# Attraction of charged particulates in plasmas with finite flows

Sergey V. Vladimirov* and Mitsuhiro Nambu

*Department of Applied Physics, Faculty of Engineering, Kyushu University, Ropponmatsu, Fukuoka 810, Japan*

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It is demonstrated that charged dust particulates in plasmas with finite ion flows can attract each other due to collective interactions involving the ion oscillations in the flow. The possibility of lattice formation in electrostatic sheaths at the boundary of a dusty plasma is discussed.

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Recently, there is much interest in low-temperature plasmas containing micrometer-sized highly charged particulates or "dust" [1–19]. The interest is motivated by numerous industrial applications, such as material processing [1–4], as well as by observations in space and the Earth's environment [5–7].

In a series of recent experiments [8–12], the formation of microscopic Coulomb crystals [13] of solid particles as well as particulate coagulation [14,15] have been demonstrated. Associated with the Coulomb crystallization in strongly coupled dusty plasmas, there have been indications of formation of pairs of particulates with small separations [8] as well as of the presence of enhanced low-frequency fluctuations [9]. In a typical experiment, the dust is embedded in the sheath region [20] where the balance between the gravitational and electrostatic forces is established (see, e.g., [8,10]).

The physics of the sheath region of two-component plasmas has been qualitatively understood for many years [20]. It was demonstrated that strong electric fields, plasma density gradients, and ion flows are established there. According to the Bohm criterion, the average velocity of the plasma ion flow in the sheath region must exceed that of ion sound [20]. The effects of dust grains in the plasma-wall region have been investigated in Refs. [16–18]. In particular, it was found that, similar to the Bohm criterion for two-component plasmas, the ions entering the sheath region must have a velocity exceeding the critical one (the latter depends on the dust concentration and is not less than the ion sound velocity) [18].

In this Rapid Communication, we point out a possibility of charged particle attraction in dust plasma systems with finite ion flows. The mechanism is similar to the recently proposed scenario of attractive forces between *moving* charged particulates which involves the collective interactions via low-frequency electrostatic fluctuations [19] of dusty plasmas. The effect is analogous to the Cooper pairing [21] of electrons in superconductors, and has been studied for two-component electron-ion plasmas by Nambu and Akama [22] in which the possibility of electron attraction has been demonstrated. The important feature of the present investigation is that we consider the situation close to laboratory experiments, namely, *static* dust particulates which can interact through the low-frequency oscillations in the ion flow whose velocity $v_{i0}$ exceeds the ion-acoustic velocity $v_s$. Physically, the generation of the wake acoustic fields in this case is the same as the generation of the electromagnetic fields for the charge at rest placed in a medium moving faster than light [23].

The electrostatic potential around the isolated test dust particle can be written as

$$\Phi(\mathbf{x},t)=\int \frac{q_t}{2\pi^2 k^2}\frac{\delta(\omega-\mathbf{k}\cdot\mathbf{v}_t)}{\varepsilon(\mathbf{k},\omega)}\exp(i\mathbf{k}\cdot\mathbf{x}-i\omega t)d\mathbf{k}d\omega, \quad (1)$$

where $q_t$ and $v_t \ll v_{Ti}$ are the charge and velocity of the test dust particle, respectively. The dielectric response function of the plasma in the presence of finite ion flow with the speed $v_{i0}$ [$v_{Ti} \ll v_{i0} \ll v_{Te}$, where $v_{Ti,e}=(T_{i,e}/m_{i,e})^{1/2}$ is the ion (electron) velocity, $T_{i,e}$ is the ion (electron) temperature, $T_i \ll T_e$, and $m_{i,e}$ is the ion (electron) mass] is calculated under the condition

$$kv_{Ti} \ll |\omega-k_z v_{i0}| \ll kv_{Te}, \quad (2)$$

where the $z$ axis is directed along the ion flow.

Furthermore, we consider two cases. The first one corresponds to the situation when influence of the dust on dielectric properties of the plasma can be neglected; the polarization of plasma is determined by the contribution of electrons and ions. The second case corresponds to the sufficiently high dust density. For the latter situation, we consider the test particulate at rest, i.e.,

$$|\omega=\mathbf{k}\cdot\mathbf{v}_t| \ll kv_{Td}, \quad (3)$$

where $v_{Td}=(T_d/m_d)^{1/2}$ is the dust thermal velocity ($T_d$ and $m_d$ are the dust temperature and mass, respectively).

For the plasma dielectric response, we have

$$\varepsilon(\mathbf{k},\omega)=1+\frac{1}{k^2\lambda_D^2}-\frac{\omega_{pi}^2}{(\omega-k_z v_{i0})^2}, \quad (4)$$

where $\lambda_D=\lambda_{De}\equiv(T_e/4\pi n_e e^2)^{1/2}$ is the electron Debye radius for the first case (of isolated dust test particle), $\lambda_D^{-2}=\lambda_{De}^{-2}+\lambda_{Dd}^{-2}$ for the second situation [when condition

*Permanent address: Theory Department, General Physics Institute, Vavilova 38, Moscow 117942, Russia. Present address: Department of Theoretical Physics, School of Physics, the University of Sydney, NSW 2006, Australia. Electronic address: vladimi@physics.usyd.edu.au

 

(3) takes place], $\lambda_{Dd}=(T_d/4\pi n_d q_d^2)^{1/2}$ is the dust Debye radius, $q_d$ is the charge of the dust particles, $\omega_{pi}=(4\pi e^2 n_i/m_i)^{1/2}$ is the ion plasma frequency, and $n_{e,d,i}$ are the electron, dust, and ion number densities, respectively. We stress here that the negative contribution due to the ion flow is the most important for the present consideration. From Eq. (4), we can also see that the relative contribution of plasma electrons is of less importance.

