Symmetric and asymmetric solitons in twin-core nonlinear optical fibers

B. A. Malomed,* I. M. Skinner, P. L. Chu, and G. D. Peng
School of Electrical Engineering, University of New South Wales, Sydney 2052, Australia

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Static soliton states in twin-core nonlinear optical fibers are examined using an improved variational approximation: the soliton's width is an additional varying parameter, together with the ratio of the energies in, and the phase difference between, the two cores. For the symmetric coupler, results agree well with numerical ones; in particular, the bifurcation between symmetric and asymmetric solitons is shown to be slightly subcritical. For the asymmetric coupler, the control parameters are the difference between the cores' dispersion coefficients and the phase velocity mismatch. Soliton states in the asymmetric coupler show a strong and easily controlled bistability. The soliton exists for some energies even when one of the cores has normal dispersion.

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I. INTRODUCTION

Since the pioneering works by Jensen [1] and Maier [2], twin-core nonlinear fibers (couplers) have been one of priority topics in fiber optics research. These couplers are expected to find important applications in photonics, e.g., as all-optical switches [3], and they give rise to a number of challenging physical problems. Many works analyzed soliton dynamics in models of the nonlinear couplers (see, e.g., the review [4]), although they have not yet been observed experimentally.

More recently, there have appeared detailed analyses of static properties of solitons in the twin-core fiber. First of all, it was noted [5,6] that a symmetric soliton, i.e., one having energy equally divided between the cores, becomes unstable when its energy exceeds a certain critical value (see also Ref. [7]); it may be pertinent to mention that this instability has some similarity to the known instability of certain types of solitons in models of the single-core birefringent optical fiber, which are based on a pair of the nonlinear Schrödinger (NLS) equations with a *nonlinear* coupling [8]. Next [9], it was pointed out that this instability gives rise to a *bifurcation*: a pair of new, stable asymmetric solitons (symmetric with respect to each other) appears when the symmetric state loses its stability. Later [10–12], this bifurcation was analyzed in detail by means of numerical methods. In Ref. [9], the bifurcation was considered analytically, using essentially the same variational approximation that was employed in Ref. [6] for analysis of the soliton's switching in the coupler. As a result, a so-called *forward* bifurcation was predicted, when the asymmetric stable states appeared exactly at the point where the symmetric state lost its stability. On the other hand, looking at the bifurcation diagram obtained numerically in Ref. [12], one notices that, strictly speaking, this is a *backward* (subcritical) [13] bifurcation: the new stable asymmetric states appear at a value of the soliton's energy slightly smaller than that at which the symmetric soliton becomes unstable, and there is a bistability over a tiny interval of energies above the bifurcation (forward and backward bifurcations are analogous to classical phase transitions of the second and first order, respectively).

In all these works, the twin-core fiber was assumed absolutely symmetric. However, fabrication of asymmetric couplers is quite feasible too, and, accordingly, analysis of static soliton configurations in the asymmetric couplers is also of interest, both in itself and for applications. Generally, there are two different ways to produce an asymmetric dual-core fiber: to make diameters of the two cores different, or, keeping their effective cross section areas equal, to deform them differently. The model to be considered in the present work will, primarily, assume the latter mode of the asymmetry.

In this work, we develop a modified analytical approach to analyze the soliton configurations, and, especially, their bifurcations in symmetric and asymmetric couplers. This approach is based on an improvement to the variational approximation used in Refs. [6] and [9]. Therein, the shape of the soliton was assumed fixed, and the only variational parameters were the division of the soliton's energy and the relative phase between the two cores. In our approach we introduce the soliton's width as a third variational parameter. We show that this amendment to the variational approximation, rendering the wave forms more flexible, dramatically improves agreement between the analytical and numerical results. In particular, we find that the approximation identifies the bifurcation point fairly close to the known exact value [10], and, in accord with the numerical findings mentioned above, the bifurcation proves to be a backward one, with a narrow bistability region attached to it.

After testing the improved analytical approximation on the known case of the symmetric coupler, we proceed to the new case of the asymmetric twin-core fiber. The asymmetry includes mismatching the dispersions, mean phase velocities, and mean group velocities of the two cores, while their effective nonlinear constants are kept equal. The latter condition implies that the effective areas of the cores remain equal (at least, approximately), while the differences in the phase

