Emittance optimization in three- and multiple-bend achromats

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The necessary condition for minimizing the emittance of the three- and multiple-bend achromat lattices is derived. For isomagnetic three- or multiple-bend achromat lattices, the minimum emittance can only be attained if the length of the dipoles is ϕ a factor of $3^{1/3}$ longer than that of outer dipoles. For the three- or multiple-bend achromat with equal length dipoles the minimum emittance can also be achieved by increasing the magnetic field of middle dipoles by a factor of $\sqrt{3}$ larger than that of outer dipoles. The minimum emittance formula for the isomagnetic three-bend achromat with equal length dipole has also been derived. [S1063-651X(96)11408-2]

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Recently, electron storage rings have been used frequently as light sources for research in condensed matter physics, chemistry, cell biology, microbiology, industrial processing, etc. For many experiments, it is desirable to use a high brightness light, which requires high brightness electron beams. The amplitudes of the betatron and synchrotron oscillations are determined by the equilibrium processes of the quantized emission of photons and the rf acceleration fields used in compensating the energy loss of the synchrotron radiation [1]. The horizontal emittance of electron beams in storage rings is given by

$$\epsilon_x = C_q \gamma^2 \frac{\langle \mathcal{H}/|\rho|^3 \rangle}{J_x \langle 1/\rho^2 \rangle}, \tag{1}$$

where $C_q = 3.84\times 10^{-13}$ m, ρ is the bending radius of the dipole, $J_x \approx 1$ is the damping partition number, and

$$\mathcal{H} = \frac{1}{\beta_x}[D^2 + (\alpha_x D + \beta_x D')^2] \tag{2}$$

is the dispersion action. Here α_x, β_x are the Courant-Snyder betatron amplitude functions, and D and D' are the dispersion function and its derivative [2].

The design of low emittance optics is to minimize $\langle \mathcal{H} \rangle / J_x$ in dipoles, where possible lattices are regularly spaced focusing and defocusing quadrupole (FODO) cells, the Chasman-Green lattice [3], and three-bend achromat (TBA), etc. [4]. FODO cells, composed of inter-spacing quadrupole and dipole magnet units, are used mostly in the collider design due to their simplicity and high packing factor. A Chasman-Green (CG) lattice is composed of cells with two dipoles to form an achromat, i.e., a zero dispersion function at both ends. Thus the CG lattice is also called the double-bend achromat (DBA). The three-bend achromat is composed of three dipoles with zero dispersion function at both ends.

Since 1980, accelerator physicists have realized that there is an achievable minimum emittance, which can serve as a guideline for realistic lattice design [5–7]. Since $\mathcal{H}$ is proportional to $L\theta^2$, where θ is the bending angle and $L=\rho\theta$ is the length of the dipole, the horizontal emittance obeys a scaling law:

$$\epsilon_x = \mathcal{F} \frac{C_q \gamma^2 \theta^3}{J_x}, \tag{3}$$

where the scaling factor $\mathcal{F}$ depends on the storage ring lattice arrangement. For the minimum emittance separate function DBA, we have $\mathcal{F}_{\mathrm{MEDBA}} = 1/4\sqrt{15}$ in small bending angle approximation [5,6]. If one removes the constraint of the achromat condition, the achievable minimum emittance factor is $\mathcal{F}_{\mathrm{ME}} = 1/12\sqrt{15}$.

It is generally believed that the achievable minimum emittance in the TBA lattice is the arithmetic mean of the minimum emittance (ME) and the minimum emittance DBA (MEDBA), i.e., $\mathcal{F}_{\mathrm{METBA}} = \frac{1}{3}(2\mathcal{F}_{\mathrm{MEDBA}} + \mathcal{F}_{\mathrm{ME}})$. The four-bend achromat (QBA) is expected to have an even smaller emittance. In reality, all existing TBA lattices have $\mathcal{F}_{\mathrm{TBA}} \geqslant 2\mathcal{F}_{\mathrm{MEDBA}}$. It is argued that this ‘‘may be simply due to the TBA being a more recent design, of which the capabilities have not yet been throughly explored’’ [8]. Understanding the fundamental limit can relieve the troubles of lattice designers.

