Interface fluctuations on a hierarchical lattice

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We consider interface fluctuations on a two-dimensional layered lattice where the couplings follow a hierarchical sequence. This problem is equivalent to the diffusion process of a quantum particle in the presence of a one-dimensional hierarchical potential. According to a modified Harris criterion, this type of perturbation is relevant and one expects anomalous fluctuating behavior. By transfer-matrix techniques and by an exact renormalization-group transformation we have obtained analytical results for the interface fluctuation exponents, which are discontinuous at the homogeneous lattice limit. [S1063-651X(96)01808-9]

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I. INTRODUCTION

Recently, there has been a growing interest in natural and artificial systems that are organized in a hierarchical way. Examples can be found in economical organizations [1] and stock-market exchanges [2], in geological processes before major earthquakes [3], and in studies of relaxation phenomena of proteins [4], spin glasses [5], and computer architectures [6]. Theoretically, much effort has been devoted to the understanding of the linear dynamics (i.e., the diffusion process) in a system with hierarchically organized energy barriers. According to numerical [7,8] and exact [9,10] results, the diffusion in such systems can be anomalous (which is often called ''ultradiffusion'' [7]). Furthermore, in several models there is a dynamical phase transition [8] separating regions with normal and anomalous diffusion. For a comprehensive review on the subject see Ref. [11].

Another subject of theoretical interest is the properties of (static) phase transitions on hierarchical lattices. For these and other nonperiodic (quasiperiodic or more generally aperiodic) systems a relevance-irrelevance criterion has recently been proposed [12], in analogy to the Harris criterion [13] for random magnets. The crossover exponent corresponding to a nonperiodic perturbation is given by

$$\Phi = 1 + \nu D(\Omega - 1) \tag{1}$$

in terms of the ν correlation length exponent of the unperturbed system and the wandering exponent of the sequence Ω [14]. Here D denotes the number of coordinates on which the couplings depend, e.g., for a layered system $D=1$. The perturbation is then expected to be relevant (irrelevant) if $\Phi>0$ ($\Phi<0$), which was indeed found in a series of exact studies on two-dimensional layered Ising models [15,16]. For marginal sequences, where $\Phi=0$, continuously varying critical exponents and anisotropic scaling behavior were observed [17].

As far as the critical behavior on hierarchical lattices is concerned, mainly the two-dimensional layered Ising model with a one-dimensional Huberman-Kerszberg (HK) sequence [7] and the corresponding Ising quantum chain were studied. In numerical [18] and exact [19,20] calculations, nonuniversal critical behavior was found in accordance with the vanishing crossover exponent in Eq. (1), which follows from the fact that the fluctuation exponent of the HK sequence is $\Omega=0$ [20].

In this paper we consider the interface fluctuation problem on a layered lattice, where the couplings between the layers follow the HK hierarchical sequence. As far as interface wandering on nonperiodic lattices is concerned we should mention the work by Henley and Lipowsky [21], who considered the interface roughening in two-dimensional quasicrystals. On a layered lattice with Fibonacci-type quasiperiodicity, nonuniversal interface fluctuations were observed, with a continuously varying interface wandering exponent. This behavior is again in accord with the relevance-irrelevance criterion, since with $\Omega=-1$ and $\nu=\nu_\perp=\frac{1}{2}$ the crossover exponent in Eq. (1) is $\Phi=0$. In our problem, on the HK lattice $\Omega=0$, thus $\Phi=\frac{1}{2}>0$ and the perturbation is relevant. Therefore one expects anomalous interface fluctuations on this lattice.

The structure of the paper is as follows. We define the model in Sec. II. The results of the transfer-matrix calculations and that of an exact renormalization-group (RG) transformation are presented in Secs. III and IV, respectively. The results are discussed in Sec. V.

II. FORMALISM

We consider a diagonally layered ferromagnetic spin model (cf. the Ising model) on the square lattice with hierarchically organized interactions. The couplings in the hth diagonal $K_h=J_h/k_BT$ are selected from a set $(\kappa_0,\kappa_1,\kappa_2,...)$ and $\kappa_n=n\kappa_0$, such that

