RAPID COMMUNICATIONS

PHYSICAL REVIEW E VOLUME 53, NUMBER 3 MARCH 1996

Electron cavitation and acceleration in the wake of an ultraintense, self-focused laser pulse

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(Received 15 November 1994; revised manuscript received 24 July 1995)

A particle code is used to calculate the plasma response to the relativistic ponderomotive force of a short intense laser pulse propagating in a tenuous plasma. The regime of total cavitation of the plasma is investigated. It is found that stable propagation over a long distance is possible in this regime, and that energetic electrons are produced with a simple characteristic dependence of their angle of deflection on energy.

PACS number(s): 52.40.Db, 42.65.Jx, 52.35.Mw, 52.40.Nk

Channeling of intense optical fields in plasmas is an important challenge, with possible applications in the context of laser plasma accelerators and x-ray lasers. It has been shown in recent papers that self-channeled intense laser pulses are subject to severe instabilities of the Raman type which modulate the laser pulse and erode its tail [1–7] or cause the laser pulse to veer off its axis [8,9]. The simulations and theory of Refs. [1–4] were based on laser-plasma fluid models corresponding to a cold plasma. This prevents one from treating situations where the plasma motion reaches the wavebreaking limit and where fast electrons are generated in the interaction. In addition, these models contain a mathematical singularity at zero plasma electron density which prevents their use when the electrons are totally expelled from the axis of the laser propagation (electron cavitation). Such features are strong limitations of the fluid models in the high intensity regime. An alternative to the fluid models is the particle in cell (PIC) technique [6,7]. This technique follows the evolution of the laser radiation on the short time scale associated with the laser period, and thus is computationally intensive and restrictive in the parameters that can be studied.

In the following, a particle model is used to describe the long-time plasma behavior under the action of an ultrahigh intensity (of the order of 10^{18} W/cm^2 or more), short laser pulse (1 psec or less). Among the results we obtain, we emphasize the following. (i) Relativistic focusing for short laser pulses is possible over a long distance ($>30l_R$) with total electron cavitation in the laser channel and strong reduction of the Raman type plasma instabilities ($l_R=\omega_0 r_L^2/2c$ is the Rayleigh length, ω_0 is the laser frequency, r_L is the laser spot size, and c is the light velocity). This self-focused propagation does not need the help of a performed plasma channel [10] which is necessary at moderate intensities. (ii) Plasma electrons can be ejected with relativistic energies in the wake of the laser pulse. The angle of ejection is related simply to the electron energy as a consequence of momentum and energy conservation.

The model is fully nonlinear, relativistic, and two dimensional [11]. It uses the quasistatic approximation (QSA) [12], which assumes that the electron transit time through the laser pulse is short compared with the characteristic laser pulse deformation time. We assume that the laser wavelength (frequency) is much smaller (greater) than all the other characteristic lengths (times) in the system. In particular, $\omega_p \ll \omega_0$ and $r_L \gg c/\omega_0$, where $\omega_p=(4\pi q^2 n_0/m)^{1/2}$ is the plasma frequency based on the ambient density n_0, and $q=-e$ and m are the charge and mass of an electron. These inequalities enable one to expand the equation of motion of the electrons and the wave equation in powers of the small parameter ω_p/ω_0.

The envelope approximation is used to describe the laser pulse propagation. We separate the rapidly varying phase of the radiation vector potential and its slowly varying amplitude,

$$\mathbf{A}(\mathbf{r},t)=\mathbf{A}(\mathbf{r}_\perp,\xi,t)\exp[-i\omega_0(t-z/c)]+\text{c.c.}, \qquad (1)$$

where $\xi=ct-z$. The wave equation is

$$\left(2i\frac{\omega_0}{c^2}\frac{\partial}{\partial t}-\frac{2}{c}\frac{\partial^2}{\partial t\partial\xi}+\nabla_\perp^2\right)\mathbf{A}=\frac{\omega_p^2}{c^2}\frac{n}{n_0}\overline{\gamma^{-1}}\mathbf{A}, \qquad (2)$$

