Nonlinear lower hybrid vortices

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Using a two-fluid description, we derive a set of equations describing the nonlinear interaction of a lower hybrid pump wave, propagating almost perpendicularly to the external magnetic field, with low frequency density perturbations associated with a drift wave. In the strongly nonlinear regime, when the system is dominated by convective vector-product-type nonlinearities on the slow time scale, we find a stationary, localized dipole vortex solution driven by the lower hybrid ponderomotive force. These types of vortices can be driven both by a long-wavelength pump, in an oscillating two-stream parametric process, and by the modulation of a short-wavelength pump wave. In most cases, all vortex parameters (amplitude, radius, and velocity) are determined by the amplitude of the pump.

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I. INTRODUCTION

Plasma heating at the lower hybrid frequency [1] is connected with a number of undesirable processes occurring when a strong, lower hybrid pump wave interacts with the background plasma during its propagation towards the resonance layer. An important role is played by the relatively strong nonlinear effects accompanying the wave propagation, which can give rise to the emergence of various high- and low-frequency modes and eventually modify the character of the pump itself. One of these processes is the parametric decay [2] when the pump wave energy goes into the parametrically amplified decay wave. This decay wave does not penetrate successfully into the plasma, thus most of the pump energy is deposited near the plasma surface and consequently an anomalous heating of the plasma surface takes place. Other processes include the filamentation due to the nonlinear interaction with electrostatic ion-cyclotron perturbations [3], decay through induced scattering by particles (i.e., nonlinear electron and ion Landau damping) [4], nonlinear excitation of electrostatic and electromagnetic zero-frequency modes (eddies) [5], oscillating-two-stream (OTS) instability [6], and interaction with nonresonant density and temperature perturbations [7].

It has been shown [8,9] that lower hybrid waves, in the interaction with low-frequency density perturbations, are modulationally unstable and give rise to Langmuir-type disklike solitons. These one-dimensional structures are further unstable to perpendicular perturbations and eventually collapse, producing small-scale structures, or cavitons, with the lower hybrid waves trapped inside. This process is very similar to the Langmuir collapse [8], as the short-wavelength electromagnetic part of the lower hybrid spectrum becomes dominant within the collapsing structure.

In the case of a long-wavelength lower hybrid pump wave, however, effects associated with its electromagnetic component may be neglected compared to some nonlinear effects, such as the self-interaction on the low-frequency scale. These low-frequency terms, arising from the dominance of the convective nonlinearity $\vec{v}_{\mathrm{LF}}\cdot\vec{\nabla}$, are responsible for the creation of plasma vortices, which are robust, two-dimensional coherent nonlinear structures [10,11].

It is known that vortices may arise in processes of self-organization in laboratory [10] and in astrophysical plasmas [12]. Since they can carry plasma particles effectively, the investigation of vortices may be of great importance in the problems of plasma fusion [13]. Apart from the self-organization, vortices can be produced also by the interaction of a high-frequency pump wave [14] injected into a plasma, with low-frequency density perturbations normally existing in the plasma. In such cases they appear to be better defined, being determined by the pump amplitude and its group velocity only.

In our previous work [15] we discussed the parametric interaction of a spatially nonuniform, lower hybrid pump with low-frequency density perturbations. Although we derived a complete set of equations, only the parametric case was studied, i.e., self-consistent perturbations of the pump were not included in the analysis. Nonuniformity of the pump, leading to the formation of vortices, was expressed through a ponderomotive force term. We obtain an equation for the low-frequency potential, similar to the Hasegawa-Mima equation, and found a particular solution in the form of a double vortex.

A hypothesis about the existence of self-consistent, driven lower hybrid drift vortices was put forth, without proof, in Ref. [7]. In the present paper we solve analytically a system of equations similar to Eqs. (48) and (49) in Ref. [7], describing perturbations of the lower hybrid pump propagating almost perpendicularly to the external magnetic field lines and interacting with density perturbations on the drift-wave time scale. In the strongly nonlinear regime, when convective nonlinear terms in the low-frequency momentum equations are of the same order as the time derivatives, we find a well localized, stationary, and moving solution in the form of a dipolar vortex, driven by the lower hybrid pump. All the vortex parameters, including its velocity and the core radius, are completely determined by the pump amplitude, in contrast to the case of free vortices [10,12].

II. BASIC EQUATIONS

We investigate quasi-three-dimensional turbulence of a homogeneous plasma immersed in the external homogeneous magnetic field $\vec{B}_0=B_0\vec{e}_z$, driven by a lower hybrid pump wave propagating almost perpendicularly to the magnetic field. Two different regimes will be studied: the oscillating two-stream case, where the characteristic scale size of the low-frequency perturbations is much shorter than the pump wavelength, and the opposite case, when the conditions for the modulation instability are fulfilled. The nonlinear interaction of the pump with the slow scale plasma perturbations leads to the perturbation of the pump itself. To describe this process, we decompose the field quantities into their high- and low-frequency components and write two sets of equations for the slow (drift) scale and the rapid (lower hybrid) scale.

