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Dark and bright vector spatial solitons in biased photorefractive media

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We show that the vector beam evolution equations in properly oriented biased photorefractive media can exhibit bright-dark soliton pair solutions under steady-state conditions. These wave pairs are obtained perturbatively provided that the intensities of the two optical beams are approximately equal. Our analysis indicates that these bright-dark vector solitons exist irrespective of the polarity of the external bias field. The stability of these vector pairs has been investigated numerically and it has been found that they are stable only in the regime of positive bias polarity.

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Since their first experimental observation [1], optical spatial solitons in photorefractive (PR) media have been a topic of considerable interest [2–11]. To date, three different types of PR solitons have been considered in the literature. The first kind involves the so-called quasi-steady-state solitons, or transient solitons, which are typically observed over a time interval, i.e., during the screening process of the external bias field [1–4]. The other two types, better known as screening [8,9] and photovoltaic solitons [6], are possible only under steady-state conditions and have been recently observed experimentally [5,7,11]. In particular, photovoltaic steady-state planar solitons can exist in PR materials with appreciable photogalvanic coefficients [6]. Screening solitons, on the other hand, require the application of an external bias field [8,9]. Very recently, vector solitons have also been predicted in biased PR media by Segev *et al.* [12]. Depending on the symmetry class of the appropriate crystal and its orientation, these solitary beams were found to obey a self-coupled or a cross-coupled system of nonlinear evolution equations. Thus far, bright-bright and dark-dark, self- or cross-coupled vector solitons have been predicted [12].

In this Rapid Communication we show that the self-coupled vector beam evolution equations in biased PR media can also exhibit bright-dark soliton pair solutions under steady-state conditions. It is interesting to note that these vector solitons are also reminiscent of those previously predicted in birefringent $\chi^{(3)}$ media [13]. These wave pairs are obtained perturbatively provided that the intensities of the two optical beams are approximately equal. Moreover, our analysis indicates that these bright-dark vector solitons exist irrespective of the polarity of the external bias field. The stability of our solutions has been investigated numerically and we have found that they are stable only in the regime of positive bias polarity. Conversely, when the polarity is negative, the pair tends to disintegrate as a result of modulational instability.

To start, let us consider an optical beam that propagates in a PR material along the z axis and is allowed to diffract only along the x direction. Moreover, let us assume that the external bias electric field is also applied along x. For demonstration purposes, let the PR crystal be $LiNbO_3$ ($3m$ class, $m \perp x_2$). As previously pointed out by Segev *et al.* [12], this crystal is a good candidate for the observation of such planar self-coupled or cross-coupled vector solitons. More specifically, for the self-coupled case, the permittivity changes in $LiNbO_3$ along the extraordinary and ordinary components of the optical beam are equal, i.e., $\Delta\varepsilon_{ee}=\Delta\varepsilon_{oo}$, provided that the optical c axis of this crystal makes an angle $\theta\approx 11.9°$ with respect to the z axis. $\Delta\varepsilon_{ee}$ and $\Delta\varepsilon_{oo}$ represent the diagonal perturbations on the relative permittivity tensor. Moreover, in this case the off-diagonal elements, i.e., $\Delta\varepsilon_{eo}$ and $\Delta\varepsilon_{oe}$, are zero. By associating slowly varying envelopes with the extraordinary and ordinary polarizations, $\phi_e(x,z)$ and $\phi_o(x,z)$, then one quickly finds the following set of self-coupled nonlinear evolution equations [12]:

$$2ik_e\frac{\partial\phi_e}{\partial z}+\frac{\partial^2\phi_e}{\partial x^2}+k^2\Delta\varepsilon\,\phi_e=0, \tag{1a}$$

$$2ik_o\frac{\partial\phi_o}{\partial z}+\frac{\partial^2\phi_o}{\partial x^2}+k^2\Delta\varepsilon\,\phi_o=0, \tag{1b}$$

