Scroll breakup in a three-dimensional excitable medium

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We show that a scroll wave in a homogeneous three-dimensional (3D) excitable medium can spontaneously break up into an irregular spatial pattern. This occurs in a FitzHugh-Nagumo model with modified dynamics of the slow variable (shortened relative refractory period of the excitable medium). The mechanism of the scroll breakup in 3D is similar to the mechanism of the spiral breakup in 2D and is associated with the instability of wave propagation under high frequency forcing.

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I. INTRODUCTION

Complicated spatiotemporal regimes play an important role in excitable media of various types. If such a regime occurs in cardiac tissue it causes cardiac fibrillation, which is the main cause of death in the industrialized world [1]. So far little is known about the nature of cardiac fibrillation. One recent hypothesis connects ventricular fibrillation to complicated three-dimensional (3D) dynamics of a vortex filament that wiggles around and encounters the surface in complicated ways [2]; another shows that a quasi-two-dimensional mechanism (rapid motion of a rotor) gives rise to electrocardiographic patterns of activity indistinguishable from fibrillation [3].

One of the most generally accepted mathematical models of spatiotemporal chaos in excitable media explains it as a process of spiral breakup. It has been shown in cellular automata models [4–6] and in reaction diffusion models of excitable media [7–14] that a 2D spiral wave can spontaneously break up into a complicated spatiotemporal regime because of the special kinetics of excitable media without any heterogeneities. This process is an important example of spatiotemporal chaos in two dimensions. However, several experimental studies suggest that in the normal heart cardiac fibrillation is three dimensional and does not occur in two dimensions [2,15].

Here we model a complicated spatiotemporal pattern caused by a breakup of a scroll wave in 3D. We show that the breakup in 3D is associated with instabilities of wave propagation in one-dimensional media under high frequency forcing. We compare the characteristics of the scroll breakup in 3D and the spiral breakup in 2D. We show that they have different amplitudes of oscillation of an excitable gap and different ranges of existence in parametric and in physical space.

II. MODEL

For numerical computation we use FitzHugh-Nagumo-type equations with piecewise linear kinetics [16,11]:

$$\partial e/\partial t = \nabla^2 e - f(e) - g \;,$$
$$\partial g/\partial t = \varepsilon(e,g)(ke - g) \;, \qquad (1)$$

with $f(e)=C_1 e$ when $e<e_1$, $f(e)=-C_2 e + a$ when $e_1 \le e \le e_2$, $f(e)=C_3(e-1)$ when $e>e_2$, and $\varepsilon(e,g)=\varepsilon_1$ when $e<e_2$, $\varepsilon(e,g)=\varepsilon_2$ when $e>e_2$, and $\varepsilon(e,g)=\varepsilon_3$ when $e<e_1$ and $g<g_1$. The parameters determining the shape of the function $f(e)$ are $e_1=0.0026$, $e_2=0.837$, $C_1=20$, $C_2=3$, $C_3=15$, $a=0.06$, and $k=3$. With these parameter values the function $f(e)$ is continuous. The shape of the function $f(e)$ specifies fast processes such as the initiation of the action potential. The dynamics of the recovery variable g in (1) is determined by the function $\varepsilon(e,g)$. In $\varepsilon(e,g)$ the parameter ε_3^{-1} specifies the recovery time constant for small values of e and g. This corresponds approximately to the relative refractory period. Similarly, ε_1^{-1} specifies the recovery time constant for relatively large values of g and intermediate values of e. This corresponds approximately to the wave front, wave back, and the absolute refractory period. The main difference between model (1) and the previous models [16] is that model (1) uses two independent constants ε_1^{-1} and ε_3^{-1} for the refractory state. The values of these parameters were fixed at $\varepsilon_1^{-1}=75.0$, $\varepsilon_2^{-1}=1.0$, $g_1=1.8$, and $0.5<\varepsilon_3^{-1}<10.0$.

For numerical computations we used the explicit Euler method with Neumann boundary conditions, and the rectangular grid contained $120\times120\times120$ elements. To initiate the first scroll we used initial data corresponding to a 3D broken wave front. Numerical integration was performed with a space step $h_s=0.5$ and a time step $h_t=0.0222$. The error in these computations, estimated using the difference between the computed and the saturated value for the velocity of plane wave propagation, is less than 5%.