$$K_h=\kappa_n, \quad h=2^n(2m+1). \tag{2}$$

This type of structure of the couplings (Fig. 1), which shows

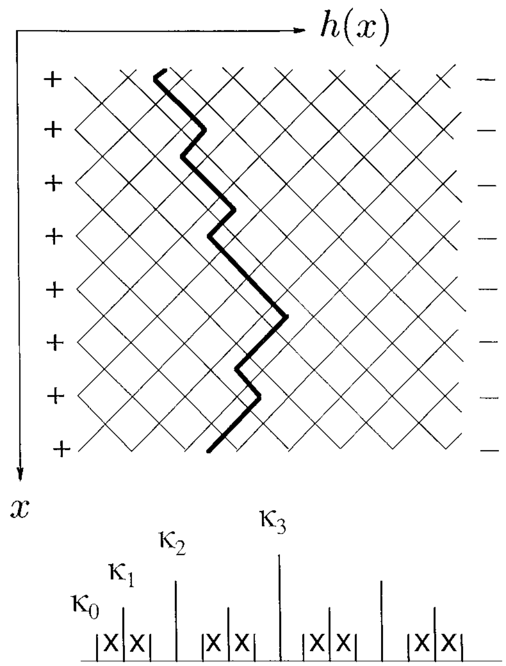

FIG. 1. Structureless interface on a diagonally layered square lattice. The values of the couplings, which follow the hierarchical HK sequence in Eq. (2), are indicated below. Sites to be decimated in the RG transformation are marked by X.

the typical features of ultrametric topology [5], was introduced by Huberman and Kerszberg [7] following the work in Ref. [1].

The boundary spins on the (1,1) surfaces are fixed in different orientations (Fig. 1) and we are interested in the fluctuations of the interface separating the positive and negative regions. The interface is considered as a continuous structureless string and complicated interface configurations, such as overhangs and bubbles, are omitted. It is generally accepted that to study interfacial fluctuations it is sufficient to keep only solid-on-solid (SOS) -type interface configurations. In this so-called SOS model the interface is geometrically represented by a directed walk or polymer [22].

In the SOS model the interface is characterized by its height $h(x)$ at site x and the interfacial energy is specified by the Hamiltonian

$$-H/k_BT=\sum_x 2K_{h(x)}, \tag{3}$$

where surface effects are omitted. The thermodynamic properties of the interface are conveniently studied in the transfer-matrix formalism [23,24]. For our model the transfer matrix in the x direction, parallel to the boundaries, is given by

$$T_{h,l}=\delta_{h,l-1}e^{-2K_h}+\delta_{h,l+1}e^{-2K_l}. \tag{4}$$

Here, according to Eq. (2), the matrix elements are from a set $(\epsilon_0,\epsilon_1,\epsilon_2,\dots)$ and the ratio of successive terms is constant: $\epsilon_{n+1}/\epsilon_n=R<1$. For the homogeneous system $R=1$, whereas for hierarchical lattices R measures the strength of inhomogeneity. The interface is not likely to visit sites with a matrix element ϵ_n, $n\gg 1$, since the corresponding probability is weighted by a factor of R^n.

The interfacial free energy σ and the longitudinal correlation length $\xi_\parallel$, which is measured parallel to the boundaries, are given in terms of the leading and the next to leading eigenvalues of the transfer matrix λ_0 and λ_1 as

$$\sigma=-\log\lambda_0 \tag{5}$$

and

$$\xi_\parallel^{-1}=\log(\lambda_0/\lambda_1). \tag{6}$$

The fluctuations of the interface grow on a power-law scale

$$\langle[h(0)-h(x)]^2\rangle\sim x^{2w}, \tag{7}$$

where w is the wandering or fluctuation exponent, which is $w=\frac{1}{2}$ for homogeneous two-dimensional systems [22].

Another quantity of interest is the probability $P_0(x)$ that the interface after x steps has the same position, i.e., $h(0)=h(x)$. For a walk or diffusion problem, where x plays the role of the time, $P_0(x)$ is the autocorrelation function, which has the asymptotic behavior $P_0(x)\sim x^{-\gamma}$. For homogeneous two-dimensional lattices $\gamma=\frac{1}{2}$ and generally $w=\gamma$ [8]. It could be shown by slightly modifying the derivation in Ref. [8] that the autocorrelation function averaged over the starting positions of the interface can be expressed through the spectrum of the transfer matrix as

$$\overline{P_0}(x)=\frac{1}{L}\sum_i\left(\frac{\lambda_i}{\lambda_0}\right)^x=\int_{-\infty}^{1}g(\lambda)\left(\frac{\lambda}{\lambda_0}\right)^x d\lambda, \tag{8}$$

where $g(\lambda)=1/L\Sigma_i\delta(\lambda-\lambda_i)$ is the density of states and L denotes the width of the system in the h direction, thus being the dimension of the transfer matrix.