where $k=2\pi/\lambda$ and λ is the free-space wavelength of the lightwave used, and $\Delta\varepsilon=\Delta\varepsilon_{ee}=\Delta\varepsilon_{oo}$. The wave numbers k_e and k_o are defined as $k_e=k\hat{n}_e$ and $k_o=kn_o$, where $\hat{n}_e$ and n_o are the refractive indices seen by the extraordinary and ordinary components when $\theta=11.9°$. Here, n_o is taken to be 2.286 and $n_e=2.200$ at $\lambda=0.633$ μm [14]. Using these values it can be readily shown that the effective refractive index for the extraordinary wave is $\hat{n}_e=2.282$ when $\theta=11.9°$. The relative permittivity changes $\Delta\varepsilon_{ee}$ and $\Delta\varepsilon_{oo}$ can be expressed as $\Delta\varepsilon_{ee}=-r_{\mathrm{eff},e}\hat{n}_e^4E_{\mathrm{SC}}$ and $\Delta\varepsilon_{oo}=-r_{\mathrm{eff},o}n_o^4E_{\mathrm{SC}}$, where $r_{\mathrm{eff},e}$ and $r_{\mathrm{eff},o}$ are the effective electro-optic coefficients for the extraordinary and ordinary polarizations, respectively. E_{SC} represents the space-charge electric field. For a wave propagating in $LiNbO_3$ at an angle θ with respect to

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the c axis, and provided that the space-charge electric field is x directed [15], the effective electro-optic coefficients are given by $r_{\mathrm{eff},e}=[r_{11}\cos^3(\theta)-(r_{13}+2r_{51})\cos^2(\theta)\sin(\theta)-r_{33}\sin^3(\theta)]$ and $r_{\mathrm{eff},o}=-[r_{11}\cos(\theta)+r_{13}\sin(\theta)]$. By using typical values for the electro-optic coefficients of $LiNbO_3$ [14] and for $\theta=11.9°$, $\Delta\varepsilon$ can be easily evaluated and is given by $\Delta\varepsilon=235.85\times10^{-12}E_{\mathrm{SC}}$, where E_{SC} is measured in units of volts per meter. Moreover, under strong bias conditions and for relatively broad beam configurations, the steady-state space-charge electric field is approximately given by [9]

$$E_{\mathrm{SC}}(x,z)=E_0\frac{I_d+I_\infty}{I_d+I(x,z)},\tag{2}$$

where $I=I(x,z)$ is the power density of the optical wave front and it is related to the slowly varying envelopes ϕ_e and ϕ_o through Poynting's vector, i.e., $I=(\hat{n}_e/2\eta_0)|\phi_e|^2+(n_o/2\eta_0)|\phi_o|^2$. In Eq. (2), I_d is the so-called dark irradiance which phenomenologically accounts for the thermal generation of electrons in the conduction band, I_∞ represents the constant power density the vector pair attains away from the center of the PR crystal, i.e., $I_\infty=I(x\to\pm\infty,z)$, and E_0 is the value of the space-charge electric field also at $x\to\pm\infty$. If the spatial extent of the optical waves involved is much less than the x width W of the PR crystal, then under a constant voltage bias V, E_0 is approximately given by $\pm V/W$ [9]. Moreover, for simplicity, let us adopt the following dimensionless variables and coordinates, i.e., let $\xi=z/(k_o x_0^2)$, $s=x/x_0$, $\phi_e=(2\eta_0 I_d/\hat{n}_e)^{1/2}U$ and $\phi_o=(2\eta_0 I_d/n_o)^{1/2}V$. x_0 is an arbitrary spatial width, and the power densities of the optical beams have been scaled with respect to the dark irradiance I_d. By employing these latter transformations and by substituting Eq. (2) in Eq. (1), the normalized planar envelopes U and V are found to satisfy

$$i(\hat{n}_e/n_o)\frac{\partial U}{\partial\xi}+\frac{1}{2}\frac{\partial^2 U}{\partial s^2}-\beta(1+\rho)\frac{U}{1+|U|^2+|V|^2}=0,\tag{3a}$$

$$i\frac{\partial V}{\partial\xi}+\frac{1}{2}\frac{\partial^2 V}{\partial s^2}-\beta(1+\rho)\frac{V}{1+|U|^2+|V|^2}=0,\tag{3b}$$

where $\rho=I_\infty/I_d$ and under this orientation ($\theta=11.9°$ in $LiNbO_3$) $\beta=-(235.85\times10^{-12}/2)(kx_0)^2E_0$. For simplicity any loss effects have been omitted in Eq. (3).