III. RESULTS AND DISCUSSION

To generate scroll breakup in three dimensions we started with parameter values identical to those at which the process of 2D spiral breakup was observed [11]. In the first set of computations we initiated the simplest type of 3D scroll, an untwisted scroll wave with a straight filament orthogonal to the boundaries of the excitable medium, and we studied the dynamics of its rotation.

We found that the scroll wave broke up. However, the picture was quasi-two-dimensional, i.e., in each section

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orthogonal to the filament we obtained identical pictures of deterioration of one spiral wave into five scroll waves (similar to Fig. 1 in [11]).

We found that a 3D breakup occurred if the filament of the initial scroll wave was not a straight line. To obtain such a scroll wave, we initiated a wave by stimulating a thin region located in the middle of the left face of the cube which represented our excitable tissue. (The dimensions of the plate were 3×3 points.) When this wave was approximately at the middle of the cube we replaced half of the excitable medium by a normal recovered medium (similar to experiments [17]). This wave evolved into a scroll wave with a curved filament [Fig. 1(a)]. The dynamics of its rotation was as follows. The wave rotated several times and then began to fragment close to the filament of the scroll [Fig. 1(b)]. Here we can see an oval-shaped break in the middle of the wave surface. This wave break generated an extra scroll ring. However, soon afterwards the next major breakup occurred [Fig. 1(c)]. In Fig. 1(c) we can see that two more wave breaks occur; they extend throughout the medium, from top to bottom. After these breaks, the scroll wave rotation becomes very complicated, and within 15 time units (about a half of a period of the scroll wave) it evolves into multiple 3D wavelets; these persisted in the medium all the time we were doing our computations.

The initial curvature of the scroll filament was not a necessary condition for a 3D breakup. However, in some way we needed to break the transversal symmetry of the system. We studied the rotation of a scroll wave with an initial straight filament in a medium with a little transversal heterogeneity. (The refractory periods for the layers orthogonal to the filament were changed randomly within 5% of their average value.) In this case too we observed a 3D breakup of the scroll wave similar to the breakup in Fig. 1. Note that computer round-off errors are not sufficient to break the symmetry.

We studied the range of existence of the scroll breakup under changes in the parameter ε_1^{-1} which accounts for the absolute refractory period of an excitable medium [18]. We found that at $\varepsilon_1^{-1}\geq 54$ the scroll breakup in 3D disappears. We also found that the spiral breakup in 2D disappears at $\varepsilon_1^{-1}\geq 58$. So in our model there is a 7.5% range of parameter values in which we can have a breakup in 3D but normal rotation of a spiral wave in 2D.

The complicated spatiotemporal pattern generated by the breakup has a finite lifetime in a medium of relatively small spatial size [10,14,18]. This lifetime increases with the increase of the size of the medium and at some critical size the lifetime can become infinite. We computed the value of this critical size for a breakup in 2D and in 3D. In 2D we were able to obtain a sustained activity