This paper studies the theoretical minimum emittance attainable in storage rings without using wigglers or undulators. Minimum emittance can be examined through minimizing the $\langle \mathcal{H} \rangle / J_x$ function with respect to lattice functions. For separate function (dipoles without field gradient) storage rings, the dispersion function in the dipole region is given by

$$D = \rho(1-\cos\phi) + D_0\cos\phi + \rho D_0'\sin\phi,$$
$$D' = \left(1 - \frac{D_0}{\rho}\right)\sin\phi + D_0'\cos\phi, \tag{4}$$

where $\phi = s/\rho$ with $s=0$ corresponding to the entrance of the dipole, and D_0 and D_0' are the values of the dispersion function and its derivative at $s=0$, respectively. For the double-bend achromat we set $D_0=0$ and $D'_0=0$ to attain the achromatic condition. Using Eq. (4), the evolution of the $\mathcal{H}$ function in a dipole is given by [9]

$$\mathcal{H}(\phi)=\mathcal{H}_0+2(\alpha_0 D_0+\beta_0 D'_0)\sin\phi -2(\gamma_0 D_0+\alpha_0 D'_0)\rho(1-\cos\phi)+\beta_0\sin^2\phi +\gamma_0\rho^2(1-\cos\phi)^2-2\alpha_0\rho\sin\phi(1-\cos\phi), \quad (5)$$

where $\mathcal{H}_0=\gamma_0 D_0^2+2\alpha_0 D_0 D'_0+\beta_0 D'^2_0$, α_0,β_0, and γ_0 are Courant Snyder parameters at $s=0$. Averaging the $\mathcal{H}$ function in the dipole, one obtains

$$\langle\mathcal{H}\rangle=\mathcal{H}_0+(\alpha_0 D_0+\beta_0 D'_0)\theta E(\theta) -\frac{1}{3}(\gamma_0 D_0+\alpha_0 D'_0)\rho\theta^2 F(\theta)+\frac{\beta_0}{3}\theta^2 A(\theta) -\frac{\alpha_0}{4}\rho\theta^3 B(\theta)+\frac{\gamma_0}{20}\rho^2\theta^4 C(\theta). \quad (6)$$

Here θ is the bending angle of the dipole and

$$E(\theta)=\frac{2(1-\cos\theta)}{\theta^2}, \quad F(\theta)=\frac{6(\theta-\sin\theta)}{\theta^3},$$

$$A(\theta)=\frac{6\theta-3\sin 2\theta}{4\theta^3}, \quad B(\theta)=\frac{6-8\cos\theta+2\cos 2\theta}{\theta^4},$$

$$C(\theta)=\frac{30\theta-40\sin\theta+5\sin 2\theta}{\theta^5}.$$

In the small angle limit, $A\to 1$, $B\to 1$, $C\to 1$, $E\to 1$, and $F\to 1$. Using the normalized scaling parameters,

$$d_0=\frac{D_0}{L\theta}, \quad d'_0=\frac{D'_0}{\theta}, \quad \tilde{\beta}_0=\frac{\beta_0}{L}, \quad \tilde{\gamma}_0=\gamma_0 L, \quad \tilde{\alpha}_0=\alpha_0, \quad (7)$$

where $L=\rho\theta$ is the length of the dipole, the averaged $\mathcal{H}$ function is given by

$$\langle\mathcal{H}\rangle=\rho\theta^3\left\{\tilde{\gamma}_0 d_0^2+2\tilde{\alpha}_0 d_0 d'_0+\tilde{\beta}_0 d'^2_0+\left(\tilde{\alpha}_0 E-\frac{\tilde{\gamma}_0}{3}F\right)d_0 +\left(\tilde{\beta}_0 E-\frac{\tilde{\alpha}_0}{3}F\right)d'_0+\frac{\tilde{\beta}_0}{3}A-\frac{\tilde{\alpha}_0}{4}B+\frac{\tilde{\gamma}_0}{20}C\right\}. \quad (8)$$

In a special case with achromat condition, $d_0=0$ and $d'_0=0$, the average $\mathcal{H}$ function is given by

$$\langle\mathcal{H}\rangle=\rho\theta^3\left\{\frac{\tilde{\beta}_0}{3}A-\frac{\tilde{\alpha}_0}{4}B+\frac{\tilde{\gamma}_0}{20}C\right\}. \quad (9)$$