The eigenvalues of the transfer matrix are dense at the top of the spectrum and one can develop a scaling theory in terms of these critical eigenvalues. We consider a critical level λ_i of a system with a finite width L and denote by $\Delta\lambda_i=\lambda_0-\lambda_i$ its difference from the top of the spectrum. Changing lengths by a factor of $b=2$, i.e., with $L'=L/2$, the ith eigenvalue will be λ_i' and the difference $(\Delta\lambda_i)'$ will scale with a factor of b^{y_λ}; thus

$$(\Delta\lambda_i)'=2^{y_\lambda}\Delta\lambda_i, \tag{9}$$

where y_λ is the gap exponent. We stress that the statement in Eq. (9), that all critical levels scale with the same factor, is a scaling hypothesis, which will be verified by actual calculations in the following sections.

Using Eq. (9), the transformation law for the density of states is given by

$$g(\Delta\lambda)=2^{y_\lambda-1}g'[(\Delta\lambda)'], \tag{10}$$

which is compatible with a power-law dependence of the density of states at the top of the spectrum:

$$g(\Delta\lambda)\sim(\Delta\lambda)^{1/y_\lambda-1}. \tag{11}$$

Now putting this expression into Eq. (8) and evaluating the autocorrelation function, one gets $\gamma=1/y_\lambda$.

From the scaling behavior of the spectrum in Eq. (9) one obtains, for the finite-size corrections to the largest eigenvalues,

$$\lambda_0 - \lambda_i(L) \sim L^{-y_\lambda}. \tag{12}$$

Thus, from Eqs. (6) and (12), the longitudinal correlation length is $\xi_\parallel \sim L^{y_\lambda}$. In a finite system the correlation length perpendicular to the (1,1) surface is limited by the width of the strip $\xi_\perp \sim L$; therefore the interface wandering exponent in Eq. (7), which can be alternately defined as $\xi_\perp \sim \xi_\parallel^w$, is given by

$$w = 1/y_\lambda . \tag{13}$$

Thus, indeed, $w = \gamma$, as expected from scaling considerations.

In the following we calculate the interface fluctuations on the HK lattice by two methods. First, we study numerically the spectrum of the transfer matrix, verify the validity of the scaling hypothesis, and determine the interfacial tension and the wandering exponent. Then we apply an exact renormalization-group transformation and calculate analytical expressions for the critical exponents.

III. NUMERICAL STUDY OF THE TRANSFER MATRIX

The transfer matrix of the interface problem in Eq. (4) is tridiagonal and could be diagonalized by powerful methods [25]. In the specific problem, however, due to the hierarchical structure of the transfer matrix, one can implement a very fast algorithm to calculate the roots of the corresponding determinant.

We consider a finite system of size $L = 2^l$ and express the corresponding determinant $D(2^l)$ by two subdeterminants of sizes 2^{l-1} and $2^{l-1}-1$, respectively, in the form

$$D(2^l) = D(2^{l-1})D(2^{l-1}) - D(2^{l-1}-1)D(2^{l-1}-1)\epsilon_{l-1}^2. \tag{14a}$$

The symmetric determinant $\tilde{D}(2^l-2)$ of size 2^l-2, which is obtained from $D(2^l)$ by leaving out the first and last rows and columns, can be similarly expressed as

$$\tilde{D}(2^l-2) = D(2^{l-1}-1)D(2^{l-1}-1) - \tilde{D}(2^{l-1}-2)\tilde{D}(2^{l-1}-2)\epsilon_{l-1}^2. \tag{14b}$$

Finally,

$$D(2^l-1) = D(2^{l-1})D(2^{l-1}-1) - D(2^{l-1}-1)\tilde{D}(2^{l-1}-2)\epsilon_{l-1}^2. \tag{14c}$$

These relations supplemented with $D(1) = -\lambda$, $D(2) = \lambda^2 - \epsilon_0^2$, and $\tilde{D}(2) = \lambda^2 - \epsilon_1^2$ define a fast procedure to calculate the value of the determinant for very large sizes. For example, we could treat with this method slightly perturbed systems with $R \approx 1$ up to sizes $L = 2^{30}$–2^{40}.