where we have dropped the term $\partial^2/\partial t^2$ which is here of order $(\omega_p/\omega_0)^2$. Here n is the slowly varying component of the electron density and $\overline{\gamma^{-1}}$ is the averaged value of the inverse of the slowly varying component of the relativistic factor. The average is taken on the distribution function of the electrons. Note that retaining the $\partial^2/\partial t\partial\xi$ term enables one to treat forward Raman scattering and pump depletion which are important phenomena when ω_p/ω_0 is not too small. On the other hand, the envelope approximation precludes the treatment of backward Raman scattering which, however, is seen to saturate rapidly in full PIC simulations [6].

The density appearing in Eq. (2) results from the forced excitation of a plasma disturbance by the ponderomotive potential of the radiation. Averaging the equation of motion of the electrons over the laser period results in

$$\frac{d\mathbf{p}}{dt}=q\left(\mathbf{E}_s+\frac{\mathbf{v}}{c}\times\mathbf{B}_s\right)-\frac{q^2}{2\gamma mc^2}\nabla\langle A^2\rangle, \qquad (3)$$

where all quantities except the radiation vector potential **A** are slowly varying quantities. Here $\langle \rangle$ denotes the average over the short period of the laser light. Due to the QSA, the validity of this equation is limited to the electrons whose axial velocity is smaller than the group velocity of the laser light. In particular, for high energy electrons accelerated in the forward direction the limitation is $\gamma < \omega_0/\omega_p$.

The slowly varying component of the relativistic factor simplifies to [11]

$$\gamma = \left(1 + \frac{p^2}{m^2c^2} + \frac{q^2\langle A^2\rangle}{m^2c^4}\right)^{1/2}. \tag{4}$$

Equation (3) can be deduced from the averaged (ponderomotive) Hamiltonian

$$H(\mathbf{r},\mathbf{P},t) = [m^2c^4 + (\mathbf{P}_c - q\mathbf{A}_s)^2 + q^2\langle A^2\rangle]^{1/2} + q\Phi_s, \tag{5}$$

where $\mathbf{A}_s$ and Φ_s are the slowly varying vector and scalar potentials associated with the wake fields, and **P** is the canonical momentum, $\mathbf{P} = \mathbf{p} + q\mathbf{A}_s/c$. In the QSA H depends on z and t only via $\xi = ct - z$. This leads to the constancy of $H - cP_z$ via

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} = c\frac{\partial H}{\partial \xi} = -c\frac{\partial H}{\partial z} = c\frac{dP_z}{dt}, \tag{6}$$

so for electrons initially at rest

$$\gamma mc^2 + q\Phi_s - cp_z - qA_{s,z} = mc^2. \tag{7}$$

Therefore it is sufficient to solve for the radial component of the electron momentum for which one can write (for a two-dimensional axisymmetric laser pulse)

$$\frac{dp_r}{d\xi} = \frac{1}{1+\Psi}\left(\gamma\frac{\partial}{\partial r}\Psi - \frac{1}{2}\frac{\partial}{\partial r}\langle a^2\rangle\right) - b_\theta, \tag{8a}$$

$$\frac{dr}{d\xi} = \frac{p_r}{1+\Psi}, \tag{8b}$$

where we use normalized quantities, $\mathbf{a}(r,t) = q\mathbf{A}/mc^2$, $\mathbf{p} = \mathbf{p}/mc$, $\Psi = e(\Phi_s - A_{s,z})/mc^2$, $r = k_p r$, $\xi = k_p\xi$ (where $k_p = \omega_p/c$), and $\mathbf{b} = e\mathbf{B}_s/m\omega_p c$ (note Ψ is independent of the gauge). The slowly varying component of the field quantities satisfies the Maxwell equations which can be written (in the QSA)

