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Existence of long-range order in the steady state of a two-dimensional, two-temperature XY model

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Monte Carlo simulations are used to show that the steady state of the $d=2$, two-temperature, diffusive XY model displays a continuous phase transition from a homogeneous disordered phase to a phase with long-range order. The long-range order exists although both the dynamics and the interactions are local, thus indicating the failure of a naive extension of the Mermin-Wagner theorem to nonequilibrium steady states. It is argued that the ordering is due to effective dipole interactions generated by the nonequilibrium dynamics.

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Fluctuations increase as the spatial dimension of a system is decreased and low-dimensional, *equilibrium* systems with finite-range interactions cannot maintain long-range order. This observation is known as the Landau-Peierls (LP) argument [1] or, for systems with continuous symmetry, as the Mermin-Wagner (MW) theorem [2]. Since phase transitions and emerging long-range order in *nonequilibrium* steady states have attracted a lot of interest recently [3,4], it is natural to ask if similar arguments are valid for nonequilibrium systems.

Clearly, the LP argument or the MW theorem must be generalized if we want to apply them to a nonequilibrium steady state. The reason is that, while interactions and temperature define an equilibrium state, a nonequilibrium steady state is the result of the interplay of interactions and dynamics. Thus, apart from restricting the range of interactions, we must also qualify the dynamics in a nonequilibrium process. To understand this point, consider a kinetic Ising model in which two dynamical processes compete with each other [5]. The elementary processes are (i) spin flips generated by nearest-neighbor interactions coupled to a heat bath at temperature T and (ii) random spin exchanges generated by a $T=\infty$ heat bath. In this model, the interactions are short ranged but the dynamics is long ranged because spins are exchanged at arbitrarily large distances. As a result of the long-range dynamics, effective long-range interactions are generated and long-range order appears in this flip-and-exchange kinetic Ising model [6] even in $d=1$. This result is in contradiction with the LP argument which states that long-range order is impossible for a $d=1$ equilibrium Ising system with short-range interactions. Thus a simple extension of the LP argument and of the MW theorem to nonequilibrium systems without restricting the dynamics clearly does not work.

One might conjecture that by restricting both the interactions and the elementary dynamical processes to be local, the generation of effective long-range interactions would be prevented and an extension of the LP or MW arguments to nonequilibrium systems could be made. This conjecture, however, is not necessarily true since local interactions combined with anisotropic local dynamics have been shown to lead to effective long-range (dipole) interactions in a two-temperature kinetic Ising model [7]. Furthermore, a spherical approximation to the above model has shown [8] that effective long-range interactions are generated not only in the Ising case but also in the case of infinite-component order-parameter field. Thus one may expect that neither the LP argument nor the MW theorem can be extended to nonequilibrium steady states even if both the interactions and the dynamics are local. Of course, this expectation should be checked carefully since the details of the dynamics do play an important role in determining the ordering properties of nonequilibrium steady states, and the details, such as the presence or absence of topological defects, the type of defects determining the long-time relaxation of order, are indeed very different in the Ising model, in the spherical limit and in the case of an n-component order-parameter field with $1<n\leq d$.

Our aim here is to confirm the failure of the naive extension of the MW argument by studying the existence of long-range order in the $d=2$, two-temperature, diffusive kinetic XY model. According to the MW theorem, long-range order cannot exist in the $d=2$ equilibrium XY model with short-range interactions. However, we will present the results of Monte Carlo simulations which show that a continuous transition from the homogeneous disordered state to a state with long-range order exists in the nonequilibrium two-temperature model. We will also construct a field-theoretic description of the transition that is consistent with the Monte Carlo results. The theory indicates that, just as in the two-temperature Ising case [7], the critical properties are in the universality class of an equilibrium system with long-range interactions, supporting our conjecture that the generation of effective long-range interactions is the mechanism responsible for the breakdown of the MW theorem.