The largest eigenvalues calculated by this method all have the same type of finite-size dependence, thus the scaling hypothesis in Sec. II is indeed satisfied. The leading eigenvalues calculated on the largest finite lattices are accurate at least up to 10–12 digits. The gap exponents describing the finite-size dependence of $\lambda_i(L)$ in Eq. (12), however, could be obtained from the raw data with a comparatively smaller accuracy, up to 5 digits. In this case, to increase accuracy we used sequence extrapolation methods, such as the van den Broeck-Schwartz and the Bulirsch-Stoer methods [26].

TABLE I. Leading eigenvalue and the corresponding interface fluctuation exponent from numerical diagonalization of the transfer matrix for different values of the hierarchical parameter.

R	λ_0/ϵ_0	$w = 1/y_\lambda$
1	2	0.5
0.999	1.998 008 94	0.456 719 9
0.9	1.828 532 74	0.455 109 2
0.75	1.622 186 48	0.445 143 8
0.5	1.352 860 81	0.400 454 0
0.25	1.149 486 52	0.311 057 7
0.1	1.053 814 56	0.227 297 1
0.001	1.000 500 38	0.091 186 7

The leading eigenvalue of the transfer matrix, which is connected to the interfacial tension in Eq. (5), and the extrapolated values of the interface wandering exponent are listed in Table I. One can see that both the leading eigenvalue and the wandering exponent are monotonically decreasing as R goes from one to zero. In the limit $R \to 0$ the interfacial tension in Eq. (5), together with the wandering exponent, goes to zero, which is due to the fact that the system tends to be separated into disconnected parts. More interesting is the behavior of the wandering exponent around the homogeneous lattice point. As the value of R is lowered below one the wandering exponent jumps by a finite amount of $\Delta w = 0.043\,279\,9$ from $w = \frac{1}{2}$. In renormalization-group (RG) language, such type of behavior corresponds to a relevant perturbation, which brings the system into another stable fixed point. In the next section we shall explicitly construct the RG transformation and determine exactly the wandering exponent.

IV. RENORMALIZATION-GROUP CALCULATION

We are going to study the scaling behavior of the largest eigenvalues of the transfer matrix in Eq. (4), which satisfy the second-order difference equation

$$0 = T_{i,i+1}\psi_{i+1} - \lambda\psi_i + T_{i-1,i}\psi_{i-1}, \tag{15}$$

where in the thermodynamic limit the boundary terms are omitted. The structure of the couplings that are connected to $T_{i,i+1}$ in Eq. (4) are shown in Fig. 1. To construct an exact recursion we decimate those sites that are connected to a κ_1 coupling or equivalently to an ϵ_1 matrix element (denoted by crosses in Fig. 1). We note that the same type of decimation was used by Maritan and Stella in their study of the diffusion problem on the HK lattice [10]. One can see that after a decimation step the $(\epsilon_0, \epsilon_1, \epsilon_0)$ triplet will play the role of the renormalized ϵ_0', whereas the other couplings will renormalize as $\epsilon_n' = \epsilon_{n+1}$, keeping the value of R and the structure of the transfer matrix unchanged.

Performing the RG transformation we first denote the two neighboring sites to be decimated by i and $i+1$ and express ψ_i and ψ_{i+1} as

$$\psi_i = A\psi_{i-1} + B\psi_{i+2},$$

$$\psi_{i+1} = B\psi_{i-1} + A\psi_{i+2}, \tag{16}$$

where $A = \epsilon_0\lambda/(\lambda^2 - \epsilon_1^2)$ and $B = \epsilon_0\epsilon_1/(\lambda^2 - \epsilon_1^2)$. Then the difference equations in terms of the remaining, nondecimated spins have the same form as Eq. (15), provided that the eigenvalue and the couplings transforms as

$$\lambda' = \frac{\lambda - A\epsilon_0}{B}, \quad \epsilon_n' = \frac{\epsilon_{n+1}}{B}, \quad n = 1,2,\ldots, \tag{17}$$

and $\epsilon_0' = \epsilon_0$. Thus the ratio of the sequence remains invariant $R' = R$, as expected. As a consequence, in the RG transformation, besides λ, it is sufficient to consider only one coupling, say ϵ_1, and the RG transformation can be written as a two-parameter recursion

$$\lambda' = \lambda\,\frac{\lambda^2 - \epsilon_1^2 - \epsilon_0^2}{\epsilon_0\epsilon_1}, \quad \epsilon_1' = R\,\frac{\lambda^2 - \epsilon_1^2}{\epsilon_0}, \tag{18}$$

where ϵ_0 is the input value of the largest matrix element.