To find the bright-dark solitary pair solutions of Eq. (3) let us express the normalized envelopes U and V in the following way: $U=r^{1/2}f(s)\exp[i(n_o/\hat{n}_e)\mu\xi]$ and $V=\rho^{1/2}g(s)\exp(i\nu\xi)$, where the beam profiles f and g are normalized real functions. Here we have assumed, without any loss of generality, that the extraordinary envelope U is bright, whereas the ordinary one, V, is darklike. Hence, $f(0)=1$, $\dot{f}(0)=0$, $f(s\to\pm\infty)=0$, $g(0)=0$, $g(s\to\pm\infty)=\pm1$, and moreover, all the derivatives of f and g are assumed to vanish at infinity. The positive variable r represents the ratio of the maximum power density of the bright wave with respect to the dark irradiance I_d. By substituting these forms of U and V in Eqs. (3) we find

$$\ddot{f}=2\left(\mu+\frac{\beta(1+\rho)}{1+rf^2+\rho g^2}\right)f,\tag{4a}$$

$$\ddot{g}=2\left(\nu+\frac{\beta(1+\rho)}{1+rf^2+\rho g^2}\right)g,\tag{4b}$$

where $\ddot{f}=d^2f/ds^2$, etc. At this point, let us search for particular solutions which also satisfy the condition $f^2+g^2=1$. In this case, Eqs. (4) take the form

$$\ddot{f}=2\left(\mu+\frac{\beta}{1+\delta f^2}\right)f,\tag{5a}$$

$$\ddot{g}=2\left[\nu+\frac{\beta}{1+\delta(1-g^2)}\right]g,\tag{5b}$$

where the parameter δ is defined as $\delta=(r-\rho)/(1+\rho)$. From Eq. (5) and by the use of the f-g boundary conditions, the values of the constants μ and ν can be readily obtained and are given by $\mu=-(\beta/\delta)\ln(1+\delta)$ and $\nu=-\beta$ [9]. Equations (5a) and (5b) can now be solved perturbatively provided that $|\delta|\ll1$, that is when the peak intensities of the two vector components are approximately equal. In this case,

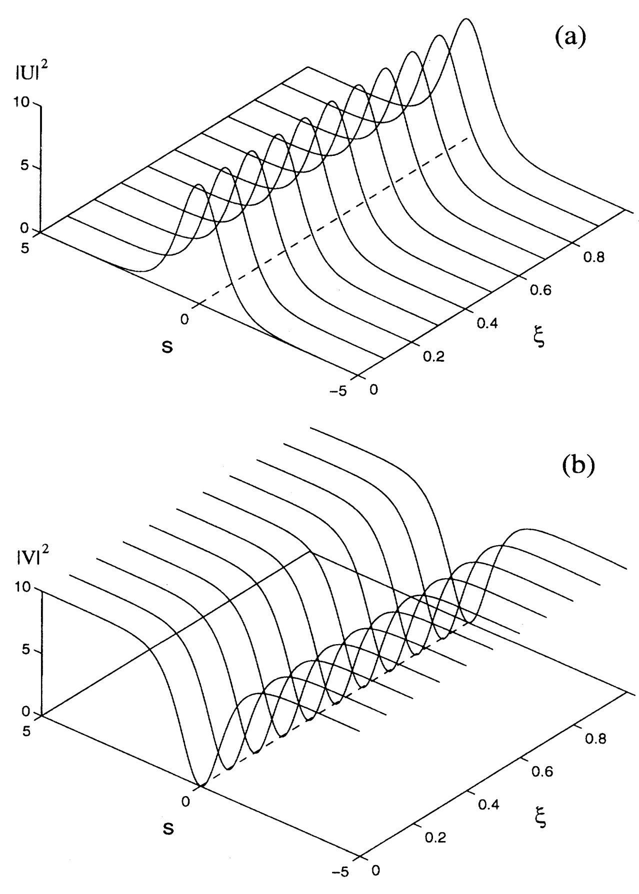

FIG. 1. Stable propagation of the (a) bright and (b) dark components of the vector soliton pair when $\beta=-27.9$, $r=9.45$, and $\rho=10$.

μ is approximately given by $\mu \cong -\beta[1-(\delta/2)]$. Moreover, in this limit, a first order expansion of the term associated with β yields

$$\ddot{f}=\beta\delta f-2\beta\delta f^3, \tag{6a}$$

$$\ddot{g}=-2\beta\delta g+2\beta\delta g^3. \tag{6b}$$

It can be directly shown that Eqs. (6) exhibit closed form solutions of the form $f=\text{sech}[(\beta\delta)^{1/2}s]$ and $g=\tanh[(\beta\delta)^{1/2}s]$. Note that these solutions are also consistent with our previous condition $f^2+g^2=1$. Thus, the approximate vector pair solutions of Eqs. (3) (for $|\delta|\ll 1$) are given by

$$U=r^{1/2}\text{sech}[(\beta\delta)^{1/2}s]\exp\{-i\beta(n_o/\hat{n}_e)[1-(\delta/2)]\xi\}, \tag{7a}$$