$$(\Delta_\perp - 1)\frac{\partial^2}{\partial \xi^2}\Psi = \frac{1}{r}\frac{\partial}{\partial r}r\left(\frac{\partial j_r}{\partial \xi} - b_\theta\right) + j_z, \tag{9a}$$

$$\frac{1}{r}\frac{\partial}{\partial r}rb_\theta = \frac{\partial^2}{\partial \xi^2}\Psi + j_z, \tag{9b}$$

where $\mathbf{j} = \mathbf{j}/n_0ec$ is the normalized current density. Equations (2), (4), and (7)–(9) fully describe the laser plasma interaction in the QSA. These equations reduce to the model described in Ref. [2] when the electron flow is laminar.

We have solved the set of equations on both a two-dimensional Cartesian grid (ξ,x) and a cylindrical grid $(\xi,r_\perp)$ with a numerical method which is implicit as far as weakly nonlinear terms are concerned [1] and where the terms $1+\Psi$ and b_θ appearing in Eqs. (8) and (9a) are treated with a predictor-corrector scheme. Once the particle trajectories are computed, we collect the terms contributing to the right-hand side of Eq. (2) to advance the laser field in time. Finally we also treat the ions' motion and contribution to the source in Maxwell equations. However, except for quite long laser pulses, when the ions tend to be expelled from the laser channel by the ambipolar field due to the electron expulsion, the simulation results are almost insensitive to whether or not the ions are allowed to move.

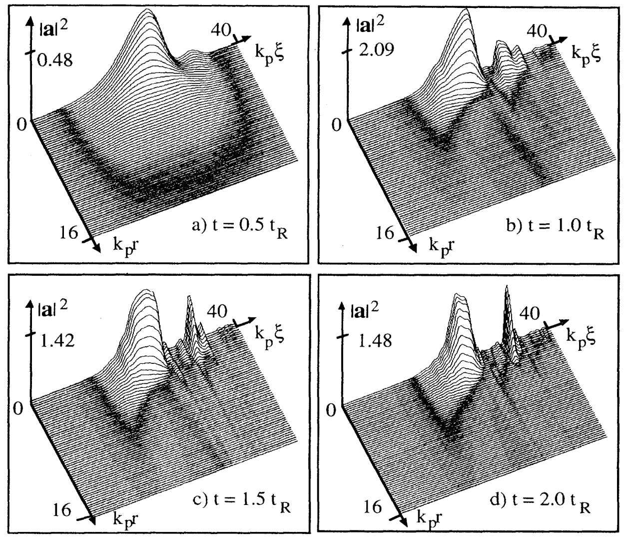

FIG. 1. Time evolution of the laser intensity for the case of a moderate intensity and large focal spot ($a_0 = 0.25$, $k_pr_L = 16$).

We verified that the code gives identical results with Ref. [1] (slab geometry) in the weakly relativistic limit and with Ref. [2] (cylindrical geometry) when the electron flow is laminar. We also compared the code with the results of a PIC code [6] for values of ω_p/ω_0 as large as 0.2, and in situations where the QSA is only marginally valid, and the code gave very close results, with a lower noise in our case.

We now show the result of a high intensity laser pulse propagating in a tenuous plasma in cylindrical geometry. Initially, the radiation is of the form

$$a(r,\xi,t=0) = a_0\sin(\pi\xi/L)\exp(-r^2/r_L^2).$$

The following parameters were chosen: $a_0 = 0.25$, $k_pL = 40$, $k_pr_L = 16$, and $\omega_p/\omega_0 = 0.03$, which corresponds to a plasma of density 10^{18} cm^{-3} for a 1 μm laser light. Figure 1 shows surface plots of the laser intensity at four times during the simulation. The pulse is about twice the critical power for relativistic self-focusing [13,14] [$P_c = 16.2(\omega_0/\omega_p)^2 \times 10^9$ W]. As expected, we observe that the pulse is subject to the Raman self-focusing instability [1]. However, we observe that the instability develops slightly slower than predicted by the weakly nonlinear model of Ref. [1]. After one Rayleigh length, we observe that the pulse has been separated into two pulses which are still focused and propagate on about two Rayleigh lengths, and then diffracts quickly after a strong pump depletion [7]. The same case run with $\omega_p/\omega_0 = 0$