The model we study consists of XY spins $\mathbf{m}_i$ (i.e., of two-dimensional vectors of unit magnitude) at sites i of a square lattice with periodic boundary conditions imposed. There are ferromagnetic interactions of strength J between nearest-neighbor spins, so the energy of a configuration is $H=-J\Sigma_{\langle ij\rangle}\mathbf{m}_i\cdot\mathbf{m}_j$, where $\langle ij\rangle$ indicates nearest-neighbor

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pairs. The system evolves by Kawasaki exchanges [9] of nearest-neighbor spins, occurring with Metropolis rates [10]. Exchanges along one axis of the lattice, which we call the "parallel" direction, satisfy detailed balance at an inverse temperature $\beta_{\parallel}=J/T_{\parallel}$, while exchanges in the other direction, which we call the "perpendicular" direction, satisfy detailed balance at an inverse temperature $\beta_{\perp}=J/T_{\perp}$. When the temperatures of the two heat baths are equal, the dynamics satisfies detailed balance with the nearest-neighbor XY Hamiltonian, and the model reduces to the equilibrium kinetic XY model with Kawasaki dynamics. However, when the two temperatures are not equal, an energy current flows from the high-T heat bath to the low-T heat bath, and detailed balance is not satisfied. The critical behavior of the system when the two temperatures are not equal is very interesting in the Ising version of the above model [7]. Its universality class coincides with that of another equilibrium model: The Ising ferromagnet with dipolar interactions [7]. As we shall see below, an analogous situation develops in case of the XY model.

For simplicity, in our simulations, we considered only the case $\beta_{\parallel}=0$, i.e., exchanges in the parallel direction were random. Furthermore, the distribution of the angles of the spins (which is conserved by the dynamics) was chosen to be uniform: for a system with $N=L_{\perp}L_{\parallel}$ lattice sites the angles of the N spins $\{\mathbf{m}_j\}$ were $2\pi j/N$ with $j=1,2,\ldots,N$. A difficulty in studying the phase transition in this system is that the spatial anisotropy requires an analysis using anisotropic finite-size scaling [11]. That is, one must compare systems whose shapes scale such that $L_{\perp}/L_{\parallel}^{1+\Delta}$ is constant, where Δ is an anisotropy exponent. As was done for the two-temperature Ising system [12], and based on renormalization group results (discussed below) which indicate that $\Delta\approx1$ in the present model, we choose to simulate systems with sizes $L_{\perp}\times L_{\parallel}=12\times9$, 16×16, 24×36, and 32×64, which are related by the naive scaling $4L\times L^2$.

The simulations revealed a homogeneous disordered state at small $\beta_{\perp}$, and an ordered state with long-range order at large $\beta_{\perp}$. A typical ordered configuration is shown in Fig. 1. Because the spin distribution is conserved by the dynamics, the ordering occurs as a phase separation resulting in a steady-state configuration with a spin wave in the perpendicular direction. To study the transition with the type of ordering shown in Fig. 1, we defined the order parameter Ψ as the following average of long-wavelength limits of structure factors:

$$\Psi=\frac{1}{2}[C_1(2\pi/L_{\perp},0)+C_2(2\pi/L_{\perp},0)],$$

where $C_{\alpha}(q_{\perp},q_{\parallel})$ is the normalized Fourier transform of the αth component of the magnetization density. In the simulations, we measured the time evolution of Ψ and, after producing a rough estimate of their relaxation times, determined the time averages $\langle\Psi\rangle$ and $\langle\Psi^2\rangle$ in the steady state. The runs typically ranged in length from 4×10^5 Monte Carlo sweeps (MCS) for the 12×9 systems to 4×10^6 MCS for the 32×64 systems.

Figures 2 and 3 display the results of our simulations. The data for Ψ (Fig. 2) clearly show a continuous transition to a phase-separated ordered state at $\beta_{\perp c}\approx0.68$. (Note that the