A. Lower hybrid equations

First, we derive the equations describing processes on the rapid time scale. We assume cold, linear, and unmagnetized ions and purely electrostatic perturbations, whose direction of propagation is almost perpendicular to the magnetic field. On the rapid time scale, the electron momentum equation has the form

$$\left[\frac{\partial}{\partial t}+\vec{v}_{es}\vec{\nabla}\right]\vec{v}_{er}+(\vec{v}_{er}\vec{\nabla})\vec{v}_{es} = -\frac{e}{m_e}\left[\vec{E}_r+\vec{v}_{er}\times B_0\vec{e}_z+\frac{1}{e}\left[\frac{1}{n}\vec{\nabla}p\right]_{er}\right] . \quad (1)$$

Here subscripts r and s denote rapidly and slowly varying quantities, respectively, and other notations are standard. We also assume an adiabatic process on the rapid time scale, for which we have

$$\left[\frac{1}{n}\vec{\nabla}p\right]_{er}\simeq\frac{\gamma T}{n_0}\left[\vec{\nabla}n_{er}-\frac{n_{es}}{n_0}\vec{\nabla}n_{er}-\frac{n_{er}}{n_0}\vec{\nabla}n_{es}\right] , \quad (2)$$

where γ is the ratio of the specific heats. From Eq. (1) for the electron motion parallel and perpendicular to the magnetic field lines, respectively, we obtain

$$\frac{\partial v_{erz}}{\partial t}=-\frac{e}{m_e}\left[E_{rz}+\frac{T_e}{en_0}\frac{\partial n_{er}}{\partial z}+E_{\mathrm{NL}z}\right] , \quad (3)$$

$$\frac{\partial\vec{v}_{er\perp}}{\partial t}-\Omega_e\vec{e}_z\times\vec{v}_{er\perp}=-\frac{e}{m_e}\left[\vec{E}_{r\perp}+\frac{\gamma T_e}{en_0}\vec{\nabla}_\perp n_{er}+\vec{E}_{\mathrm{NL}\perp}\right] , \quad (4)$$

where

$$\vec{E}_{\mathrm{NL}}=-\frac{T_e}{en_0^2}(\gamma n_{es}\vec{\nabla}n_{er}+\gamma n_{er}\vec{\nabla}n_{es}) +\frac{m_e}{e}[(\vec{v}_{es}\cdot\vec{\nabla})v_{er}+(\vec{v}_{er}\cdot\vec{\nabla})v_{es}] , \quad (5)$$

We use also the Poisson equation for the electrostatic potential on the rapid scale

$$\vec{\nabla}^2\Phi_r\equiv-\vec{\nabla}\cdot\vec{E}_r=-\frac{e}{\varepsilon_0}(n_{ir}-n_{er}) . \quad (6)$$

Here e is the ion charge and the subscripts e,i stay for electrons and ions, respectively. Using the continuity equations for electrons and ions and for ions following straight line orbits, the Poisson equation (6), describing the variation of the lower hybrid potential, can be rewritten in the form

$$-\left[\frac{\partial^2}{\partial t^2}+\omega_{pi}^2\right]\vec{\nabla}^2\Phi_r =\frac{en_0}{\varepsilon_0}\vec{\nabla}\cdot\frac{\partial\vec{v}_{er}}{\partial t}+\frac{e}{\varepsilon_0}\frac{\partial}{\partial t}\vec{\nabla}\cdot(n_{es}\vec{v}_{er}+n_{er}\vec{v}_{es}) . \quad (7)$$

Here ω_{pi} denotes the ion plasma frequency. For almost perpendicular perturbations $\partial/\partial z\ll\vec{\nabla}_\perp$, with $\omega/\Omega_e\ll1$, treating thermal effects as higher-order corrections in the nonlinear terms (which is justified [4] if $v_{Ti}^2\vec{\nabla}^2$, $v_{Te}^2\vec{\nabla}_\parallel^2\ll\omega^2$, and $v_{Te}^2\vec{\nabla}_\perp^2\ll\Omega_e^2$), and using Eqs. (3)–(5), Eq. (7) can be rewritten as

$$\left\{\left[\frac{\partial^2}{\partial t^2}+\omega_{pi}^2+\frac{\omega_{pe}^2}{\Omega_e^2}\left[1+\frac{c_s^2\vec{\nabla}_\perp^2}{\Omega_i\Omega_e}\right]\frac{\partial^2}{\partial t^2}\right]\vec{\nabla}_\perp^2+\omega_{pe}^2\vec{\nabla}_\parallel^2\right\}\Phi_r =-\frac{\omega_{pe}^2}{\Omega_e n_0}\vec{\nabla}_\perp\left[n_{es}\vec{e}_z\times\frac{\partial}{\partial t}\vec{\nabla}_\perp\Phi_r\right] . \quad (8)$$

Here subscripts $\perp$ and $\parallel$ refer to directions perpendicular and parallel to the magnetic field lines. In the above notations ω is the lower hybrid frequency, $c_s^2=\gamma T_e/m_i$, and v_{Ti},v_{Te} are the ion and electron thermal velocity, respectively. The derivative $\partial/\partial t$ corresponds to the time variation on the lower hybrid scale. Linearizing Eq. (8), we obtain the standard dispersion equation for the lower hybrid waves

$$\omega^2=\Omega_i\Omega_e+c_s^2k_\perp^2 . \quad (9)$$

B. Low-frequency response

On the slow scale we use the standard drift scaling, which corresponds to a slow time variation, compared to the ion gyrofrequency, and strong nonlinearities:

$$\partial/\partial t\sim\vec{v}_{es}\vec{\nabla}\sim\vec{v}_{is}\vec{\nabla}\ll\Omega_i . \quad (10)$$