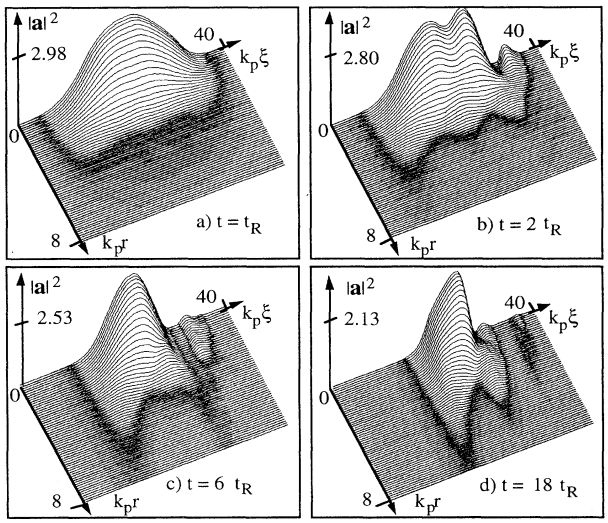

FIG. 2. Time evolution of the laser intensity for the case of a large intensity and small focal spot ($a_0=1.0$, $k_pr_L=4$).

would propagate on about 10 Rayleigh lengths, which shows the importance of the forward Raman term here.

Figure 2 corresponds to the same laser power, but to a smaller focal spot, i.e., $a_0=1.0$, $k_pr_L=4$. In this case, we observe a total expulsion of the electrons from the laser channel which stabilizes the Raman instability in the bulk of the laser pulse. As a result, the laser pulse propagates over a large distance (more than 30 Rayleigh lengths) with a radial profile similar to the one predicted in the model of Sun *et al.* [14]. The front part of the pulse is eroded as predicted by Sprangle *et al.* [12]. The fact that self-focused propagation on distances much larger than the Rayleigh length is possible is in agreement with recent experimental results [15]. We verified here that the role of the forward Raman term was small in this case, in contrast with the case of Fig. 1.

Figure 3 shows the electron density after one Rayleigh length for the same case as Fig. 2. One observes total electron cavitation where the laser intensity is large, enhancement of the density on the sides of the self-focused channel, and a strong peak behind the laser pulse due to electrons which come back towards the center of the channel under the action of the charge separation field. This behavior is similar to the one observed in the case of a nonlinear plasma wake

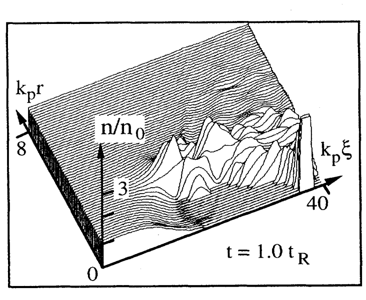

FIG. 3. Electron density after one Rayleigh length (same case as Fig. 2). The electron density has been limited to $3n_0$ on this curve.

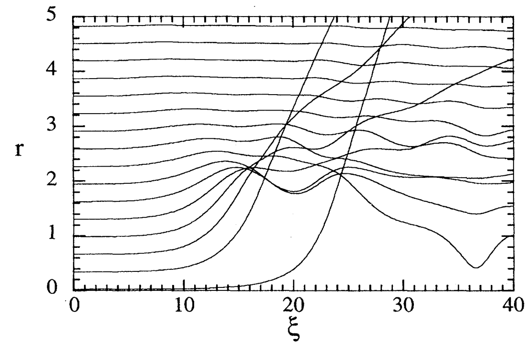

FIG. 4. Sample electron trajectories for the case of Figs. 2 and 3, after the laser pulse has propagated 1 Rayleigh length in the plasma.

field [16]. Suppression of Raman instabilities may be attributed to a number of effects. The electron density is reduced where the laser intensity is greatest, the plasma channel is inhomogeneous which disturbs the plasma wave resonance [17], and the channel density has a maximum at a radius just greater than the spot size. This last effect contributes to enhancing the diffraction of radiation which is sidescattered and thus suppresses sidescattering instabilities [18].