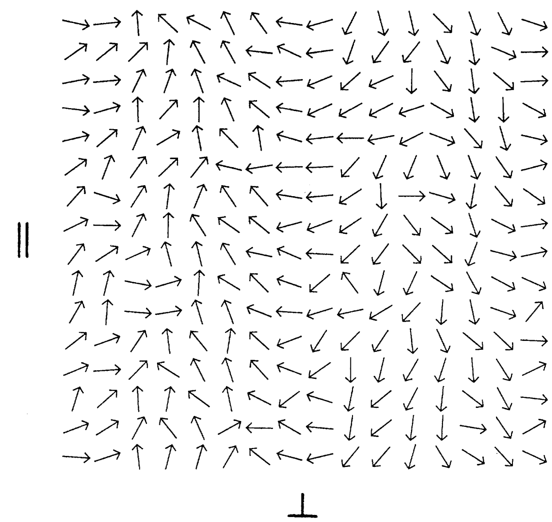

FIG. 1. A typical ordered configuration with long-range order. The $\perp$ and $\parallel$ directions correspond to the directions of the $\perp$ and $\parallel$ bonds, respectively. The configuration shows a spin wave in the perpendicular direction.

equilibrium Kosterlitz-Thouless transition [13] occurs at a lower temperature $\beta_c=8/9$ [14].) The presence of a continuous transition is further supported by the cumulants for various finite-size systems (Fig. 3), $g_L\equiv3(1-\frac{2}{3}\langle\Psi^2\rangle/\langle\Psi\rangle)$. In the limit of large sizes, they cross at the critical point $\beta_{\perp c}$, as usual for continuous transitions [15].

Since Figs. 2 and 3 show convincingly that the system does order, we now turn to the question of how the two-temperature XY model can have an ordered state with long-range order. To address this issue we construct a field-theoretic description of the model that indicates that the MW theorem fails because of the presence of effective long-range interactions generated by the two-temperature, diffusive dynamics. Although not rigorous, similar arguments have pre-

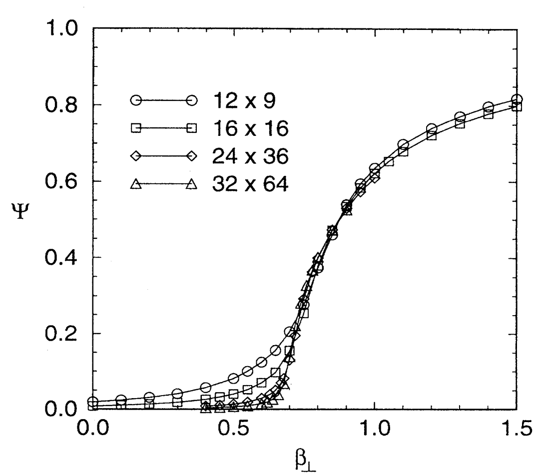

FIG. 2. Monte Carlo results for the order parameter $\langle\Psi\rangle$, showing a transition to long-range order at $\beta_{\perp c}\approx0.68$. The size and shape ($L_{\perp}\times L_{\parallel}$) of the systems is indicated in the legend. Error bars are much smaller than the symbol size.

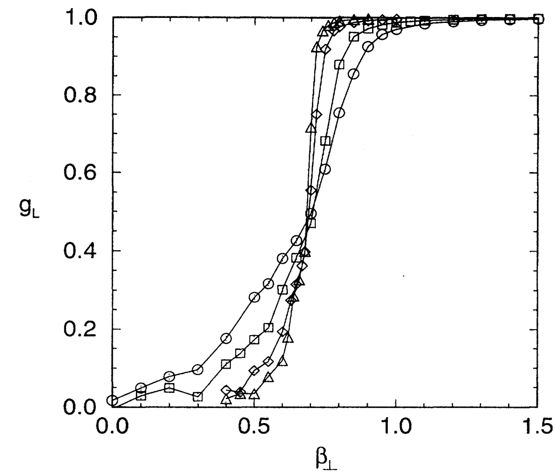

FIG. 3. Monte Carlo results for the cumulant g_L defined in the text. The data for the different finite-size systems cross asymptotically at $\beta_{\perp c} \approx 0.68$, in agreement with the data for $\langle \Psi \rangle$, shown in Fig. 2. The size and shape, $L_\perp \times L_\parallel$, of the systems is the same in Fig. 2. Error bars indicate one standard deviation of statistical errors.