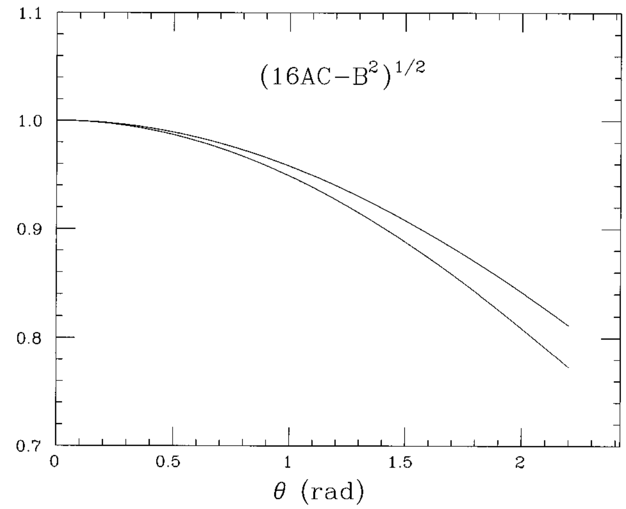

FIG. 1. The minimum $\langle\mathcal{H}\rangle$ factors $\sqrt{16AC-15B^2}$ for the DBA (the lower curve) and $\sqrt{16\tilde{A}\tilde{C}-15\tilde{B}^2}$ for the ME (the upper curve) lattices are plotted as a function of the bending angle θ. Note that $\langle\mathcal{H}\rangle$ is slightly smaller in long dipoles.

Using the condition $\tilde{\beta}_0\tilde{\gamma}_0=(1+\tilde{\alpha}_0^2)$, the minimum of $\langle\mathcal{H}\rangle$ is given by

$$\langle\mathcal{H}\rangle_{\rm MEDBA}=\frac{G}{4\sqrt{15}}\rho\theta^3, \quad (10)$$

where $G=\sqrt{16AC-15B^2}$. The corresponding betatron amplitude functions are $\tilde{\beta}_0=6C/\sqrt{15}G$, $\tilde{\alpha}_0=\sqrt{15}B/G$, and $\tilde{\gamma}_0=8\sqrt{5}A/\sqrt{3}G$. The factor $G=\sqrt{16AC-15B^2}$ depends slowly on the dipole bending angle θ shown in Fig. 1, where $\langle\mathcal{H}\rangle$ is slightly smaller due to the horizontal focusing of the bending radius. In the small angle approximation, one obtains easily $\langle\mathcal{H}\rangle_{\rm MEDBA}=(1/4\sqrt{15})\rho\theta^3$. The corresponding minimum emittance is $\epsilon_{\rm MEDBA}=C_q\gamma^2\theta^3/4\sqrt{15}J_x$, and the $\mathcal{H}$ function at the dispersive end of the dipole is $\mathcal{H}(\theta)=(1/\sqrt{15})\rho\theta^3$.

Without the achromat constraint, the lattice can be considered as a single dipole lattice. Thus the dispersion and the betatron amplitude functions, which minimize $\langle\mathcal{H}\rangle$, will be symmetric with respect to the center of the dipole. The minimization procedure can be accomplished through the following steps. First, $\langle\mathcal{H}\rangle$ can be minimized by finding the optimal dispersion function with $\partial\langle\mathcal{H}\rangle/\partial d_0=0$, $\partial\langle\mathcal{H}\rangle/\partial d'_0=0$ to obtain $d_{0,\min}=\frac{1}{6}F$, $d'_{0,\min}=-\frac{1}{2}E$, and

$$\langle\mathcal{H}\rangle=\frac{1}{12}\rho\theta^3\left(\tilde{\beta}_0\tilde{A}-\tilde{\alpha}_0\tilde{B}+\frac{4\tilde{\gamma}_0}{15}\tilde{C}\right), \quad (11)$$

where $\tilde{A}=4A-3E^2$, $\tilde{B}=3B-2EF$, and $\tilde{C}=\frac{9}{4}C-\frac{5}{4}F^2$. Using the relation that $\tilde{\beta}_0\tilde{\gamma}_0=1+\tilde{\alpha}_0^2$, we obtain

$$\langle\mathcal{H}\rangle_{\rm ME}=\frac{\tilde{G}}{12\sqrt{15}}\rho\theta^3, \quad (12)$$

where $\widetilde{G}=\sqrt{16\widetilde{A}\widetilde{C}-15\widetilde{B}^2}$ is also shown in Fig. 1. Thus the minimum $\langle\mathcal{H}\rangle$ without achromatic constraint is a factor of 3 smaller than that with the achromat condition [5–7]. The minimum condition corresponds to $\widetilde{\beta}_0=8\widetilde{C}/\sqrt{15}\widetilde{G}$, $\widetilde{\alpha}_0=\sqrt{15}\widetilde{B}/\widetilde{G}$, and $\widetilde{\gamma}_0=2\sqrt{15}\widetilde{A}/\widetilde{G}$.