Figure 4 shows sample electron trajectories after the laser has propagated 1 Rayleigh length in the plasma. In this example the electron flow is laminar until $k_p\xi\approx 15$. For $k_p\xi>15$, the radial wavebreaking of the plasma disturbance results in a multipeaked electron distribution function and the ejection of fast electrons in the MeV range. The validity of fluid codes would be limited to $k_p\xi<15$ here. Figure 5 shows the cross section $\sigma(E)$ for the generation of electrons with energy greater than E. This cross section is generated after the laser pulse has propagated 30 Rayleigh lengths in the plasma. It is averaged over the propagation length and normalized to $\sigma_0=2\pi(c/\omega_p)^2$. The rate of production R of electrons with energy greater than E is given by

$$R=\sigma(E)n_0c=5.3\times 10^{22}\frac{\sigma(E)}{\sigma_0}\ \sec^{-1}. \tag{10}$$

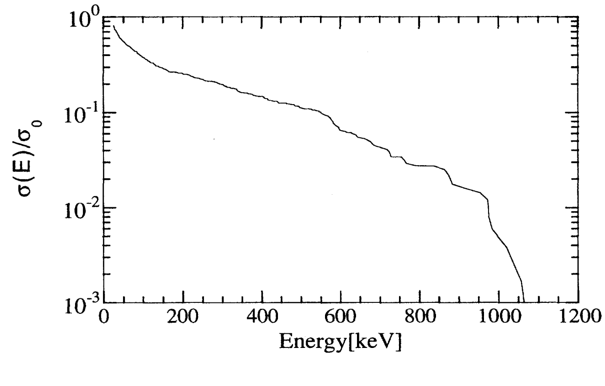

FIG. 5. Cross section for the generation of electrons with energy greater than E in the case of Figs. 2–4. The fast electrons are collected over the first 30 Rayleigh lengths.

In the example shown in Fig. 5 the maximum Lorentz factor of the accelerated electrons is of the order of 3, which is well below the validity limit of the ponderomotive approach [Eq. (3)] since here $\omega_0/\omega_p = 33$. Due to the constraint, Eq. (7), the ejected electrons (for which $\psi=0$) satisfy the relation $p_z = \frac{1}{2}p_r^2$. Note that this relation applies to the electrons accelerated by the ponderomotive force (and self-consistent plasma fields), and that it has the same form as the relation between the axial and the radial jitter momentum of an electron in a *plane* wave [19]. The relation maybe reexpressed in terms of the angle θ made by the electron with respect to the axis of the laser,

$$\cos\theta = \left(\frac{\gamma-1}{\gamma+1}\right)^{1/2}. \quad (11)$$

A simple interpretation of this result can also be given by the analysis of the collision of a photon packet (of total energy $E \gg mc^2$) with a single electron when the energy loss of the radiation is small (multiphoton Compton effect with small photon deflection) [20]. It is interesting to note that the relation holds even in the presence of large wake fields, once the electron has escaped from the region of the wake. This relation can be expected to apply so long as the radiation wave vector remains in the forward direction. Electrons satisfying this relation were recently observed by Meyerhofer *et al.* in a laser-gas experiment [21].

We have studied stable propagation of ultraintense laser pulses through tenuous plasma in the region of total electron cavitation with a particle code calculating the particle trajectories on the plasma period time scale. Relativistic electrons are ejected from the wake of the pulse in a cone whose angle decreases with energy.

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