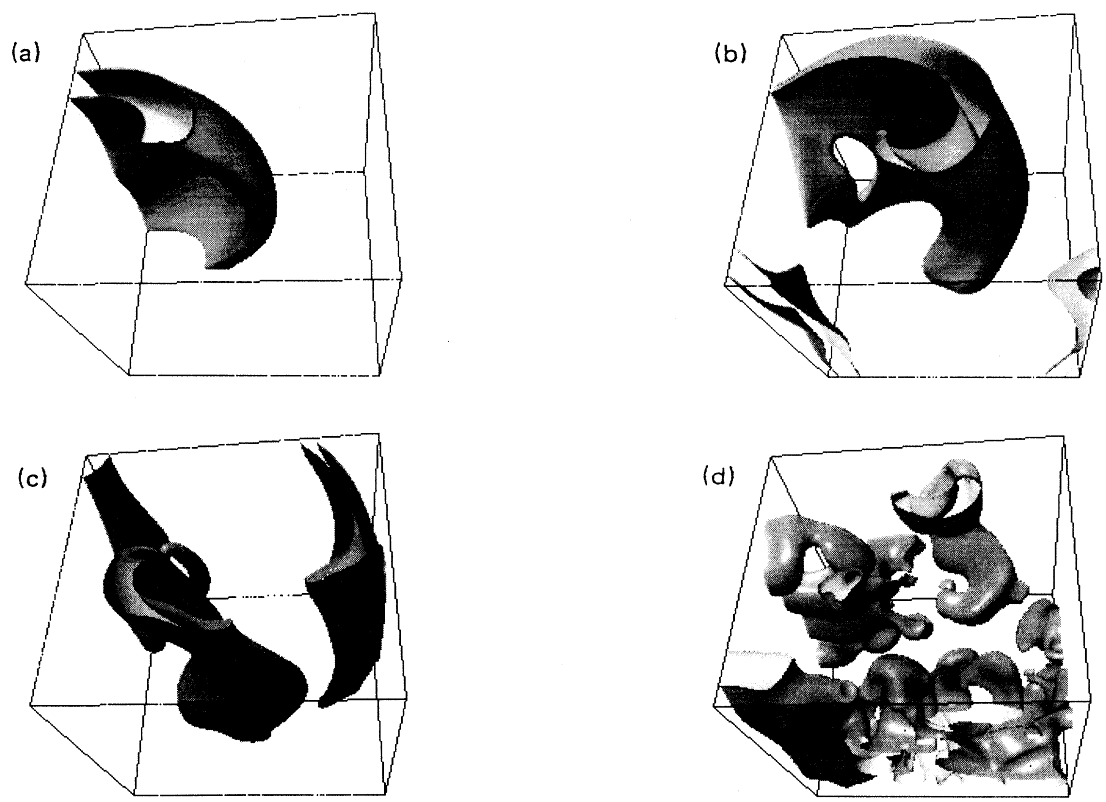

FIG. 1. The breakup of a scroll wave in model (1). The pictures are at times (a) $t=46$, (b) $t=160$, (c) $t=168$, and (d) $t=347$; numerical integration with space step $h_s=0.5$ and time step $h_t=0.0222$ on the grid of $120\times120\times120$ elements. The gray surface depicts the excited region of the tissue ($e>0.6$).

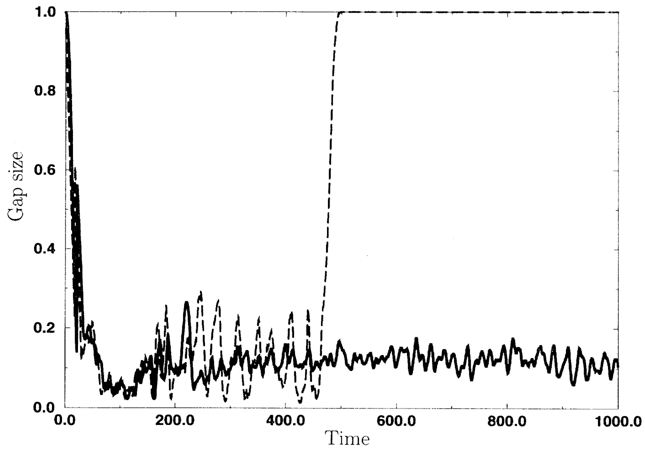

FIG. 2. The relative size of the excitable gap vs time during the process of spiral breakup. The thick solid line represents 3D computations on the grid of $120 \times 120 \times 120$ elements, the dashed line represents 2D computations on the grid of 120×120 elements.

(longer than 100 cycles) in a square of size 2.46λ (where λ is the average wavelength of a spiral wave in the excitable medium; for our model it is 65 spatial points). In 3D a similar estimate for the critical size of the medium with cubic geometry yields the size $=1.15\lambda$.

One of the important characteristics of wave patterns in excitable tissue is the size of the excitable gap, i.e., the size of the recovered part of the tissue. The excitable gap shows how densely rotors are packed in excitable media. We computed the excitable gap for a breakup in 2D and in 3D. To do this, at each moment in time we counted the total number of points that have been recovered [we considered the point to be recovered if $g<0.3$ and $e<0.1$, where g and e are the variables from Eq. (1)] and divided this number by the total number of points in our excitable media (Fig. 2). We see (Fig. 2) that the average size of the excitable gap is about the same in 3D and 2D excitable media. However, we see that the 2D pattern on the grid of 120×120 elements shows oscillations with a wider amplitude and in this case spirals disappear after the time $t=450$.