We use the momentum equation for plasma components in the form

$$\left[\frac{\partial}{\partial t}+\vec{v}_s\vec{\nabla}\right]\vec{v}_s+\langle(\vec{v}_r\cdot\vec{\nabla})\vec{v}_r\rangle = \frac{q}{m}\left[\vec{E}_s+\vec{v}_s\times B_0\vec{e}_z-\left[\frac{1}{nq}\vec{\nabla}p\right]_s\right], \quad (11)$$

where $\langle\ \rangle$ denotes a time average over the high-frequency period. For purely potential perturbations on the slow scale and cold and two-dimensional ions we have

$$\vec{v}_{is}=\frac{1}{B_0}\vec{e}_z\times\vec{\nabla}\Phi_s -\frac{1}{\Omega_i B_0}\left[\frac{\partial}{\partial t}+\frac{1}{B_0}(\vec{e}_z\times\vec{\nabla}\Phi_s)\cdot\vec{\nabla}\right]\vec{\nabla}_\perp\Phi_s . \quad (12)$$

Using the quasineutrality condition $n_{is}=n_{es}$ and Eq. (12), we readily obtain from the ion continuity

$$\left[\frac{\partial}{\partial t}+\frac{1}{B}(\vec{e}_z\times\vec{\nabla}_\perp\Phi_s)\cdot\vec{\nabla}_\perp\right]\left[\frac{\omega_{pi}^2}{\Omega_i^2}\vec{\nabla}_\perp^2\Phi_s-\frac{en_{es}}{\varepsilon_0}\right]=0 . \quad (13)$$

The validity of the assumption of purely potential perturbations used here can be checked by comparing leading-order nonpotential and potential terms f_{NP},f_P in the general expression for the ponderomotive force [16]. It can be shown that in the regime studied in this paper we have

$$\frac{f_{NP}}{f_P}\sim\frac{1}{\omega}\frac{\partial}{\partial t}\frac{k_r}{k_s}\frac{\delta E_r}{E_r} ,$$

where k_r,k_s are the intensities of wave vectors on the rapid and slow scales, respectively, and δE_r is the perturbation of the pump electric field. In the lower hybrid case, for small perturbations of the pump, and using conditions (10) on the slow scale, the above ratio is small enough and the assumption of purely potential perturbations is justified.

In the zero electron mass limit, the parallel electron momentum equation yields the Boltzmann distribution

$$\Phi_s+\Phi_p-\frac{T_e}{e}\frac{n_{es}}{n_0}=0 , \quad (14)$$

where Φ_p is the ponderomotive potential

$$\Phi_p=-\frac{i}{\omega B_0}\vec{e}_z\cdot(\vec{\nabla}_\perp\Phi_r^*\times\vec{\nabla}_\perp\Phi_r) \quad (15)$$

and the asterisk denotes a complex conjugate term.

C. Integrals of motion

Besides the obvious (and some not so obvious) linear integrals of motion, Eqs. (8), (13), and (14) possess also the following quadratic conserved quantities.

1. Energy

We multiply Eqs. (13) and (8) respectively by Φ_s and $(\partial/\partial t-i\omega)\Phi_r^*$, calculate the real part, and integrate for the whole space with the standard requirement that both the "slow" and "rapid" electric fields and charges vanish in the infinity. Making use of Eqs. (12), (14), and (15) and after some algebra, we arrive at

$$\frac{\partial}{\partial t}\int dV\left[\frac{c_s^2m_i^2}{2e^2}(\vec{v}_{is})^2+\frac{1}{2}\left[\frac{n_{es}T_e}{n_0e}\right]^2 +\frac{c_s^4}{\Omega_e^2\Omega_i^2}|\vec{\nabla}_\perp^2\Phi_r|^2+\frac{c_s^2}{\Omega_i^2}|\vec{\nabla}_\parallel\Phi_r|^2-\Phi_p\frac{n_{es}T_e}{n_0e}\right]=0 . \quad (16)$$

2. Generalized plasmon number

We multiply Eq. (8) by $\vec{\nabla}_\perp^{2n}\partial\Phi_r^*/\partial t$, find the real part, and integrate for the whole space. Due to the symmetry properties of the vector product, the right-hand-side integrates to zero and we have

$$\frac{\partial}{\partial t}\int dV\left[\left[1+\frac{\Omega_e^2}{\omega_{pe}^2}\right]\left|\vec{\nabla}_\perp^{n+1}\frac{\partial\Phi_r}{\partial t}\right|^2+\Omega_e\Omega_i|\vec{\nabla}_\perp^{n+1}\Phi_r|^2 +\Omega_e^2|\vec{\nabla}_\perp^n\vec{\nabla}_\parallel\Phi_r|^2-\frac{c_s^2}{\Omega_e\Omega_i}\left|\vec{\nabla}_\perp^{n+2}\frac{\partial\Phi_r}{\partial t}\right|^2\right]=0 . \quad (17)$$