*Permanent address: Department of Applied Mathematics, School of Mathematical Sciences, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Ramat Aviv, Tel Aviv 69978, Israel. Electronic address: malomed@leo.math.tau.ac.il

velocities and dispersions can be produced by deformation of the cores (the group-velocity difference will be actually neglected). The mismatch in the dispersion coefficients can be additionally contributed to by a small difference in the areas of the cores, which does not essentially affect the nonlinear coefficients, but may alter the effective dispersion coefficient of the fiber operating near the zero-dispersion point [14]. Using the same variational approximation, we produce a set of bifurcation diagrams for the asymmetric coupler, and these clearly demonstrate effects of the different mismatch factors. Consideration of the solitonic states in the asymmetric coupler is of obvious interest both from the academic viewpoint and for applications, as the presence of the additional parameters should allow one to obtain solitons with essentially novel properties. In particular, we show that, using the asymmetry between the cores, it is easy to produce a broad region of bistability, which may find use in optical switching and other applications. In this relation, it is relevant to mention the recent work [15], in which switching properties of the asymmetric coupler were considered in detail in the cw (continuous-wave, i.e., time-independent) regime. It was shown that the asymmetry may help to reduce the switching power, which is of direct interest for the applications.

It is also interesting to consider the case when the two cores in the asymmetric coupler are so widely different that their dispersion coefficients have opposite signs; i.e., one core exhibits anomalous dispersion, while the other one has normal dispersion (of course, in this case our assumption of the equal nonlinear coefficients becomes a serious technical limitation). Using our approximation, we consider this case too. We show that, above a certain value of the soliton's energy, the two coupled cores with opposite dispersions can support a soliton, which is an essentially novel result (in this work, we do not consider dark solitons).

The paper is organized as follows. The mathematical model of the coupler is presented in Sec. II, where we detail our analytical approximation. In Sec. III, we employ this approximation for the symmetric case, and compare the results, when possible, with known numerical data. In Sec. IV, new results for the asymmetric couplers are displayed. In Sec. V, we deal with the case when the dispersion coefficients in the two cores have opposite signs. Concluding remarks are collected in Sec. VI.

II. THE ANALYTICAL APPROXIMATION

The asymmetric coupler is described by a system of coupled NLS equations, which is a straightforward generalization of the well-known system for the symmetric case [4], as well as of that governing the cw dynamics of the asymmetric coupler [16]:

$$iu_z+qu+icu_\tau+\tfrac{1}{2}D_1u_{\tau\tau}+|u|^2u+Kv=0, \tag{1}$$

$$iv_z-qv-icv_\tau+\tfrac{1}{2}D_2v_{\tau\tau}+|v|^2v+Ku=0, \tag{2}$$

where K is the coupling constant. When writing the equations in this form, we assume that the effective cross section areas of the two cores are nearly equal, so that the corresponding effective nonlinear coefficients, determined by the areas [14], are (approximately) equal too. In this case, the phase and group velocity differences between the two cores, measured, respectively, by the parameters q and c, can be produced by different deformations of the two cores. This implies the same polarization of light in both cores, and that their birefringence axes are aligned. The deformation of the cores, affecting their modal structure, also gives rise to a difference in their dispersion coefficients. Notice that this difference can be essentially augmented by a relatively small mismatch in the effective areas between the cores, which produces a negligibly weak effect on the nonlinear coefficients. Indeed, since the nonlinear optical fibers usually operate near the zero-dispersion point, at which the terms produced by the material and waveguide dispersions nearly compensate each other [14], even a weak change of the latter term may significantly alter the effective dispersion coefficient. Note that the phase velocity mismatch between the two otherwise identical cores was mentioned, but not analyzed, in Ref. [17].

To translate the velocity differences into explicit z and τ dependence of the coupling constant, it is convenient to transform the fields u and v as follows:

$$u(z,\tau)\equiv U(z,\tau)\exp\left(-icD_1^{-1}\tau+iqz+\frac{1}{2}ic^2D_1^{-1}z\right); \tag{3}$$

$$v(z,\tau)\equiv V(z,\tau)\exp\left(icD_2^{-1}\tau-iqz+\frac{1}{2}ic^2D_2^{-1}z\right). \tag{4}$$

As well as Eqs. (1) and (2), the transformed equations for the variables U and V admit a variational representation with the Lagrangian

$$L=\int_{-\infty}^{+\infty}\mathscr{L}(\tau)d\tau, \tag{5}$$

where the Lagrangian density is

$$\mathscr{L}=K\{U^*V\exp[-2iQz+ic(D_1^{-1}+D_2^{-1})\tau]+UV^*\exp[2iQz-ic(D_1^{-1}+D_2^{-1})\tau]\}+\tfrac{1}{2}i(U^*U_z-UU_z^*)$$
$$+\tfrac{1}{2}i(V^*V_z-VV_z^*)-\tfrac{1}{2}D_1|U_\tau|^2-\tfrac{1}{2}D_2|V_\tau|^2+\tfrac{1}{2}|U|^4+\tfrac{1}{2}|V|^4. \tag{6}$$

Here,

$$Q \equiv q + \tfrac{1}{2}c^2(D_1^{-1} - D_2^{-1}) \tag{7}$$

is an *effective* phase velocity mismatch.