The mechanism of scroll breakup described in this paper is similar to the mechanism of spiral breakup in 2D excitable tissue, and is associated with the interaction of the front of the wave with its refractory tail. Figure 3 shows several cross sections of a scroll at the moment of the first [Fig. 1(b)] and the second [Fig. 1(c)] major breakups. We see that the breaks occurred due to the interaction of the front of the wave with its refractory tail. The difference between Figs. 3(a)–3(c) and 3(d)–3(f) was that at $t=160$ the breaks occurred in several sections only; this picture corresponds to the oval break in Fig. 1(b). At

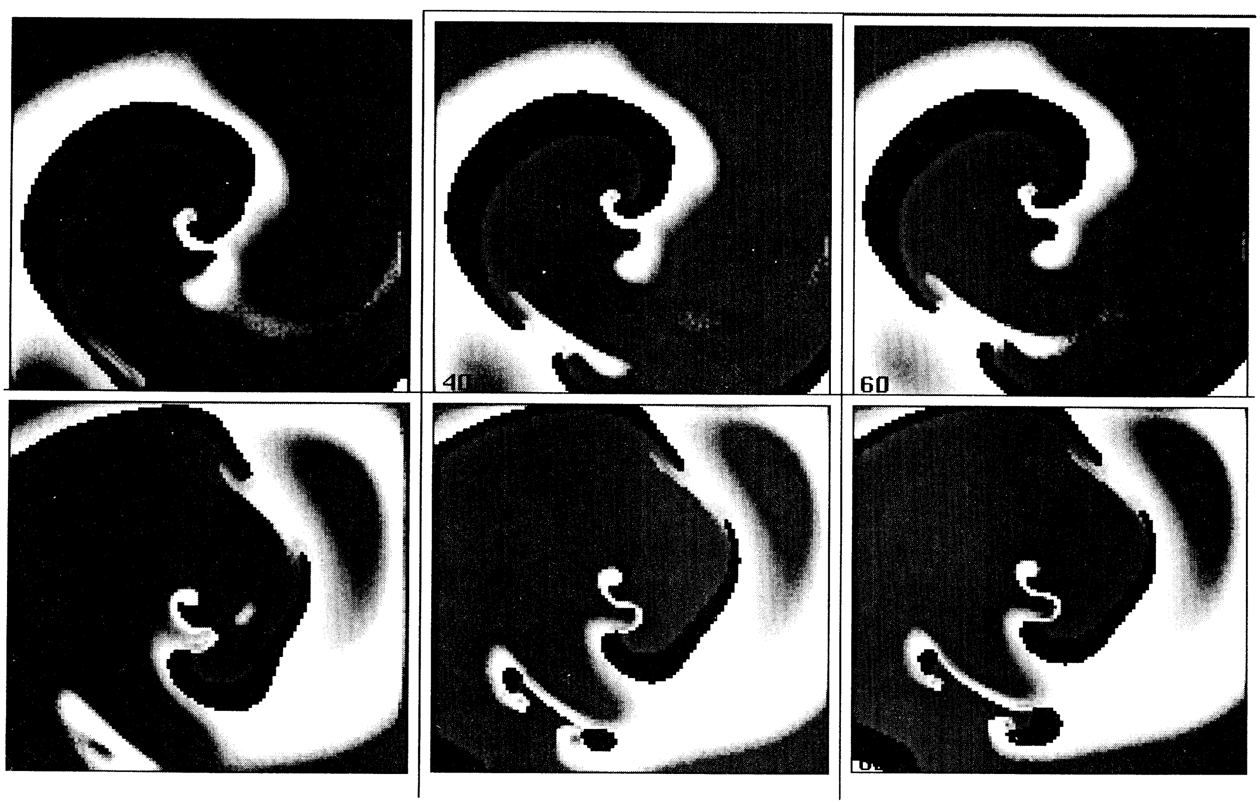

FIG. 3. The cross sections of Fig. 1 at time $t=160$ [(a)–(c), upper panel] and at time $t=168$ [(d)–(f), lower panel]. The black area represents the excited state of the tissue ($e>0.6$), dark gray shows the region where $g>1.8$ (close to the absolute refractory state), and intermediate shading from dark gray to white shows different levels of $g, 0<g<1.8$ (estimate for the relative refractory period). The number of the section of the excitable medium is 20 (a,d), 40 (b,e), and 60 (c,d).

$t = 168$ the breaks occurred in all sections of the media; this corresponds to the second major breakup in Fig. 1(c).

The mechanism which accounts for such a complicated interaction of the front with the refractory tail of the wave was discussed in [11]. It was found that under high frequency forcing the propagation of a pulse in one-dimensional excitable tissue becomes oscillating, which results in functional heterogeneities in refractoriness and a breakup of the spiral in 2D.