Here n is an arbitrary non-negative integer. The above expression may be generalized also to negative values of n, provided that the boundary conditions are such that the function Ψ, defined by $\Phi_r=\vec{\nabla}_\perp^{2|n|}\Psi$, and its derivatives vanish in the infinity. For a lower hybrid wave with a slowly varying amplitude $\Phi_r=\phi_r\exp(-i\omega t)$, Eq. (17) gives, with the accuracy to second order, the generalized plasmon number conservation law

$$\frac{\partial}{\partial t}\int dV\left[2|\vec{\nabla}_\perp^{n+1}\phi_r|^2+\frac{m_i}{m_e}|\vec{\nabla}_\parallel\vec{\nabla}_\perp^n\phi_r|^2-\frac{c_s^2}{\Omega_e\Omega_i}|\vec{\nabla}_\perp^{n+2}\phi_r|^2 -\frac{1}{i\omega}\left[\vec{\nabla}_\perp^{n+1}\frac{\partial\phi_r}{\partial t}\vec{\nabla}_\perp^{n+1}\phi^*-\text{c.c.}\right]\right]=0 . \quad (18)$$

3. Enstrophy

Low-frequency ion continuity Eq. (13) conserves the quantity

$$\frac{\partial}{\partial t}\int dx\,dy\,f\left[\frac{c_s^2}{\Omega_i^2}\vec{\nabla}_\perp^2\Phi_s-\frac{n_{es}T_e}{n_0e}\right]=0 , \quad (19)$$

where f is an arbitrary function. Adopting f to be quadratic and subtracting the energy Eq. (16), we obtain the enstrophy conservation law in the form

$$\frac{\partial}{\partial t}\int dV\left[\frac{c_s^4}{2\Omega_i^4}(\vec{\nabla}_\perp^2\Phi_s)^2+\frac{c_s^2}{2\Omega_i^2}(\vec{\nabla}_\perp\Phi_s)^2+\frac{c_s^2}{\Omega_i^2}\vec{\nabla}_\perp\Phi_s\cdot\vec{\nabla}_\perp\Phi_p +\Phi_s\frac{n_{es}T_e}{n_0e}-\frac{c_s^4}{\Omega_e^2\Omega_i^2}|\vec{\nabla}_\perp^2\Phi_r|^2-\frac{c_s^2}{\Omega_i^2}|\vec{\nabla}_\parallel\Phi_r|^2\right]=0 . \quad (20)$$

4. Momentum

Multiplying Eq. (8) by $\vec{\nabla}\Phi_r^*$, repeating a similar procedure as above, and making use of the slow ion continuity and momentum equations, we obtain

$$\frac{\partial}{\partial t}\int dV\,\vec{P}=\int dV\, n_{es}\vec{v}_{is}\times\vec{e}_z\frac{eB_0}{m_i}\ , \tag{21}$$

where

$$\vec{P}=n_{es}\vec{v}_{is}+\frac{i\varepsilon_0\omega_{pe}^2}{m_i(\Omega_e\Omega_i)^{1/2}\Omega_e^2}\left[\frac{\partial\Phi_r}{\partial x_i}\vec{\nabla}_\perp\frac{\partial\Phi_r^*}{\partial x_i}-\text{c.c.}\right]. \tag{22}$$

5. Angular momentum

In the same fashion we obtain also

$$\frac{\partial}{\partial t}\int dV\,\vec{P}\times\vec{r}=\int dV(n_{es}\vec{v}_{is}\times\vec{e}_z\Omega_i)\times\vec{r}\ . \tag{23}$$

It is worth noting that expressions of the same form are applicable for all high-frequency waves coupled with slow density perturbations and whose dispersion relations are similar to Eq. (9). Thus, substituting $c_s^2\to 3v_{Te}^2$, $\Omega_e\Omega_i\to\omega_{pe}^2$, $\omega_{pe}\to\Omega_e$, $\vec{\nabla}_\parallel\to 0$, $B_0\to 0$, and $\Phi_p\to -e|\vec{\nabla}\Phi_r|^2/m_e\omega_{pe}^2$, our Eqs. (16), (22), and (23) recover the well known expressions for the energy, momentum, and angular momentum in the Langmuir turbulence of a two-dimensional, cold ion plasma. Likewise, for $n=0$, the leading-order (i.e., the first) term in Eq. (18) recovers the Langmuir plasmon number.

The energy conservation Eq. (16) reveals an instability of the nonlinear modes whose density perturbation is of the same sign as the ponderomotive potential. That condition is equivalent to the Lighthill criterion for the modulation and/or oscillating-two-stream instability of high-frequency waves with positive dispersion, Eq. (9). It is well known that in the one-dimensional case these parametric instabilities saturate to a disklike soliton. In two dimensions these structures are unstable [8,9], but the additional constraints discussed above (such as the enstrophy conservation) may lead to the formation of another type of coherent structures, having the form of dipole vortices, which are characteristic for the undriven drift wave turbulence [10]. The existence of such structures will be studied below.