In the small angle approximation with $\theta\ll 1$, where $\widetilde{A}\to 1$, $\widetilde{B}\to 1$, $\widetilde{C}\to 1$, and $\widetilde{G}\to 1$, the waist of the optimal betatron amplitude function for the minimum $\langle\mathcal{H}\rangle$ is located at the middle of the dipole. The values of the dispersion $\mathcal{H}$ function at both sides of the dipole are important to determine the beam size in straight sections, where insertion devices such as the undulators are located. At the ME condition, we have

$$\mathcal{H}(0)=\mathcal{H}(\theta)=\frac{1}{3\sqrt{15}\widetilde{G}}\rho\theta^3\left\{6\widetilde{C}E^2-\frac{15}{2}\widetilde{B}EF+\frac{5}{2}\widetilde{A}F^2\right\}. \tag{13}$$

In the small bending angle approximation, we have $\mathcal{H}(\theta)=(1/3\sqrt{15})\rho\theta^3$.

The brilliance of the photon beam from the undulator depends on the electron beam width, which depends on the emittance and momentum spread of the beam. Now we define the dispersion emittance as [10]

$$\begin{aligned}\epsilon_d &\equiv \gamma_x(D\delta)^2-\beta_x'(D\delta)(D'\delta)+\beta_x(D'\delta)^2=\mathcal{H}(0)\delta^2\\ &=\frac{1}{3\sqrt{15}}\frac{C_q\gamma^2\theta^3}{J_E},\end{aligned} \tag{14}$$

where $\delta^2=(\sigma_E/E)^2=C_q\gamma^2/J_E\rho$ is the equilibrium energy spread in the beam, J_E is the damping partition in synchrotron phase space. For a separated function lattice, $J_E\approx 2$, $J_x\approx 1$, or $J_E\approx 2J_x$. The total emittance for a bi-Gaussian distribution is given by

$$\epsilon=\epsilon_\beta+\epsilon_d=\frac{1}{4\sqrt{15}}\frac{C_q\gamma^2\theta^3}{J_x}=\epsilon_{\mathrm{MEDBA}}. \tag{15}$$

Thus the decrease in the betatron emittance is consumed by the dispersion beam size. The brilliance of the photon beam from insertion devices or equivalently the size of the electron beam in the ‘‘dispersion free’’ straight section, is not affected by the dispersion introduced to minimize the betatron emittance. The total electron beam size in the straight section remains unchanged. Thus the minimization procedure does not increase the beam brilliance of undulators.

Now we are ready to discuss the minimum emittance for three-bend achromat lattices, which have often been used in synchrotron radiation sources such as the Advanced Light Source in Lawrence Berkeley National Laboratory, the Synchrotron Radiation Research Center in Taiwan, etc. The TBA is a combination of DBA lattice with a single dipole cell at the center. If all dipoles have equal length, the minimum emittance is usually quoted to be $\epsilon_{\mathrm{METBA}}=\frac{2}{3}\epsilon_{\mathrm{MEDBA}}+\frac{1}{3}\epsilon_{\mathrm{ME}}=\frac{7}{9}\epsilon_{\mathrm{MEDBA}}$.