We modeled the propagation plane waves in 2D under the same high frequency forcing as in 1D. We found the same oscillating instability of 1D pulse propagation, but it did not have a lateral mode in 2D. In Fig. 4 we clearly see the oscillations in a wavelength of the pulses; however, the wave front shape is an undisturbed straight line (Fig. 4). In our model, we do not have lateral instabilities of wave propagation; therefore we need an initial break in transversal symmetry of our system in order to observe a purely three-dimensional breakup. However, we can suggest that for systems which show lateral instability one can expect a purely three-dimensional breakup without any initial breaking of symmetry.

Our paper shows that in model (1) a breakup in 3D occurs in a wider range of parameters than in 2D and becomes persistent in media of smaller spatial size. These differences can be important for understanding the differences in fibrillation in 2D and in 3D.

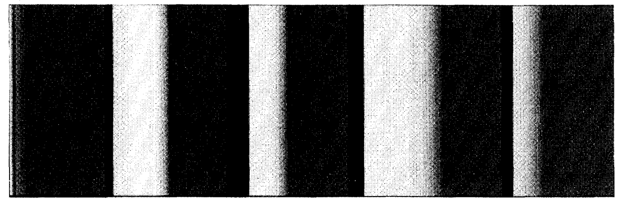

FIG. 4. Wave propagation during periodic forcing with a period $t_p = 23$ in the medium of 130×390 elements. Gray scale coding is the same as in Fig. 3.

It should be noted that the wavelength of a spiral wave in myocardium is of the order of 30–60 mm (see [2]), whereas the size of the ventricles of the canine or human heart is of the order of 100 mm. Therefore there is enough room for a 3D breakup to occur in the heart and it can generate an extremely complicated spatiotemporal pattern of excitation.

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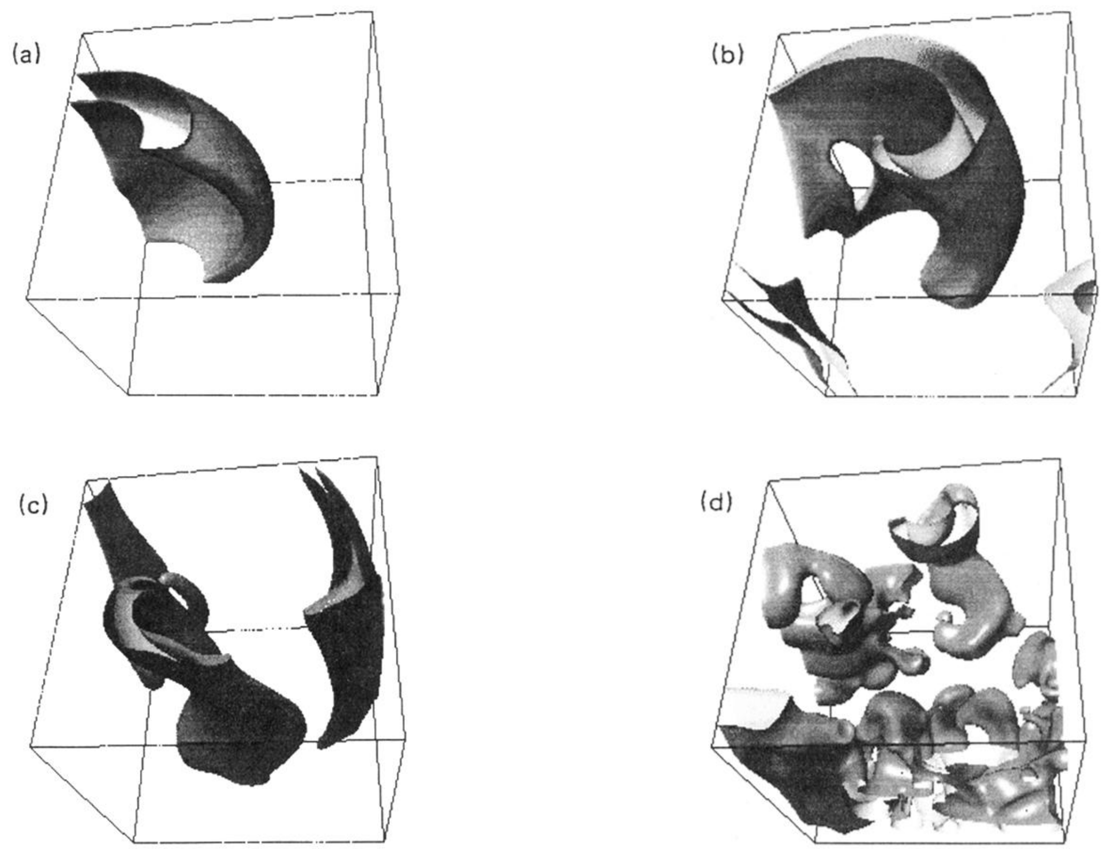