III. SOLUTIONS

If the low-frequency self-interacting term in Eq. (13) may be neglected, and using Eqs. (8), (14), and (15), we obtain an equation describing a lower hybrid potential, exponentially growing in time in the parametric regime due to the oscillating two-stream and/or modulational instability. However, the presence of the self-interacting, convective nonlinear term in Eq. (13) may produce a solution in the form of a spatially localized double vortex moving with a constant velocity. This coherent nonlinear structure will be constructed analytically, solving Eqs. (8), (13), and (14) and seeking a stationary solution propagating with the velocity v_y in the direction perpendicular to the external magnetic field,

$$\partial/\partial t=-v_y\partial/\partial y\ , \tag{24}$$

assuming a small perturbation of the pump wave, due to the interaction with low-frequency modes.

In the following, we will solve Eqs. (8), (13), and (14) assuming the electric field of the pump in the form

$$\vec{E}_r\equiv-\vec{\nabla}\Phi_r=[E_0\vec{e}_y-(i\vec{k}+\vec{\nabla})\delta\Phi_r(\vec{r},t)]\times\exp[-i(\omega t-\vec{k}\cdot\vec{r})]\ , \tag{25}$$

where $\omega,\vec{k}$ satisfy the dispersion relation (9).

A. OTS case

First, we consider the case of a long-wavelength pump, which in the one-dimensional parametric limit gives rise to the OTS instability. Using $k\to 0$ in (25), we can rewrite the high-frequency Poisson equation as

$$\left[2i\frac{\partial}{\partial t}+\frac{c_s^2}{\omega}\vec{\nabla}_\perp^2\right]\vec{\nabla}_\perp^2\delta\Phi_r=i(\vec{e}_z\times\vec{E}_0)\cdot\vec{\nabla}\frac{\Omega_e n_{es}}{n_0}\ . \tag{26}$$

For Boltzmann distributed electrons, Eq. (14), we have

$$n_{es}=\frac{n_0e}{T}\left[\Phi_s+\frac{i}{\omega B_0}(\vec{e}_z\times\vec{E}_0)\cdot\vec{\nabla}(\delta\Phi_r-\delta\Phi_r^*)\right]. \tag{27}$$

Combining Eqs. (26) and (27) we obtain

$$\frac{1}{\gamma}\left[4\frac{\partial^2}{\partial t^2}+\frac{c_s^4}{\omega^2}\vec{\nabla}_\perp^4\right]\left[\frac{en_{es}}{\varepsilon_0}-\frac{\omega_{pi}^2}{c_s^2}\Phi_s\right]=\frac{2}{B_0^2}(\vec{e}_z\times\vec{E}_0\cdot\vec{\nabla})^2\frac{en_{es}}{\varepsilon_0}\ . \tag{28}$$

In the one-dimensional case, when the vector-product-type nonlinear term in the ion continuity equation reduces to zero, from Eqs. (13) and (28) we get the equation for the slowly varying potential Φ_s,

$$\left[\frac{4\rho^2}{\gamma v_{Te}^2}\frac{\partial^2}{\partial t^2}+\rho^4\vec{\nabla}_\perp^4\right](\rho^2\vec{\nabla}_\perp^2-1)\Phi_s=-\frac{2E_0^2}{B_0^2c_s^2}\frac{m_i}{m_e}\rho^4\frac{\partial^2}{\partial y^2}\vec{\nabla}_\perp^2\Phi_s\ , \tag{29}$$

where $\rho=c_s/\Omega_i$ is the ion inertial length. From Eq. (29) we readily obtain the usual dispersion equation of the parametric process

$$\delta\omega^2=\frac{1}{4}\gamma\rho^4k_s^4\Omega_e\Omega_i\left[1-\frac{2E_0^2}{B_0^2c_s^2}\frac{m_i}{m_e}\frac{1}{1+\rho^2k_s^2}\right], \tag{30}$$

where $\delta\omega,k_s$ are the frequency and wave number of the sidebands. This expression is indicating an OTS instability $\text{Im}\delta\omega>0$ if the pump amplitude satisfies the condition

$$E_0^2>E_c^2=\frac{1}{2}\frac{m_e}{m_i}B_0^2c_s^2(1+\rho^2k_s^2)\ . \tag{31}$$

As discussed above, in a two-dimensional case, vector nonlinearity may suppress the development of the OTS instability. We assume a stationary solution, moving with the velocity $v_y\vec{e}_y$, which permits us to integrate the nonlinear equation Eq. (13) one time, yielding

$$\frac{\omega_{pi}^2}{\Omega_i^2}\vec{\nabla}_\perp^2\Phi_s-\frac{en_{es}}{\varepsilon_0}=F(\Phi_s+B_0v_yx)\ . \tag{32}$$

Here F is an arbitrary function of the given argument. The vortex solution is constructed adopting a linear function $F(\Phi_s+B_0v_yx)$, but allowing for different slopes F inside and outside the vortex core. We solve Eqs. (28) and (32) in the cylindrical coordinates (r,θ), expanding all the field quantities in terms of cylindrical harmonics as

$$f(r,\theta)=\sum f_n=F_1(r)\cos\theta+\left[\sum_{n=2}^{\infty}F_{n+2}(r)e^{i(2n+1)\theta}+\text{c.c.}\right]\ . \tag{33}$$