$$V=\rho^{1/2}\tanh[(\beta\delta)^{1/2}s]\exp(-i\beta\xi). \tag{7b}$$

Equation (7) clearly shows that these solutions are possible provided that $\beta\delta$ is a positive quantity. Furthermore, it is very interesting to note that these solutions exist irrespective of the bias polarity. More specifically, if E_0 is negative, in which case β is positive, $\delta=(r-\rho)/(1+\rho)$ can be judiciously chosen so that $\beta\delta>0$, as long as $|\delta|\ll 1$. On the other hand, when $\beta<0$, δ should be negative, which in turn implies that the intensity of the dark component is slightly higher than that of the bright. It is also noteworthy to point out that in the absence of vector pairs (i.e., when either U or V is zero), bright solitons are possible in the regime of negative bias polarity, i.e., $\beta>0$. On the other hand, dark solitons can exist alone when E_0 is positive, or $\beta<0$ [8,9]. The intensity full width at half maximum (FWHM) of these vector solitons can also be explicitly obtained and is given by FWHM $=1.76x_0(\beta\delta)^{-1/2}=1.62\times10^5(-k^2E_0\delta)^{-1/2}$. Evidently, for a given physical system, the spatial extent of these waves depends only on two variables, namely, E_0 and δ.

In order to investigate the stability of the bright-dark pair solutions, Eqs. (3) have been solved numerically using a beam propagation method. The normalized vector envelopes given by Eqs. (7) have been used as the input beam profiles. As an example, let us assume that the $LiNbO_3$ crystal, which is oriented at $\theta=11.9°$, is biased by $E_0=15$ kV/cm. If the arbitrary spatial scale x_0 is taken to be 40 μm and $\lambda=0.633$ μm, then $\beta=-27.9$. Let us also assume that $\rho=10$ and $r=9.45$, in which case $\delta=-0.05$. Figure 1 depicts the evolution of the bright-dark vector pair obtained under these conditions. The solitary behavior of this vector structure is of course evident in this figure since the pair propagates unchanged. Note that $\xi=1$ corresponds to an actual propagation distance of approximately 3.6 cm. The stability of these pairs

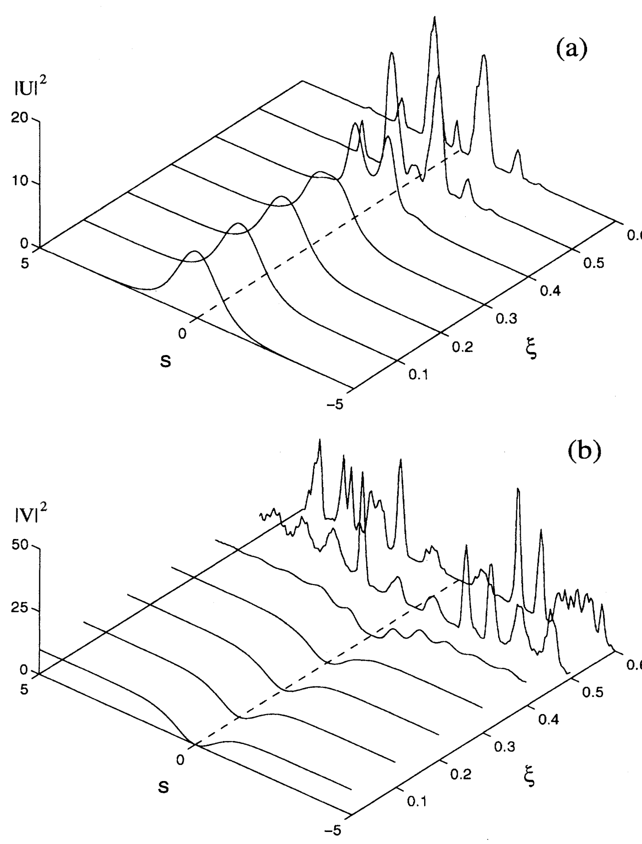

FIG. 2. Unstable propagation of the (a) bright and (b) dark components of the vector soliton pair when $\beta=27.9$, $r=10.55$, and $\rho=10$.