The physically relevant fixed point of the transformation with $\lambda > 0$ is given by

$$\left(\frac{\epsilon_1}{\epsilon_0}\right)^* = \frac{R}{1-R}, \quad \left(\frac{\lambda}{\epsilon_0}\right)^* = \frac{\sqrt{1-R+R^2}}{1-R}, \tag{19}$$

which is stable for $0 < R < 1$. The eigenvalues of the linearized fixed-point transformation are roots of a quadratic equation and are given as

$$\Lambda_{1,2} = \frac{1}{R} + R + \frac{1}{2} \pm \left[\left(\frac{1}{R} + R + \frac{1}{2}\right)^2 - 2\right]^{1/2}. \tag{20}$$

The leading eigenvalue $\Lambda_1 > 1$ determines the scaling behavior of the spectrum of the transfer matrix and the y_λ scaling dimension is given by

$$y_\lambda = \frac{\log \Lambda_1}{\log 2}. \tag{21}$$

The second eigenvalue of the RG transformation is $\Lambda_2 < 1$ and the corresponding scaling field is irrelevant; thus the fixed point in Eq. (21) is attractive and governs the critical properties of the physical model with $\epsilon_1 = R\epsilon_0$. It is seen from Eq. (19) that the fixed point of anomalous interface fluctuations does not exist at the homogeneous point $R = 1$, where the fluctuations are characterized by the normal wandering exponent $w = \frac{1}{2}$. Comparing the analytical results for $w = 1/y_\lambda$ with those obtained by finite-size calculations in Table I, we can say that the numerical results are indeed very accurate: they correspond to that in Eq. (21) at least up to six digits.

V. DISCUSSION

In this paper we studied interface fluctuations on a layered hierarchical lattice. The perturbation caused by inhomogeneous couplings is relevant according to a linear stability analysis and the observed interface fluctuations are indeed anomalous. The wandering exponent w is a monotonically decreasing function of R and discontinuous at $R = 1$. The fact that $w(R) < \frac{1}{2}$ can be understood, since the interface preferentially stays on κ_0 lines and the probability to visit a κ_n line is rapidly decreasing with n. Consequently, the interface fluctuations are damped by the inhomogeneously distributed couplings.

One can estimate $w(R)$ in the limit $R \to 0$, when the probability of a large interface fluctuation of height $h = 2^n$ is primarily given by $p_n \sim \epsilon_n$, i.e., by the probability to have one step on the κ_n line. For such a fluctuation the interface approximately takes $x \sim p_n^{-1} \sim R^{-n}$ steps, thus the wandering exponent in leading order is $w(R) = -\log 2/\log R$, which corresponds to the asymptotic behavior of the analytical result in Eq. (22). We note that in the $R \to 0$ limit the interface fluctuations can be described by a Markovian process and then our problem is equivalent to the diffusion of a particle in a hierarchical lattice, as studied in Refs. [7–11].

The HK sequence used in this paper can be generalized by having a general ν character [27] instead of the $\nu = 2$ used in Eq. (2). Then one has, in Eq. (2), $h = R^n(\nu m + \mu)$, with $\mu = 1,2,\ldots,\nu-1$. According to our numerical and analytical investigations for $\nu = 3$ and 4, the main characteristics of interface fluctuations remain the same as for $\nu = 2$: the wandering exponent has a jump at $R = 1$ and varies with R. For $\nu = 3$ we obtained the analytical result

$$w_{\nu=3} = \frac{\log 3}{\log \Lambda_{\nu=3}},$$

$$\Lambda_{\nu=3} = 2\left(\frac{1}{R} + R + 1\right) + \left[4\left(\frac{1}{R} + R + 1\right)^2 - 3\right]^{1/2}. \tag{22}$$

As mentioned before, the problem studied in this paper is related to the diffusion process on hierarchical lattices [11]. Our problem, however, can be formulated as the quantum-mechanical diffusion process of a particle that is represented by a wave packet and placed on a one-dimensional HK potential. Then x and $h(x)$ correspond to the time t and the position of the particle at a given time step, respectively, while the transfer matrix describes time evaluation. According to our results, in a one-dimensional hierarchical potential the width of the wave packet will grow in time anomalously as $t^{w(R)}$.

Our final remark concerns some similarities of our results to that of interface fluctuations in a repulsive, inhomogeneous surface potential, decaying as $\sim l^{-\omega}$, where l measures the distance from the surface [28]. In two dimensions, for $\omega < 2$, the perturbation is relevant and the interface wandering exponent takes the anomalous value $w = 1/\omega > \frac{1}{2}$ [29]. In this problem, however, the perturbation is confined to the surface; furthermore, the wandering exponent is continuous at $\omega = 2$.