The crucial step in application of the variational approximation [6,18] is the choice of an ansatz, i.e., a trial wave form, for the soliton. Here, we use the same general ansatz that was proposed for the two-component soliton in the coupler in Refs. [9] and [17] (this ansatz was also employed for analysis of the vector-soliton dynamics in the model of a birefringent single-core fiber in Ref. [19]):

$$U = A\,\cos(\theta)\,\mathrm{sech}\left(\frac{\tau}{a}\right)\exp[i(\phi+\psi)+ib\tau^2]; \tag{8}$$

$$V = A\,\sin(\theta)\,\mathrm{sech}\left(\frac{\tau}{a}\right)\exp[i(\phi-\psi)+ib\tau^2] \tag{9}$$

(in Ref. [9], the amplitudes were defined as $A/\sqrt{a}$, but this difference does not change anything in the analysis below). Here, A is the overall amplitude, θ measures distribution of energy between the two cores, a is the soliton's width, which is assumed to be the same for both cores, ϕ and ψ are, respectively, the common and relative phases of the two components, and b is the well-known chirp parameter [18].

Notice that, with regard to the transformation (3) and (4), the ansatz based on Eqs. (8) and (9) implies that, in terms of the original notation [see Eqs. (1) and (2)], the two components of the soliton will have the frequency difference $c(D_1^{-1}+D_2^{-1})$. However, it is natural to assume the absence of the frequency difference just in terms of the transformed variables $U(z,\tau)$ and $V(z,\tau)$. Indeed, we are interested in consideration of the soliton with no walkoff between its components, and this is in terms of U and V that the walkoff is proportional to their relative frequency.

The standard procedure assumes all the parameters in the ansatz [Eqs. (8) and (9)] to be arbitrary functions of z. Then, one inserts the ansatz into the Lagrangian density [Eq. (6)] and integrates it over the variable τ to arrive at an effective Lagrangian [see Eq. (5)], which depends on the free parameters of the ansatz and their first derivatives with respect to z. Next, one obtains the Euler-Lagrange variational equations from this Lagrangian, and these lead to a system of coupled ordinary differential equations that, for a sufficiently smart choice of the underlying ansatz, furnish a good approximation to the full dynamics of the soliton system.

In Ref. [9], the chirp parameter b was dropped and the width a was assumed constant, in order to obtain a simple dynamical system (which actually proves to be equivalent to the one considered earlier in Ref. [6]) amenable to a fully analytical consideration. Although this system correctly indicated gross features of the soliton's statics and dynamics in the coupler, it was oversimplified; that is why it failed to predict essential details. In particular, in the static regime this simplest approximation correctly showed the existence of the bifurcation destabilizing the symmetric soliton state (in the symmetric coupler), but did not show the bifurcation to be slightly subcritical. Agreement between the approximate value of the soliton's energy at this bifurcation point and the known exact value was not satisfactory.

The only difference between the approximation developed in Ref. [9] and our approach is that, instead of the width a of the soliton being a predetermined constant, we now allow it to vary along with two other parameters, viz., θ and ψ. As will be shown below, this simple improvement keeps the system analytically tractable and, simultaneously, it dramatically improves the results.

For dynamical regimes (switching characteristics of the coupler, etc.), the full system of the variational equations corresponding to the ansatz based on Eqs. (8) and (9) was considered recently in Ref. [20]. The ordinary differential equations were solved numerically, and the results were compared in detail with a direct numerical solution of Eqs. (1) and (2) (obtained by means of the beam-propagation method). A very good accord over a broad parametric region was found.

In the static situation, one seeks solutions to Eqs. (9) and (10) with constant (z independent) a, ψ, and θ, while the common phase ϕ may be a linear function of z. In this case, the variational approximation produces a system of algebraic (rather than differential) equations. Moreover, it is known that the chirp parameter is always zero in the static case [18]. This simplifies the situation, and a fully analytical consideration is possible.

Thus one arrives at the following static equations produced by the variation, respectively, of ψ, θ, and a:

$$\sin(2\theta)\sin\chi = 0; \tag{10}$$

$$\frac{2E}{3a}\cos(2\theta) - 2\kappa\,\cot(2\theta)\cos\chi - \frac{D_1-D_2}{6a^2} + 2Q = 0; \tag{11}$$

$$a^{-1} = E\left[1 - \frac{1}{2}\sin^2(2\theta)\right](D_1\cos^2\theta + D_2\sin^2\theta)^{-1}, \tag{12}$$

where $\chi \equiv 2(\psi + Qz)$, the parameter E is half the energy of the soliton,

$$E \equiv \frac{1}{2}\int_{-\infty}^{+\infty}(|U|^2+|V|^2)\,d\tau = A^2 a, \tag{13}$$

and the renormalized coupling constant is

$$\kappa \equiv K\frac{(\pi/2)c(D_1^{-1}+D_2^{-1})a}{\sinh[(\pi/2)c(D_1^{-1}+D_2^{-1})a]}. \tag{14}$$