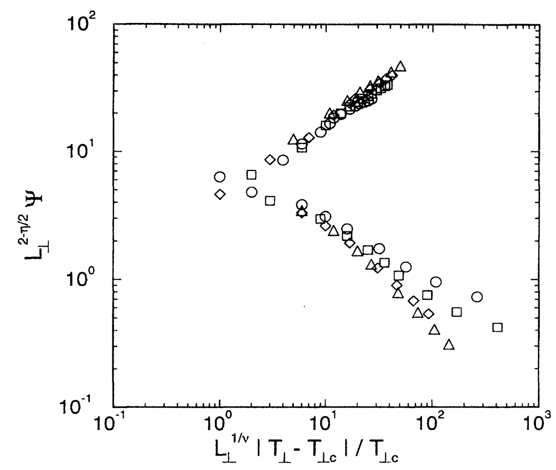

FIG. 4. Finite-size scaling of Monte Carlo results for the order parameter $\langle \Psi \rangle$, using $\beta_{\perp c}=0.68$, $\nu=0.65$, and $\eta=0.02$. The size and shape, $L_\perp \times L_\parallel$, of the systems is the same as in Fig. 2.

viously been used to discuss the generation of long-range interactions in Ising systems [7,16], and in the spherical limit of the two-temperature diffusive dynamics [8].

We start with the $O(2)$ version of model B of Halprin and Hohenberg [17] which describes critical relaxation by Kawasaki dynamics towards the equilibrium of the coarse-grained XY model:

$$\partial_t \boldsymbol{\phi} = \lambda \nabla^2 \left[(-\nabla^2 + \tau)\boldsymbol{\phi} + \frac{1}{3!} g \boldsymbol{\phi}\phi^2 \right] + \boldsymbol{\eta}.$$

Here, $\boldsymbol{\phi}(\mathbf{x},t)$ is the two-dimensional, coarse-grained order parameter, the parameters λ and g are constants, and the relevant temperature dependence is contained in τ, such that the critical point corresponds to $\tau=0$. Furthermore, $\boldsymbol{\eta}(\mathbf{x},t)$ is a Gaussian noise source which has zero mean and $\langle \eta_\alpha(\mathbf{x},t)\eta_\beta(\mathbf{x}',t')\rangle = 2\lambda\delta_{\alpha\beta}\delta(t-t')\nabla^2\delta(\mathbf{x}-\mathbf{x}')$.

Now consider the generalization of model B to a d-dimensional two-temperature model with spin exchanges that occur at one temperature in an m-dimensional "parallel" subspace and at another temperature in the remaining $d-m$ "perpendicular" dimensions. In order to account for the different temperatures of the exchanges in the different subspaces of this model, the Laplacian operators and the noise term must be split into parallel and perpendicular parts. Furthermore, parameters such as τ and g will have different values depending on whether they are associated with the diffusion in the parallel or the perpendicular directions. For example, the model B term $\tau\nabla^2\boldsymbol{\phi}$ will split into two terms: $\tau_\perp \nabla_\perp^2 \boldsymbol{\phi}$ and $\tau_\parallel \partial^2 \boldsymbol{\phi}$, where ∂ ($\nabla_\perp$) indicates a gradient over the m parallel ($d-m$ perpendicular) directions. As τ has now been split into parallel and perpendicular parts, the theory can describe various types of critical behavior depending on whether (i) $\tau_\perp=0$ and $\tau_\parallel>0$, (ii) $\tau_\parallel=0$ and $\tau_\perp>0$, or (iii) $\tau_\perp=\tau_\parallel=0$. The type of ordering ordering seen in Fig.1 corresponds to (i) and in the following we restrict our consideration to that case. Keeping only those terms which are relevant (in the renormalization group sense) near the upper critical dimension $d_c=4-m$, we arrive at the following equation:

$$\partial_t \boldsymbol{\phi} = \lambda \nabla_\perp^2 \left[\left(-\nabla_\perp^2 + \tau_\parallel \frac{\partial^2}{\nabla_\perp^2} + \tau_\perp \right) \boldsymbol{\phi} + \frac{1}{3!} g_\perp \boldsymbol{\phi}\phi^2 \right] + \boldsymbol{\eta}_\perp .$$

Note that only the perpendicular part of the noise, $\boldsymbol{\eta}_\perp$, is relevant near d_c. It has vanishing mean and $\langle \eta_{\perp\alpha}(\mathbf{x},t)\eta_{\perp\beta}(\mathbf{x}',t')\rangle = 2\lambda\delta_{\alpha\beta}\delta(t-t')\nabla_\perp^2\delta(\mathbf{x}-\mathbf{x}')$.