To simplify our discussion hereafter, we use a small angle approximation, which is a good approximation provided that the bending angle for each dipole is less than 30°. The normalized dispersion coordinates for the minimum emittance DBA and minimum emittance single dipole lattices are given, respectively, by

$$X_{\mathrm{MEDBA}}=\frac{D}{\sqrt{\beta}}=\frac{(15)^{1/4}}{8}\frac{L_1^{3/2}}{\rho_1}, \tag{16}$$

$$P_{\mathrm{MEDBA}}=\frac{\alpha D+\beta D'}{\sqrt{\beta}}=\pm\frac{7}{8(15)^{1/4}}\frac{L_1^{3/2}}{\rho_1}, \tag{17}$$

at the dispersive ends of the dipoles in the MEDBA lattice, and

$$X_{\mathrm{ME}}=\frac{D}{\sqrt{\beta}}=\frac{\sqrt{2}(15)^{1/4}}{24}\frac{L_2^{3/2}}{\rho_2}, \tag{18}$$

$$P_{\mathrm{ME}}=\frac{\alpha D+\beta D'}{\sqrt{\beta}}=\mp\frac{3}{4\sqrt{2}(15)^{1/4}}\frac{L_2^{3/2}}{\rho_2}, \tag{19}$$

at the entrance and exit locations of the dipole in the ME lattice, where ρ_1 and L_1 are the bending radius and the length of the DBA dipoles, and ρ_2 and L_2 are the bending radius and the length of the ME dipoles.

The optical matching between the MEDBA module and the ME single dipole module is accomplished with quadrupoles, where the normalized dispersion functions are transformed by the coordinate rotation, i.e.,

$$\begin{pmatrix}X_{\mathrm{ME}}\\ P_{\mathrm{ME}}\end{pmatrix}=\begin{pmatrix}\cos\Phi & \sin\Phi\\ -\sin\Phi & \cos\Phi\end{pmatrix}\begin{pmatrix}X_{\mathrm{MEDBA}}\\ P_{\mathrm{MEDBA}}\end{pmatrix}, \tag{20}$$

where Φ is the betatron phase advance. The necessary condition for achieving dispersion phase-space matching is

$$\frac{L_2^3}{\rho_2^2}=3\frac{L_1^3}{\rho_1^2}, \tag{21}$$

with a corresponding phase advance $\Phi=127.76°$. The matching condition of Eq. (21), based on the small angle approximation, requires $L_2=3^{1/3}L_1$ for isomagnetic storage rings, or $\rho_1=\sqrt{3}\rho_2$ for storage rings with equal length dipoles. For dipole angle larger than 30°, the scaling factor is slightly increased.

Thus we have proved a theorem stating that the isomagnetic TBA with equal length dipoles *cannot* be matched to attain the advertised minimum emittance. For an isomagnetic storage ring, the center dipole for the TBA should be $3^{1/3}$ longer than those of outer dipoles in order to achieve dispersion function matching. In this case, one can prove the following trivial theorem: The emittance of the matched minimum TBA (QBA, etc.) lattice is

$$\epsilon_{\mathrm{METBA}}=\frac{1}{4\sqrt{15}}\frac{C_q\gamma^2\theta_1^3}{J_x}, \tag{22}$$

where θ_1 is the bending angle of outer dipoles, provided that the length of the middle dipole is $3^{1/3}$ longer than that of outer dipoles. Although the factor $\mathcal{F}_{\mathrm{METBA}}=1/4\sqrt{15}$ is identical to that of the MEDBA, the actual emittance of the METBA is lower due to its smaller outer dipole bend angle. To provide a fair comparison of emittances, we assume that the number of dipoles in the TBA or the QBA is the same as that of the DBA lattice. Because the bend angle of the outer dipole in TBA or QBA is smaller, the resulting minimum emittances of METBA and MEQBA are related to that of the MEDBA by

$$\epsilon_{\mathrm{METBA}}=\left(\frac{3}{2+3^{1/3}}\right)^3\epsilon_{\mathrm{MEDBA}}=0.66\epsilon_{\mathrm{MEDBA}}, \tag{23}$$

$$\epsilon_{\mathrm{MEQBA}}=\left(\frac{2}{1+3^{1/3}}\right)^3\epsilon_{\mathrm{MEDBA}}=0.55\epsilon_{\mathrm{MEDBA}}. \tag{24}$$

Thus a TBA or QBA lattice can provide smaller emittance for future synchrotron radiation light sources.

At present, all TBA light sources have equal length isomagnetic dipoles; the minimum emittance can be evaluated as follows. The average $\mathcal{H}$ function is given by

$$\langle\mathcal{H}\rangle=\tfrac{1}{3}[2\langle\mathcal{H}\rangle_o+\langle\mathcal{H}\rangle_i], \tag{25}$$

where the subscripts o and i are used to identify the outer and inner dipoles, respectively. First, we minimize $\langle\mathcal{H}\rangle_o$ to obtain $\langle\mathcal{H}\rangle_o=(1/4\sqrt{15})\rho\theta^3$. Once $\langle\mathcal{H}\rangle_o$ is minimized, the $\langle\mathcal{H}\rangle_i$ is determined by the optical matching condition.