Notice here that since the renormalized coupling constant κ explicitly depends upon a according to Eq. (14), it, too, should be varied in a. This was not done above because, from examination of Eq. (14), it follows that the renormalization of the coupling constant and, hence, the variation of κ can be neglected if

$$\frac{\pi}{2}c(D_1^{-1}+D_2^{-1})a \ll 1,$$

i.e., if the group velocity mismatch c is sufficiently small. This is assumed below to be the case, and is quite realistic, since the difference between the group velocities is usually much smaller than the corresponding mismatch in the phase velocities; i.e., c may be neglected, but q must be retained in

Eqs. (1) and (2). Hence, also, in subsequent sections K and q remain, respectively, the effective coupling constant and the phase-velocity mismatch, instead of κ and Q.

Finally, variation with respect to the overall amplitude A (more convenient technically is to perform, instead, variation in E) produces the expression for $p \equiv d\phi/dz$, which gives a shift of the carrier wave's propagation constant:

$$p=-\frac{1}{6a^2}(D_1\cos^2\theta+D_2\sin^2\theta)+\frac{2E}{3a}\left(1-\frac{1}{2}\sin^2(2\theta)\right)+\kappa\,\sin(2\theta)\cos\chi. \tag{15}$$

Note that this is decoupled from the other equations.

It immediately follows from Eq. (10) that either $\sin(2\theta)=0$ or $\sin\chi=0$. The former solution implies that all the energy resides in a single core [see Eqs. (8) and (9)]. Obviously, this solution is extraneous. The latter solution, $\sin\chi=0$, implies that $\cos\chi=\pm 1$. According to the numerical findings of Ref. [11], in the symmetric case the solutions corresponding to $\cos\chi=-1$ are almost everywhere unstable, except for a narrow range of low energies. Therefore, in this work we consider only the case $\cos\chi=+1$. Substituting this and Eq. (12) into Eq. (11), we obtain

$$\frac{1}{3}E^2\frac{1-(1/2)\sin^2 2\theta}{D_1\cos^2\theta+D_2\sin^2\theta}\cos(2\theta)-\frac{1}{12}(D_1-D_2)E^2\left(\frac{1-(1/2)\sin^2 2\theta}{D_1\cos^2\theta+D_2\sin^2\theta}\right)^2 \text{[illegible]} +Q=0. \tag{16}$$

This is the basic equation of our analytical approximation for the static solitons in the coupler.

To conclude this section, notice that the simplest approximation, considered in Refs. [9,21], produced equations that were the same as analogous equations for the cw coupling (the analogy of the soliton switching with the cw case was discussed in detail in Ref. [22]). Now, by making the width of the soliton a parameter of variation, the equations become essentially different from those for the cw case. The reason is that previously the postulated width was fixed, independently of the value of θ, while now it is allowed to depend on θ.

In this relation, it is pertinent to note that a subcritical bifurcation is known [16] to be a feature of the cw modes of optical couplers in the case when the nonlinearity in the coupler is saturable rather than cubic (Kerr). Actually, the variational equations that will be derived in the present work for the symmetric coupler on the basis of the soliton ansatz, can be formally represented as cw equations for a model with a special nonlinearity, much more complicated than the cubic one [21].

III. SYMMETRIC COUPLER REVISITED

In order to test the validity of using Eqs. (10), (11), and (12) to describe static soliton configurations, we first apply them to the symmetric case $D_1=D_2\equiv 1$, and $c=q=0$. Previous analytical results were obtained in Ref. [9] in the approximation in which the soliton's width a was not used as a variational parameter; i.e., Eq. (12) was absent. Extensive numerical results for soliton states in the symmetric coupler were given in Refs. [10–12], making it possible to assess the accuracy of the present analytical approximation.

In the symmetric case, Eq. (16) simplifies to

$$\cos(2\theta)\left[\frac{E^2}{3K}\sin(2\theta)\left(1-\frac{1}{2}\sin^2(2\theta)\right)-1\right]=0. \tag{17}$$

Elementary analysis reveals that, for $0<E<E_1$, where E_1 is given by

$$E_1^2=\frac{9}{4}\sqrt{6}K, \tag{18}$$

the only solution is the symmetric one, $\cos(2\theta)=0$, with equal energies in both components. At the point at which the soliton's energy attains the value E_1, there appear asymmetric solutions with $\cos(2\theta)=\pm 1/\sqrt{3}$. When the energy further increases to the value E_2, given by

$$E_2^2=6K, \tag{19}$$

which is only slightly larger than E_1, the backward (subcritical) bifurcation occurs, rendering the symmetric solution $\cos(2\theta)=0$ unstable.