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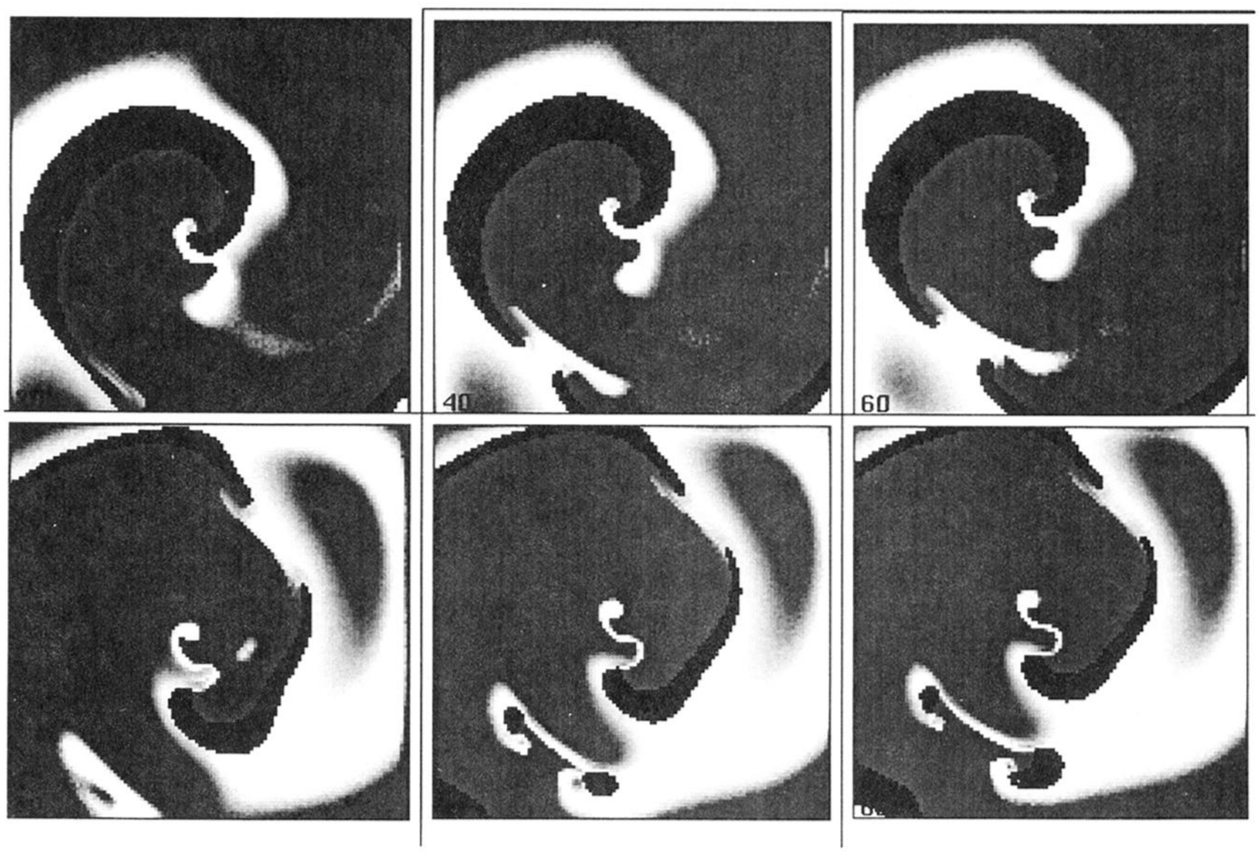

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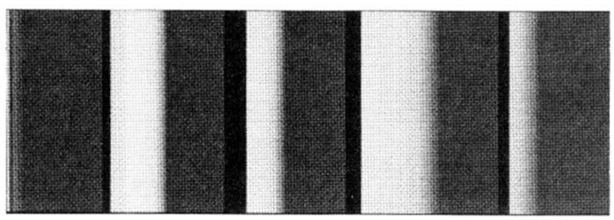

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