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[1] H. A. Simon and A. Ando, Econometrica **29**, 111 (1961).
[2] J. A. Feigenbaum and P. G. A. Freund (unpublished).
[3] H. Saleur, C. G. Sammis, and D. Sornette (unpublished).
[4] R. H. Austin, K. W. Berson, L. Eisenstein, L. H. Frauenfelder, and I. C. Gunsalus, Biochem. **14**, 5355 (1975).
[5] M. Mézard, G. Parisi, N. Sourlas, G. Toulouse, and M. Virasoro, Phys. Rev. Lett. **52**, 1156 (1984).
[6] B. A. Huberman and T. Hogg, Phys. Rev. Lett. **52**, 1048 (1984).
[7] B. A. Huberman and M. Kerszberg, J. Phys. A **18**, L331 (1985).
[8] S. Teitel, D. Kutasov, and E. Domany, Phys. Rev. B **36**, 684 (1987).
[9] M. Schreckenberg, Z. Phys. B **60**, 483 (1985); A. T. Ogielski and D. L. Stein, Phys. Rev. Lett. **55**, 1634 (1985); G. Paladin, M. Mézard, and C. de Dominicis, J. Phys. Lett. **46**, L985 (1985).
[10] A. Maritan and A. L. Stella, J. Phys. A **19**, L269 (1986).
[11] A. Giacomtti, A. Maritan, and A. L. Stella, Int. J. Mod. Phys. B **5**, 709 (1991).
[12] J. M. Luck, J. Stat. Phys. **72**, 417 (1993); F. Iglói, J. Phys. A **26**, L703 (1993); J. M. Luck, Europhys. Lett. **24**, 359 (1993).
[13] A. B. Harris, J. Phys. C **7**, 1671 (1974).
[14] M. Queffélec, in *Substitutional Dynamical Systems-Spectral Analysis*, edited by A. Dold and B. Eckmann, Lecture Notes in Mathematics Vol. 1294 (Springer, Berlin, 1987).
[15] C. A. Tracy, J. Phys. A **21**, L603 (1988); F. Iglói, *ibid.* **21**, L911 (1988); G. V. Benza, Europhys. Lett. **8**, 321 (1989).
[16] L. Turban, F. Iglói, and B. Berche, Phys. Rev. B **49**, 12 695 (1994); F. Iglói and L. Turban, Europhys. Lett. **27**, 91 (1994); L. Turban, P.-E. Berche, and B. Berche, J. Phys. A **27**, 6349 (1994).
[17] B. Berche, P.-E. Berche, M. Henkel, F. Iglói, P. Lajkó, S. Morgan, and L. Turban, J. Phys. A **28**, L165 (1995); P.-E. Berche, B. Berche, and L. Turban, J. Phys. (France) I **6**, 621 (1996); F. Iglói and P. Lajkó (unpublished).
[18] A. L. Stella, M. R. Swift, J. G. Amar, T. L. Einstein, M. W. Cole, and J. R. Banavar, Phys. Rev. Lett. **23**, 3818 (1993).
[19] Z. Lin and M. Goda, Phys. Rev. B **51**, 6093 (1995).
[20] F. Iglói, P. Lajkó, and F. Szalma, Phys. Rev. B **52**, 7159 (1995).
[21] C. L. Henley and R. Lipowsky, Phys. Rev. Lett. **59**, 1679 (1987).
[22] M. E. Fisher, J. Chem. Soc. Faraday Trans. **82**, 1569 (1986).
[23] T. W. Burkhardt, J. Phys. A **14**, L63 (1981).
[24] V. Privman and N. M. Švrakić, in *Directed Models of Polymers, Interfaces and Finite-Size Properties*, Lecture Notes in Physics Vol. 338 (Springer, Berlin, 1989).
[25] C. Lanczos, J. Res. Natl. Bur. Stand. **45**, 255 (1950).
[26] M. Henkel and G. Schütz, J. Phys. A **21**, 2617 (1988).
[27] W. P. Keirstead and B. Huberman, Phys. Rev. A **36**, 5392 (1987).
[28] R. Lipowsky and T. M. Nieuwenhuizen, J. Phys. A **21**, L89 (1988).
[29] F. Iglói, Europhys. Lett. **19**, 305 (1992).