For the symmetric coupler, the whole bifurcation diagram produced by Eq. (17) is shown in Fig. 1. Therein, the solid and dashed curves depict stable and unstable branches, respectively. Although we did not analyze stability of different solutions directly, one can readily distinguish between the

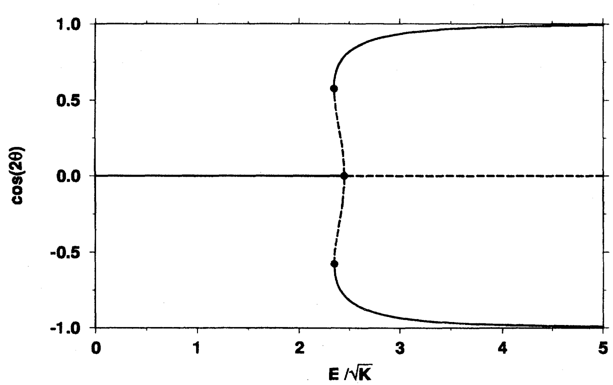

FIG. 1. The bifurcation diagram for the symmetric coupler produced by Eq. (17). Here, and in other figures, the solid and dashed curves correspond, respectively, to stable and unstable solutions, and the thick dots are used to highlight the bifurcation points (transitions between stable and unstable solutions).

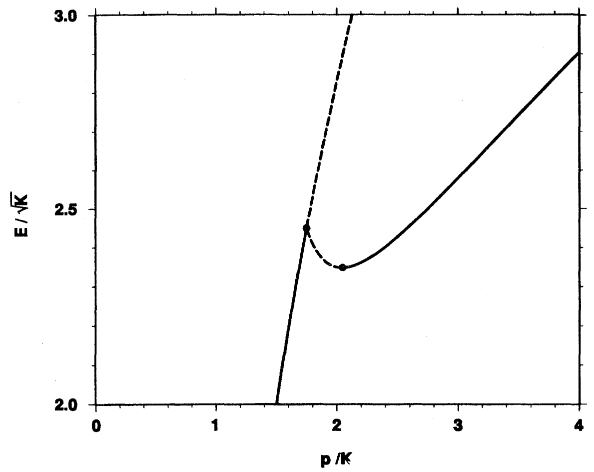

FIG. 2. An alternative bifurcation diagram for the symmetric coupler. This diagram, obtained by means of the improved variational method, should be compared with Fig. 11 from Ref. [12] obtained by numerical methods.

stable and unstable branches, using standard theorems of the bifurcation theory [13]. In describing the bifurcation (insofar as it depends on the soliton's energy), we find it most physically meaningful to use the quantity $\cos(2\theta)$, which, according to Eqs. (8) and (9), directly characterizes distribution of the soliton's energy between the two cores. This quantity is used as the vertical coordinate in Fig. 1. Notice that it can be defined independently of the describing variables:

$$\cos(2\theta)\equiv\frac{E^{(1)}-E^{(2)}}{E^{(1)}+E^{(2)}},$$

where $E^{(j)}$ is the energy in the jth core.

Seeking the value E_2 at which the backward bifurcation occurs, we have found above that $E_2=\sqrt{6K}$, whereas the known exact value is $4\sqrt{K/3}$ [5,6,12]. It follows from here that the relative error in locating the bifurcation value of the energy by means of the modified variational approximation is $\approx 5\%$. Note the dramatic improvement in comparison with the simplest approximation employed in Ref. [9], which gave $E_2=\sqrt{3K}$, i.e., the relative error of 25%. The bifurcation diagram produced by the variational technique and shown in Fig. 1 agrees with the numerical results reported in Ref. [12] in showing that the region of bistability extends over a very narrow range of energies.

For extended comparison with the results of Ref. [11], we present another version of our bifurcation diagram in Fig. 2, where we use exactly the same coordinates as in [11]: the soliton's energy E and the wave-number shift p; see Eq. (15). This choice of the coordinates allows immediate comparison with Figs. 1 and 11 of Ref. [12]. The difference between variational and numerical results can be spotted only in a neighborhood of the bifurcation, and even there it is not more than 5%. Thus, we conclude that the analytical approximation based on Eq. (15) is reliable. In the next section we apply it to the asymmetric coupler.

IV. THE ASYMMETRIC COUPLER

The general form of Eq. (16) allows consideration of the asymmetric case $D_1\neq D_2$ and $Q\neq 0$. To this end, it is convenient to use the remaining scaling invariance of the underlying equations to set

$$D_1+D_2\equiv 2, \tag{20}$$

which is true for the symmetric case, and to define the parameter

$$\Delta\equiv\frac{1}{2}(D_1-D_2), \tag{21}$$

which measures the relative mismatch in the dispersion between the cores. Recall that the parameter q, defined as per the underlying equations (1) and (2), measures the phase velocity mismatch. In what follows below, we consider effects generated by the two mismatches, q and Δ, separately. In this relation, it is relevant to recall that, as explained in the Introduction, the differences in the phase velocity and in the dispersion coefficient between the cores can be separately induced by deformation of the cores keeping their effective areas equal, and by a small difference in the areas without deformation.