To match the optical function in the TBA lattice, the betatron functions must be symmetric with respect to the center of the middle dipole. Using the symmetry condition, we obtain

$$\tilde{\beta}_0=\tilde{\beta}^*+\frac{1}{4\tilde{\beta}^*},\quad \tilde{\alpha}_0=\frac{1}{2\tilde{\beta}^*},\quad \tilde{\gamma}_0=\frac{1}{\tilde{\beta}^*}, \tag{26}$$

$$d_0=\frac{1}{6},\quad d_0'=-\frac{1}{2}, \tag{27}$$

where $\tilde{\beta}^*=\beta^*/L$ is the normalized betatron amplitude function at the center of the middle dipole. Using Eq. (8), we obtain

$$\langle\mathcal{H}\rangle_i=\rho\theta^3\left[\frac{1}{\sqrt{15}}+\left(\sqrt{\tilde{\beta}_0}-\frac{\tilde{\alpha}_0}{3\sqrt{\tilde{\beta}_0}}\right)\tilde{P}_0-\frac{1}{3\sqrt{\tilde{\beta}_0}}\tilde{X}_0+\frac{\tilde{\beta}_0}{3}-\frac{\tilde{\alpha}_0}{4}+\frac{\tilde{\gamma}_0}{20}\right], \tag{28}$$

where the normalized dispersion coordinates are given by

$$\tilde{X}_0=\frac{d_0}{\sqrt{\tilde{\beta}_0}}=\frac{1}{6(\tilde{\beta}^*+1/4\tilde{\beta}^*)^{1/2}}, \tag{29}$$

$$\tilde{P}_0=\frac{\tilde{\alpha}_0 d_0+\tilde{\beta}_0 d_0'}{\sqrt{\tilde{\beta}_0}}=-\frac{\tilde{\beta}^*+1/12\tilde{\beta}^*}{2(\tilde{\beta}^*+1/4\tilde{\beta}^*)^{1/2}}. \tag{30}$$

Using the matching condition of Eq. (20), possible solutions are given by

$\tilde{\beta}^*$	Φ	$\langle\mathcal{H}\rangle_i$
0.027635	144.79°	$(1/\sqrt{15})(1-0.79643)$
1.005160	134.01°	$(1/\sqrt{15})(1-0.67023)$

Here, we disregard the solution with $\tilde{\beta}^*=0.027635$, which is not easy attainable. The resulting emittance for the isomagnetic TBA lattices with equal length dipoles is given by

$$\epsilon_{\mathrm{METBA}}=\frac{1.1064}{4\sqrt{15}}\frac{C_q\gamma^2\theta^3}{J_x}. \tag{31}$$

In conclusion, we show that the minimum emittance isomagnetic TBA is not attainable with equal dipole length. The necessary condition for achieving a minimum emittance in the TBA (or NBA) lattice is that the length of the middle dipole(s) should be a factor of $3^{1/3}$ longer than that of the outer dipole (in small angle approximation). Further reduction in emittance can only be achieved by varying the damping partition number.

For existing TBA storage rings, the minimum emittance condition can be fulfilled by increasing the dipole field of the center dipole so that the bending radius of the center dipole is smaller by the factor $1/\sqrt{3}$ than that of the outer dipole [11]. The resulting minimum emittance factor is

$$\mathcal{F}_{\mathrm{METBA}}=\left(\frac{2+3^{3/2}}{5}\right)\frac{1}{4\sqrt{15}}.$$

In this paper we do not discuss the emittance reduction using wigglers or undulators to increase the radiation damping rate in zero dispersion straight sections, which would increase the momentum spread of the beam as well.

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[11] To maintain 2π bending angle for the existing TBA synchrotron radiation sources, the outer dipoles should decrease magnetic field by $3/(2+\sqrt{3})$ and the center dipole should increase the magnetic field by a factor of $3\sqrt{3}/(2+\sqrt{3})$. The optics should be rematched and the machine realigned.