We use

$$r=(x^2+y^2)^{1/2},\quad \tan\theta=\frac{y}{x}\ ,$$

$$\frac{\partial}{\partial y}=\sin\theta\frac{\partial}{\partial r}+\frac{\cos\theta}{r}\frac{\partial}{\partial\theta},\quad \vec{\nabla}_\perp^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\ .$$

In the cylindrical coordinates the second y derivative $\partial^2f/\partial y^2$ can be written as

$$\frac{\partial^2 f}{\partial y^2}=\frac{1}{4}\left\{\sum_{n=0}^{\infty}2\vec{\nabla}_\perp^2f_n-2\cos\theta\left[\vec{\nabla}_\perp^2-\frac{2}{r}\frac{\partial}{\partial r}\right]f_0-\vec{\nabla}_\perp^2f_1-\sum_{n=0}^{\infty}\frac{\cos n\theta}{\cos(n+2)\theta}\left[\vec{\nabla}^2+2(n+1)\left[\frac{1}{r}\frac{\partial}{\partial r}+\frac{n+2}{r^2}\right]\right]f_{n+2}\right.$$
$$\left.-\sum_{n=3}^{\infty}\frac{\cos n\theta}{\cos(n-2)\theta}\left[\vec{\nabla}^2-2(n-1)\left[\frac{1}{r}\frac{\partial}{\partial r}-\frac{n-2}{r^2}\right]\right]f_{n-2}\right\}\ . \tag{34}$$

Using the above expression, Eq. (32) breaks down to an infinite sequence of coupled differential equations for the cylindrical harmonics Φ_{sn}, which is very difficult to solve in the general case. However, in the special case $\Phi_{s3}=0$, $\Phi_{s2k}=0$, the equation for the first cylindrical harmonic Φ_{s1} decouples from the rest:

$$\left[\rho^4\vec{\nabla}_\perp^4+\rho^2\vec{\nabla}_\perp^2\left[\frac{v_y^2m_i}{\gamma c_s^2m_e}+F-1+\frac{E_0^2m_i}{2B_0^2c_s^2m_e}\right]+\frac{v_y^2m_i}{\gamma c_s^2m_e}(F-1)+\frac{E_0^2m_i}{2B_0^2c_s^2m_e}F\right](\Phi_{s1}-\alpha B_0v_yx)=0\ , \tag{35}$$

where

$$\alpha=\frac{\left[\frac{v_y^2}{\gamma}+\frac{E_0^2}{2B_0^2}\right]F}{\frac{v_y^2}{\gamma}(F-1)+\frac{E_0^2}{2B_0^2}F}\ .$$

In a similar manner we find an equation connecting the first and the fifth harmonic

$$\left[\vec{\nabla}_\perp^2+8\left[\frac{1}{r}+\frac{5}{r^2}\right]\right]f_5+\left[\vec{\nabla}_\perp^2-4\left[\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right]\right]f_1=0\ , \tag{36}$$

where

$$f_1=-\cos\theta\left[\frac{\omega_{pi}^2}{\Omega_i^2}\left[1-\frac{E_0^2m_i}{2B_0^2c_s^2m_e}\right]\left[\vec{\nabla}_\perp^2+\frac{1}{\rho^2}F\right]\times(\Phi_{s1}-B_0v_yx)-\frac{\omega_{pi}^2}{c_s^2}\Phi_{s1}\right]\ .$$

We adopt the vortex core as a circle with the radius r_0 and solve Eq. (35) separately in the regions $r<r_0$ and $r>r_0$, requesting localization of Φ_{s1} for $r\to\infty$. We allow the slope F to have different values in these two regions. Obviously, the outside value of F must be equal to zero in order to have a finite value of Φ_{s1} for $r\to\infty$.

Equation (35) is readily solved in terms of Bessel functions and a well localized vortex solution has the form

$$\Phi_{s1}(r,\theta)=\cos\theta\times\begin{cases}CK_1(\kappa_1r), & r>r_0\\ \alpha B_0v_yr+D_1J_1(k_2r)+D_2J_1(k_1r), & r<r_0\ ,\end{cases} \tag{37}$$

where J_1,K_1 are a Bessel function of first order and a modified Bessel function of first order, respectively. The constants κ_1,k_1,k_2 in (37) are related to the slopes $F_{\text{out}},F_{\text{in}}$ through the set of equations

$$-\kappa_1^2+\kappa_2^2=\frac{1}{\rho^2}\left[\frac{v_y^2m_i}{\gamma c_s^2m_e}+\frac{E_0^2m_i}{2B_0^2c_s^2m_e}-1\right]\ ,\qquad \kappa_1^2\kappa_2^2=\frac{1}{\rho^4}\frac{v_y^2m_i}{\gamma c_s^2m_e}\ ; \tag{38}$$

$$k_1^2+k_2^2=\frac{1}{\rho^2}\left[\frac{v_y^2m_i}{\gamma c_s^2m_e}+F-1+\frac{E_0^2m_i}{2B_0^2c_s^2m_e}\right]\ ,\qquad k_1^2k_2^2=\frac{1}{c_s^2\rho^4}\frac{m_i}{m_e}\left[\frac{v_y^2}{\gamma}(F-1)+F\frac{E_0^2}{2B_0^2}\right]\ . \tag{39}$$