Figure 3 displays bifurcation diagrams produced by Eq. (16) with $\Delta=0$ and different values of q: 0.01, 0.1, and 1.0. The deformation of the symmetric diagram of Fig. 1 under the action of the small phase velocity mismatch that gives rise to Fig. 3(a) is exactly the generic type of the deformation expected when a small symmetry-breaking perturbation lifts the degeneracy of the symmetric case [13].

Figure 4 illustrates the effect on the bifurcation of the dispersion mismatch Δ in the absence of the phase velocity difference (i.e., at $q=0$). It is apparent that the deformation of the symmetric diagram of Fig. 1, induced by a small Δ in Fig. 4(a), is of the same type as that shown in Fig. 3(a). However, unlike the case of the mismatched phase velocity, increasing the value of Δ keeps three bifurcation points on the diagrams. A trend of the bifurcation diagram with further increase of Δ is discussed in the next section.

The most remarkable feature of the diagrams for the asymmetric case is the well-pronounced bistability, which can find use in optical switches and other applications. Unlike the symmetric case, when bistability occurs for very restricted values of the energy (Fig. 1), in the asymmetric case the bistability may be robust, and it can be easily controlled by means of the parameters determining the asymmetry between the cores. A tristability is also possible [see Figs. 3(a) and 4(a)], but it exists only in a restricted range of energies.

V. THE CORES WITH OPPOSITE DISPERSIONS

A natural generalization of the above analysis is to consider the coupler with the two cores so different that they have opposite signs of their dispersion coefficients. Of course, in this case the assumption of the equal nonlinear coefficients is a serious limitation; however, it seems plausible that the qualitative results to be reported below will

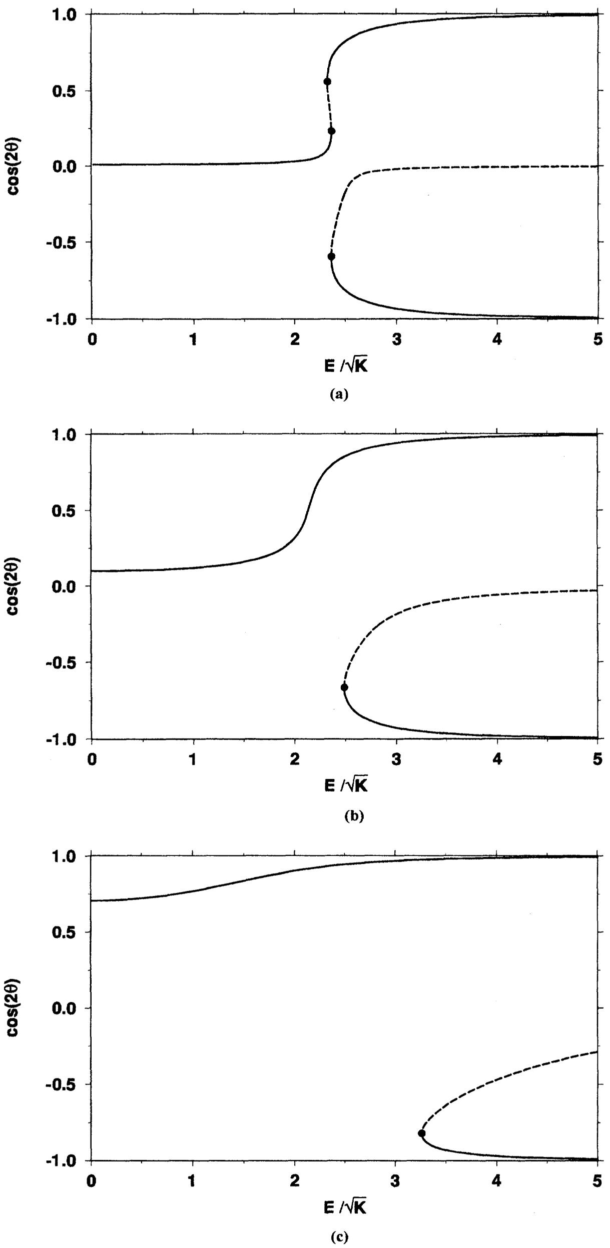

FIG. 3. The bifurcation diagrams for the asymmetric coupler with $\Delta=0$ (no dispersion mismatch): (a) $Q=0.01$; (b) $Q=0.1$; (c) $Q=1.0$.

