems? We cannot give an answer. However, as far as the two models we have investigated are concerned, the answer seems likely to be yes.

Analytical explanation of the physical origin would be interesting and we plan to study this further.

ACKNOWLEDGMENT

This work was supported in part by Tianma Microelectronics Co. Ltd. in Shenzhen, China.

[1] M. Tabor, *Chaos and Integrability in Nonlinear System* (John Wiley and Sons, New York, 1989).

[2] M.C. Gutzwiller, *Chaos in Classical and Quantum Mechanics* (Springer-Verlag, New York, 1990); F. Haake, *Quantum Signature of Chaos* (Springer-Verlag, Berlin, 1991); O. Agam, B.L. Altshuler, and A.V. Andreev, Phys. Rev. Lett. **75**, 4389 (1995).

[3] L.E. Reichl, *The Transition to Chaos in Conservative Classical System: Quantum Manifestations* (Springer-Verlag, New York, 1992).

[4] J. Zakrzewski, K. Dupret, and D. Delande, Phys. Rev. Lett. **74**, 522 (1995).

[5] M. Faas, B.D. Simons, X. Zotos, and B.L. Altshuler, Phys. Rev. B **48**, 5439 (1993), and related references therein.

[6] E.R. Mucciolo, B.D. Simons, A.V. Andreev, and V.N. Prigodin, Phys. Rev. Lett. **75**, 1360 (1995).

[7] Y. Alhassid and H. Attias, Phys. Rev. Lett. **74**, 4635 (1995); H. Attias and Y. Alhassid, Phys. Rev. E **52**, 4776 (1995).

[8] P. Gaspard, S.A. Rice, and K. Nakamura, Phys. Rev. A **42**, 4015 (1990); F. von Oppen, Phys. Rev. E **51**, 2647 (1995), and related references therein.

[9] B. Li and M. Robnik (private communication).

[10] J. Zakrzewski, D. Delande, and M. Kus, Phys. Rev. E **47**, 1665 (1993).

[11] I. Guarneri, K. Zyczkowski, J. Zakrzewski, L. Molinari, and G. Casati, Phys. Rev. E **52**, 2220 (1995).

[12] M.B. Cibils, Y. Cuche, and G. Müller, Z. Phys. B **97**, 565 (1995), and related references therein.

[13] R.H. Xie, G.O. Xu, and D.H. Liu, Phys. Lett. A **202**, 28 (1995); Chin. Phys. Lett. **13**, 97 (1996).

[14] H.J. Lipkin, N. Meshkov, and A.J. Glick, Nucl. Phys. **62**, 188 (1965).

[15] G.O. Xu, J.B. Gong, W.G. Wang, Y.T. Yang, and D.J. Fu, Phys. Rev. E **51**, 1770 (1995).

[16] G.O. Xu and Q.L. Jie, Phys. Lett. A **197**, 121 (1995), and related references therein.

[17] P. Meystre and M. Sargent III, *Elements of Quantum Optics* (Springer-Verlag, Tucson, 1990), pp. 19–25, 327–330.

[18] N. Srivastava and G. Müller, Phys. Lett. A **147**, 282 (1990).

[19] Appl. Phys. B **55**, 189 (1992), special issue on quantum noise reduction, edited by E. Giacobino and C. Fabre.

[20] J.P. Dowling, G.S. Agarwal, and W.P. Schleich, Phys. Rev. A **49**, 4101 (1994); R.H. Xie, D.H. Liu, and G.O. Xu, J. Phys. B **28**, 4697 (1995), and related reference therein.

[21] K. Zyczkowski, J. Phys. A **22**, L1147 (1989).