Interestingly, this Langevin equation is of the form

$$\partial_t \boldsymbol{\phi} = \lambda \nabla^2 \frac{\delta \mathcal{H}}{\delta \vec{\phi}} + \boldsymbol{\eta},$$

where $\boldsymbol{\eta}$ satisfies the fluctuation-dissipation theorem and therefore the above equation describes the critical dynamics of an equilibrium system. The Hamiltonian $\mathcal{H}$ of that equilibrium system is most easily expressed in Fourier space as

$$\mathcal{H} = \frac{1}{2}\int_k \frac{k_\perp^4 + \tau_\parallel k_\parallel^2 + \tau_\perp k_\perp^2}{k_\perp^2} \boldsymbol{\phi}(k)\boldsymbol{\phi}(-k) + \frac{g_\perp}{4!}\int_{k_1,\ldots,k_4} \boldsymbol{\phi}(k_1)\boldsymbol{\phi}(k_2)\boldsymbol{\phi}(k_3)\boldsymbol{\phi}(k_4) \times \delta(k_1+\cdots+k_4),$$

which is the Hamiltonian of the the XY model with dipolar interactions. Thus, as in the Ising case [7], we arrive at the conclusion that the mesoscopic critical properties of the two-temperature diffusive XY model are in the same universality class as an equilibrium model with long-range (dipole) interactions. Most importantly for the two-temperature XY model, however, we see the mechanism that apparently causes the Mermin-Wagner theorem to fail: the two-temperature diffusive dynamics generate effective long-range interactions in the steady state of the system.

The static critical properties of the equilibrium XY model with dipole interactions have been studied using renormalization group methods for the $m=1$ case [18]. The results show that the static structure function scales as

$$S(\mathbf{k}_\perp,\mathbf{k}_\parallel,\tau)=\mu^{-2+\eta}S(\mathbf{k}_\perp/\mu,\mathbf{k}_\parallel/\mu^{1+\Delta},\tau/\mu^{1/\nu}),$$

which defines the exponents η, ν, and Δ. To second order in $\varepsilon=3-d$, it has been found that $\nu\approx0.500+0.100\varepsilon+0.054\varepsilon^2$, $\eta\approx0.0177\varepsilon^2$, and $\Delta=1-\eta/2$.

Figure 4 shows the scaling plot of $\langle\Psi\rangle$ using the above values of the exponents with $\varepsilon=1$. The data collapse is not perfect, but it is consistent with the predictions of the field theory, considering that the field-theory predictions are only approximate, being to finite order in ε. We caution, however, that a range of exponents ($\nu=0.6\pm0.1$ and $\eta=0.10\pm0.15$) also produce a collapse of the data. A more accurate determination of the value of the exponents would require data for a variety of system shapes and significantly larger system sizes than were used in the current study.

In summary, our Monte Carlo simulations taken together with the field-theoretic results suggest a simple picture: Long-range order does exist in the $d=2$, two-temperature, diffusive XY model and it is caused by effective long-range interactions generated by the anisotropic diffusive dynamics coupled with the violation of the fluctuation-dissipation theorem. In light of this result, we return to the question posed at the outset: Can the LP argument or the MW theorem be extended to nonequilibrium steady states? In general, the answer to this question appears to be no, even for systems with purely local interactions and dynamics. However, it still remains possible that the LP argument or the MW theorem can be extended to nonequilibrium steady-state systems which evolve with some restricted class of dynamical rules. Unfortunately, our knowledge in this field is rather limited; we know only that long-range order can appear in low-dimensional nonequilibrium systems whose dynamics is either long ranged [6], or involves anisotropic diffusion [3]. Clearly, much more work needs to be done to fully answer the question posed in this paper.

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