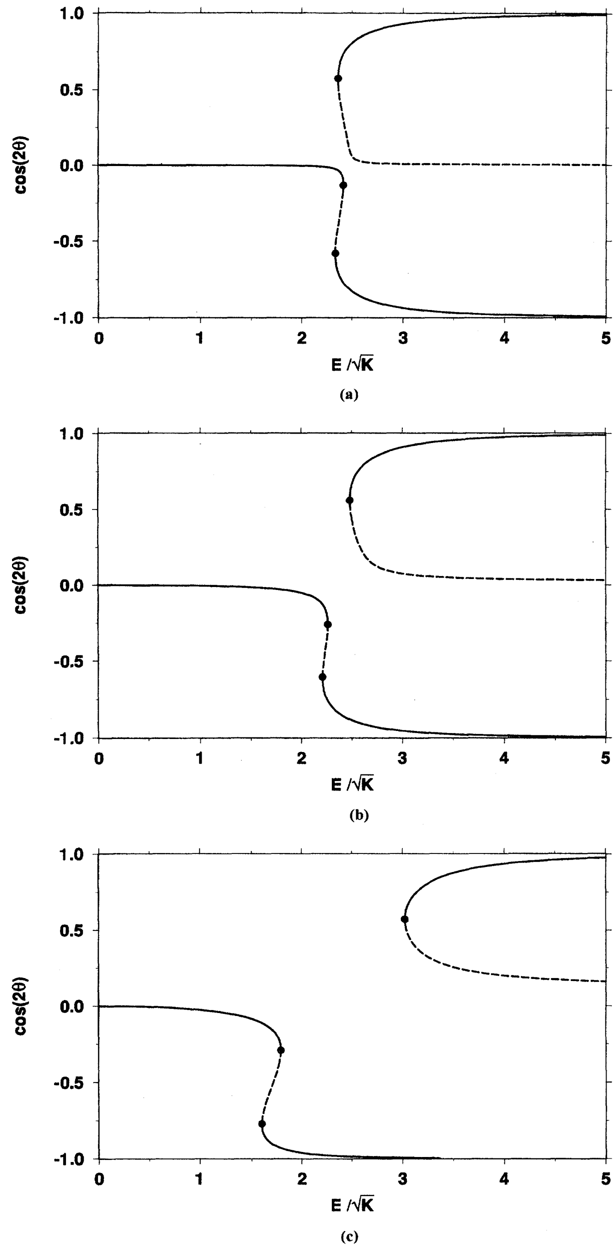

FIG. 4. The bifurcation diagrams for the asymmetric coupler with $Q=0$ (and no phase velocity mismatch $Q=0$): (a) $\Delta=0.01$; (b) $\Delta=0.1$; (c) $\Delta=0.5$.

remain relevant if one takes into account the difference in the nonlinear coefficients.

To be specific, we assume that $D_1>0$ (anomalous) and $D_2\leqslant 0$ (normal). According to Eqs. (20) and (21), in this case $\Delta\geqslant 1$. A bright soliton cannot exist in an isolated fiber with normal dispersion ($D<0$). However, the coupling between the two cores enables a soliton having components in both cores. This is clearly seen in Fig. 5, where we present increasingly negative values of D_2 (i.e., increasing values of $\Delta>1$). All the cases shown in Fig. 5 have $q=0$ to highlight the effect of competition between the opposite dispersions.

In Fig. 5, we see two portions of the solution, and two different thresholds. For energy exceeding the larger threshold value, there are stable and unstable solitons. Naturally, the stable branch is that with more energy in the core with the anomalous dispersion. For energies below a different, lower threshold value, a second pair of stable and unstable branches is found, which corresponds to more energy being in the normal-dispersion core. Examination shows that, for this solution, not only the energy but also the width a is small. Normally, for a bright soliton, the width increases as the energy decreases. The failure of the solutions found be-