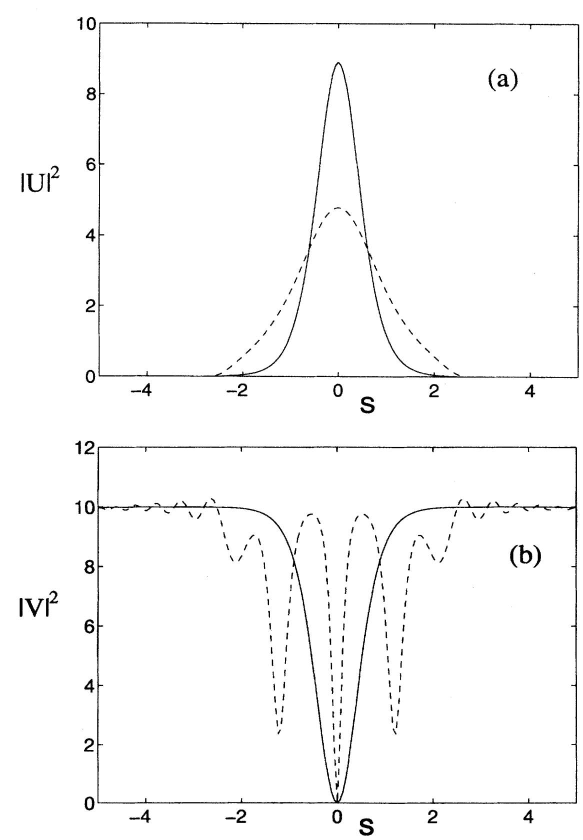

FIG. 3. Intensity profiles of the (a) bright and (b) dark components of a vector soliton pair when propagating in the absence of each other. The input (solid line) was obtained at $\beta=-27.9$, $r=8.9$, and $\rho=10$. The output, at $z=1$ cm, is depicted by the dashed line.

has also been considered numerically. In particular, we have found that these bright-dark waves are stable, against small perturbations in amplitude and width, up to distances of several centimeters. Conversely, a very different picture emerges when the polarity of the bias field is reversed. In this case, the evolution of the vector pair is illustrated in Fig. 2, when $\beta=27.9$, $\rho=10$, $r=10.55$, and $\delta=0.05$. In this regime, the plane-wave tails of the dark wave (and eventually the vector pair as a whole) disintegrate as a result of modulational instability [16]. This happens at an approximate distance of $\xi=0.4$. Note that modulational instability is possible only for positive β's or negative polarities, i.e., when single-component bright solitons can exist in this crystal system. The evolution of the bright-dark pair was also investigated when θ deviates from 11.9°. Using computer simulations, we have found that the dark-bright pair behavior still persists up to distances of 4 cm when $\theta=11.9°\pm5°$. These results were obtained for the same parameters used above and for negative β's so that modulational instability is absent.

It is also very interesting to study the evolution of a single component of such a pair in the absence of the other, that is, when either U or V is zero. This behavior may be essential if one is to confirm their existence experimentally. Figure 3(a) depicts the bright optical beam at the input and at $z=1$ cm, when the dark wave component is not present. In this case, $E_0=15$ kV/cm, $\beta=-27.9$, $r=8.9$, and $\rho=10$. The FWHM of the optical beam at the input and at $z=1$ cm is 42.2 μm and 80.8 μm, respectively. Therefore, in the absence of the dark component, the bright beam has undergone a 90% expansion at $z=1$ cm. Similarly, the evolution of the dark wave in the absence of the bright one, i.e., when $U=0$, is shown in Fig. 3(b), when $\beta=-27.9$, $r=8.9$, and $\rho=10$. In this case, the dark wave breaks up into multiple darklike waves. Thus, the above results indicate that such vector soliton behavior could be easily detected experimentally.

Finally, it can be shown that the value of the bias voltage V can be readily obtained in terms of relevant parameters. By employing $V=\int E_{SC}\,dx$ and Eqs. (2) and (7), one finds

$$V=E_0W\left[1-\delta\frac{\tanh(Q)}{Q}\right], \tag{8}$$

where $Q=5.43\times10^{-6}kW(-E_0\delta)^{-1/2}$. Keeping in mind that $|(\delta/Q)\tanh(Q)|\ll1$, then $E_0\cong V/W$, which is in accord with our previous statements.

In conclusion, we have shown that the self-coupled vector beam evolution equations in properly oriented biased PR media can exhibit bright-dark soliton pair solutions under steady-state conditions. These wave pairs were obtained perturbatively provided that the intensities of the two optical beams are approximately equal. Our analysis indicates that these bright-dark vector solitons exist irrespective of the polarity of the external bias field. The stability of our solutions has been investigated numerically and we have found that they are stable only in the regime of positive bias polarity. Conversely, when the polarity is negative, the pair tends to disintegrate due to modulational instability. Several other issues concerning their possible experimental observation have also been addressed.

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