From the physical boundary conditions at the edge of the vortex core, the potential Φ_r,Φ_s and their radial derivatives $\partial\Phi_r/\partial r,\partial\Phi_s/\partial r$ must be continuous at $r=r_0$. Furthermore, in order for the solution Eq. (37) to be valid on the core edge, we require also the continuity of $\vec{\nabla}_\perp^2\Phi_s$. From these continuity conditions all the constants of integration C,D_1,D_2,F, and the vortex parameters r_0 and

v_y, can be determined in terms of the pump amplitude E_0. From Eqs. (37)–(39), in the limit of a weak pump, we also find the standard expression of the Hasegawa-Mima mode, perturbed by the ponderomotive potential.

The number of available constants of integration is not sufficient to satisfy all the continuity conditions for the first and second radial derivatives of the fifth, Eq. (36), and higher odd-numbered cylindrical harmonics and, consequently, they will not be smooth functions at $r=r_0$. However, they produce only a small perturbation of the leading-order solution Eq. (37) and the effects of these discontinuities are expected to be negligible. It has been shown [11] that free dipole vortices associated with nonlinear drift waves are stable to small initial perturbations $\Delta\Phi(\vec{r})$, which contain only the second and higher cylindrical harmonics. Similar behavior is expected also for driven dipole vortices, while the investigation of their structural stability [i.e., stability in the presence of small perturbations $\Delta\Phi(t,\vec{r})$] would require extensive numerical calculations, which are beyond the scope of this paper.

B. Modulation instability case

Now we study the case of a short-wavelength pump. Using Eq. (25) with $k^2 \gg \vec{\nabla}_\perp^2$, analogously to Eq. (26) we have

$$\left[2i\left[\frac{\partial}{\partial t}+\vec{u}\cdot\vec{\nabla}_\perp\right]+\frac{c_s^2}{\omega}\vec{\nabla}_\perp^2\right]k^2\delta\Phi_r = -i(\vec{e}_z\times\vec{E}_0)\cdot\vec{\nabla}\frac{\Omega_e n_{es}}{n_0} , \quad (40)$$

where $\vec{u}$ is the group velocity of the pump wave

$$\vec{u}\equiv\frac{\partial\omega}{\partial k}\frac{\vec{k}}{k}=\frac{c_s^2\vec{k}}{\omega} . \quad (41)$$

It should be noted that here the nonlinear coupling is much weaker, $\sim(k^2\vec{\nabla}_\perp^2)^{-1}$, than in the case of a long-wavelength pump. As before, in the one-dimensional case we obtain the dispersion relation for the modulational instability

$$(\delta\omega-\delta\vec{k}\cdot\vec{u})^2 =\tfrac{1}{4}\gamma\rho^4k^4\Omega_e\Omega_i\left[1-\frac{2E_0^2}{B_0^2c_s^2}\frac{m_i}{m_e}\frac{\delta k^2}{k^2}\frac{1}{1+\rho^2\delta k^2}\right] , \quad (42)$$

where δk is the wave number of the envelope. Obviously, the threshold for the modulational instability is much higher than that for the OTS:

$$E_0^2>E_c^2=\frac{1}{2}\frac{m_e}{m_i}\frac{k^2}{\delta k^2}B_0^2c_s^2(1+\rho^2\delta k^2) . \quad (43)$$

From Eqs. (31) and (43) we can calculate the thresholds for OTS and modulational instability for typical plasma parameters in toroidal machines. Thus, for the work of Porkolab *et al.* [17] ($B=17$ kG and $T_e=200$ eV), threshold lower hybrid electric field E_c for the OTS instability is about 4 kV/m. In larger machines the OTS thresholds are higher: $E_c\sim20$ kV/m for the work of Liu and Tripathi [18] ($B\sim30$ kG and $T_e\sim1.5$ keV) and $E_c\sim40$ kV/m for the work of Weiland [19] ($B=27$ kG and $T_e=8.6$ keV). The corresponding modulational instability thresholds are higher by the factor $k/\delta k$. Although in realistic plasma heating experiments the thresholds are likely to be higher than our ideal plasma estimates, due to collisional effects, etc., power requirements for the OTS instability can be met with the existing rf sources.

In a two-dimensional case, we seek a traveling double vortex solution using the same procedure as for the OTS case in Sec. III A. The dipole vortex whose first cylindrical harmonic is decoupled from the higher harmonics, similarly to Eq. (35), is described by the equation

$$[(1-\alpha_1)\rho^4\vec{\nabla}^4+(\beta_1+F-1-\alpha_1F)\rho^2\vec{\nabla}^2+\beta_1(F-1)]\times\left[\Phi_{s1}-\frac{F}{F-1}B_0v_yx\right]=0 , \quad (44)$$

where

$$\alpha_1=\frac{E_0^2}{2B_0^2c_s^2}\frac{m_i}{m_e\rho^2k^2},\quad \beta_1=\frac{m_i}{\gamma m_e}\frac{(v_y-u)^2}{c_s^2} . \quad (45)$$

Obviously, localized solutions exist only if in the outer region $r>r_0$ we have $F=0$.