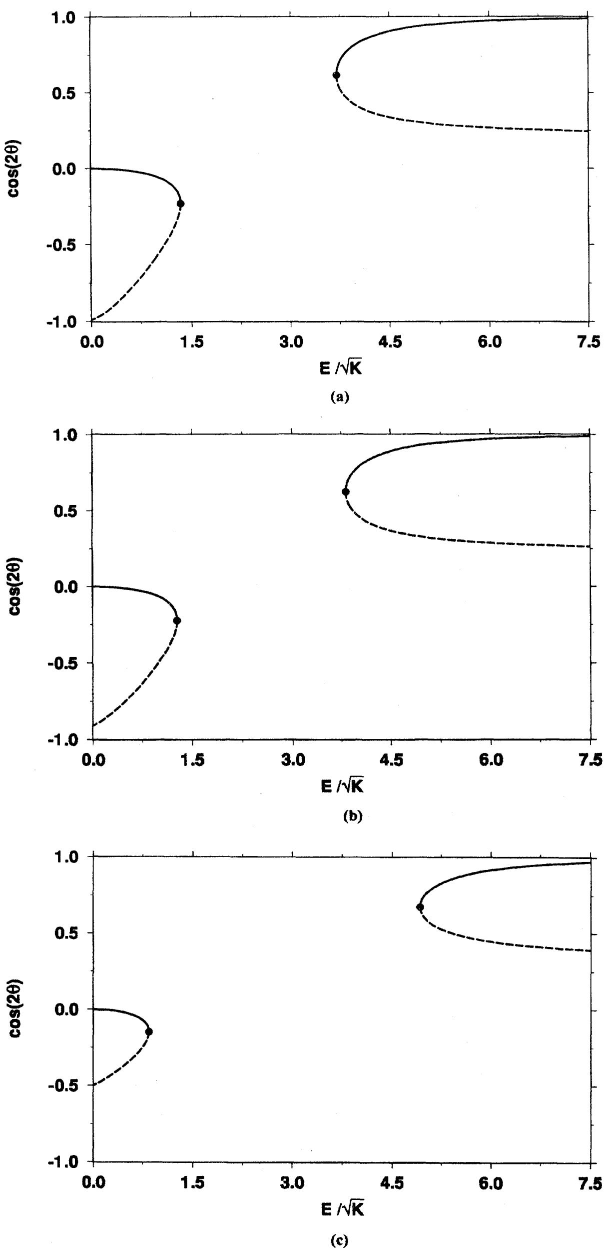

FIG. 5. The bifurcation diagrams for the coupler with the opposite signs of the dispersion in its two cores, i.e., with $\Delta > 1$ (no phase velocity mismatch): (a) $\Delta = 1.01$; (b) $\Delta = 1.1$; (c)$\Delta = 2.0$.

low the lower threshold to meet this natural condition suggests that the ansatz adopted in the present work is not a relevant approximation for those solutions. Exactly what these badly approximated solutions correspond to remains uncertain; it may be that the solutions are actually spurious.

Increasing $|D_2|$ gives rise to a natural trend also clearly seen in Fig. 5: the soliton needs more energy to support itself, and increasingly shifts into the core with the anomalous dispersion. Also, the region in which the (presumably) spurious solution exists becomes more restricted.

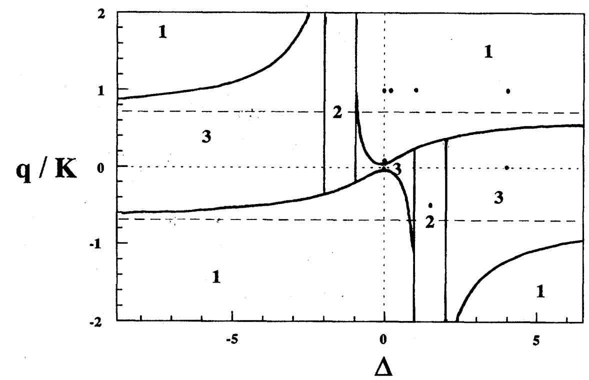

FIG. 6. The domains on the parametric plane (Δ, q) characterized by different numbers of the bifurcation points (the numbers are shown by large digits) as obtained analytically from Eq. (16). At the points shown by the large dots, the correctness of the prediction was checked against full bifurcation diagrams (some of those diagrams are shown above in Figs. 3–5; some dots are not shown to avoid overlapping with other features). The dashed horizontal lines are asymptotes of some of the domain borders.

VI. CONCLUSION

In this work, we have demonstrated that the improved version of the variational technique, based on adding the soliton's width to the set of the variational parameters, remains fully analytically tractable and, simultaneously, it produces bifurcation diagrams for the solitons in the model of the symmetric coupler that are in a fairly good agreement with numerical results. In particular, the bifurcation is correctly predicted to have the backward (subcritical) character. It is also noteworthy that this approximation produces analytical results essentially different from those for the cw (time-independent) modes.

For the asymmetric coupler, our approach has yielded new results. That strong and robust bistability can be easily obtained and controlled in the asymmetric coupler may be of practical interest for applications to photonics.

Finally, we have considered the coupler in which one core shows anomalous dispersion, while the other is normal. We have demonstrated that a two-component soliton with a sufficiently large energy, which helps it keep closer to the anomalous-dispersion core, can exist in this arrangement too.

The results obtained in the present work are summarized in the phase diagram displayed in Fig. 6, in which, on the parametric plane (Δ, q), we show borders between domains with different numbers of the bifurcation points, as obtained by direct analysis of Eq. (16). This diagram possesses an obvious symmetry with respect to inversion (simultaneous change of signs of the two coordinates), which simply implies transposition of the two cores.

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