Equation (44) has the following solution in the form of a dipole vortex:

$$\Phi_{s1}(r,\theta)=\cos\theta\times\begin{cases}C_1K_1(l_1r)+C_2K_1(l_2r), & r>r_0\\ \dfrac{F}{F-1}B_0v_yr+D_3J_1(\lambda_1r)+D_4J_1(\lambda_2r), & r<r_0 ,\end{cases} \quad (46)$$

where

$$l_1^2+l_2^2=\frac{\beta_1-1}{\rho^2(\alpha_1-1)},\quad l_1^2l_2^2=\frac{\beta_1}{\rho^4(\alpha_1-1)} , \quad (47)$$

$$\lambda_1^2+\lambda_2^2=\frac{\beta_1+F-1-\alpha_1F}{\rho^2(1-\alpha_1)},\quad \lambda_1^2\lambda_2^2=\frac{\beta_1(F-1)}{\rho^4(1-\alpha_1)} . \quad (48)$$

Using Eqs. (45), (47), and (48), we can draw the following conclusions.

(a) There are no localized solutions if $\alpha_1>1$ and $\beta_1<1$, i.e., for a very strong pump and the vortex propagation close to the group velocity.

(b) Localized solutions described by one modified Bessel function in the external region $C_2=0$ exist for arbitrary vortex speed if the pump is not too strong $0<\alpha_1<1$.

(c) Localized solutions described by two modified Bessel function in the external region exist if $\alpha_1>1$ and $\beta_1>1$.

Similarly to the long-wavelength pump case, in case (b) all parameters of the double vortex are determined by the pump amplitude E_0. In case (c) we have one constant of integration more, so that one of the vortex parameters v_y or r_0 is free, which is similar to the problem treated in Ref. [14].

The solution (46) can be visualized as a localized density perturbation, in which high-frequency short-wavelength oscillations are trapped. Its dipole structure emerges as the result of the dominance of the $\vec{E}\times\vec{B}$ convective nonlinearity on the slow time scale.

IV. CONCLUSION

In this paper we have studied the interaction of a lower hybrid pump wave, propagating almost perpendicularly to the external magnetic field, with small, low-frequency density perturbations. In the strongly nonlinear regime studied here, low-frequency, convective vector-product-type nonlinear terms cause also a modification of the pump. Consequently, apart from the equations describing perturbations on the slow time scale, we also had to solve the equation describing the pump variation, which is, in principle, very difficult to do in the general case. We proceeded by expanding all the quantities in cylindrical harmonics and decoupling the first harmonic from the rest. In this way, for the first harmonic we found a localized solution in the form of a stationary, dipole vortex moving with a constant velocity.

Small corrections due to the fifth and higher odd-numbered cylindrical harmonics, which are not smooth functions at $r=r_0$, may be considered as perturbations of the leading-order solution. As shown elsewhere [11], drift-wave dipole vortices in undriven systems are stable to small initial perturbations $\delta\Phi(\vec{r})$, which contain only the second and higher cylindrical harmonics. Thus these higher harmonics' discontinuities are not expected to be responsible for any qualitative change of the simple dipole solution Eqs. (37) and (46).

The solutions presented here are a nonlinear mode driven by the ponderomotive force, which in the limit of a weak pump becomes identical to the Hasegawa-Mima solution. In the OTS case the vortex is fully determined by the pump wave amplitude. In the modulation case, under certain conditions, it is also determined by the pump only. However, in the case of a very strong pump and of the vortex speed being significantly different from the pump group velocity, one of the vortex parameters still remains free.

As shown in Ref. [18], for the case studied here, the OTS instability is experimentally relevant for typical conditions in a tokamak. As a consequence, in tokamaks, the emergence of plasma dipole vortices whose radius is smaller than the pump wavelength is expected. Conversely, the modulation instability is not of such importance and its growth rate is rather small, even in the regime $\omega/k_z v_{Te}>1$ (which is not included in our analysis). Consequently, we expect that coherent vortices found here may represent a final stage in the development of the oscillating two-stream instability and that they can have a large influence on the particle transport in magnetically confined fusion plasmas.

Robustness of drift-wave vortices (in undriven systems) is a well established fact (see, e.g, Ref. [11] and references therein). As a consequence, strong drift-wave turbulence may be described as an ensemble of coherent vortices and weakly correlated wavelike fluctuations, which permits a relatively simple analytic calculation of the corresponding transport coefficients [13]. Emergence of drift-wave vortices in parametrically driven systems (either by an upper [14] or lower hybrid pump) thus provides a channel for the anomalous transport. Due to the coherence and good localization of plasma vortices, each of them can be regarded as independent from the others. The mutual interaction during collisions between vortices gives rise to strong, very short lived, localized, random electric fields, which are expected to be responsible for the anomalous transport [13]. The study of such important phenomena as the long-time-scale stability of isolated dipole vortices in driven systems or the evolution of a turbulent ensemble ("gas") of vortices, interacting with both the pump and individual particles, would require extensive numerical calculations, which are beyond the scope of the present paper.

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