The inverse of function (4) can be written as

$$\frac{1}{\varepsilon(\mathbf{k},\omega)}=\frac{k^2\lambda_D^2}{1+k^2\lambda_D^2}\left[1+\frac{\omega_s^2}{(\omega-k_z v_{i0})^2-\omega_s^2}\right], \tag{5}$$

where $\omega_s=kv_s/(1+k^2\lambda_D^2)^{1/2}$ is the frequency of the oscillations in the ion flow, and $v_s=\lambda_D\omega_{pi}$ is the ion sound velocity.

Substituting Eq. (5) into (1), we obtain that the total electrostatic potential can be represented as the sum of the following two potentials:

$$\Phi(\mathbf{x},t)=\Phi_D(\mathbf{x},t)+\Phi_C(\mathbf{x},t), \tag{6}$$

where

$$\Phi_D=\frac{q_t}{r}\exp(-r/\lambda_D) \tag{7}$$

is the usual static Debye screening potential, and $\Phi_C(\mathbf{x},t)$ is the additional potential involving, in particular, the collective effects caused by the oscillations in the ion flow. We have

$$\Phi_C=\int\frac{q_t}{2\pi^2k^2}\frac{k^2\lambda_D^2\omega_s^2\exp(i\mathbf{k}\cdot\mathbf{x}-i\omega t)}{(1+k^2\lambda_D^2)[(\omega-k_z v_{i0})^2-\omega_s^2]}\times\delta(\omega-\mathbf{k}\cdot\mathbf{v}_t)d\mathbf{k}d\omega. \tag{8}$$

From Eq. (8), it follows that the potential changes its sign due to the overscreening depending upon whether $|\omega-k_z v_{i0}|$ is larger or smaller than $\omega_s$. In the case when $|\omega-k_z v_{i0}|$ is close to this frequency, strong resonant interaction between the oscillations in the ion flow and the test particulate appears. The important point is that even for the static test charge, the potential "behind" it oscillates as a wake field, if the speed of the ion flow exceeds the ion-acoustic velocity. Therefore the formation of quasilattice structures can be possible since the periodic regions of attractive and repulsive forces between the particulates of the same polarity appear.

Integrating Eq. (8) over frequencies and assuming $\mathbf{k}\cdot\mathbf{v}_t\approx 0$, we obtain

$$\Phi_C(\mathbf{x},t)=\text{const}(t)=\frac{q_t}{2\pi^2\lambda_D M^2}\int\frac{k^2}{1+k^2}\frac{\exp(i\mathbf{k}\cdot\mathbf{x}/\lambda_D)}{(k_z^2+k_0^2)(k_z^2-k_1^2)}dk_z d\mathbf{k}_\perp, \tag{9}$$

where we introduced the dimensionless $k=k\lambda_D$. Furthermore, in Eq. (9), $k^2=k_z^2+k_\perp^2$, $k_{0,1}^2=\pm(1-M^{-2}+k_\perp^2)/2+[k_\perp^2M^{-2}+(1-M^{-2}+k_\perp^2)^2/4]^{1/2}$, and $M=v_{i0}/v_s$ is the Mach number. We note that contribution from the poles at $k_z=\pm ik_0$ provides the nonoscillating part which changes the effective Debye shielding scale in plasmas with finite ion flows [17].

We are interested now in the oscillating contribution to the collective potential (8) which arises from the residues at the poles at $k_z=\pm k_1$ in (9). Integration over angles in Eq. (9) can proceed using an expansion in spherical harmonics [22]. Furthermore, we assume $k_\perp^2\ll(M^2-1)$ as well as $k_\perp\ll 1$, and obtain

$$\Phi_C(z,\rho)\approx\frac{2q_t}{\lambda_D\sqrt{M^2-1}}\int_0^1 J_0(k_\perp\rho)\frac{k_\perp^2}{1-M^{-2}}\times\sin(k_\perp z/\lambda_D\sqrt{M^2-1})dk_\perp, \tag{10}$$

where $z$ and $\rho$ are the cylindrical coordinates of the field point, and $J_0$ is the Bessel function. For small distances in the perpendicular direction, $k_\perp\rho\ll 1$, and for $|z|>\lambda_D\sqrt{M^2-1}$, the main contribution to the stationary wake potential is given by

$$\Phi_C(\rho=0,z)\approx\frac{q_t}{|z|}\frac{2\cos(|z|/L_s)}{1-M^{-2}}, \tag{11}$$

where $L_s=\lambda_D\sqrt{M^2-1}$ is the effective length. From Eq. (11), we can conclude that the wake potential is attractive for $\cos(|z|/L_s)<0$.

To conclude, we presented a possibility for static dust particulate attraction in plasmas with finite ion flows. We have demonstrated that the collective interaction of the static test dust particulate with the low-frequency perturbations in the ion flow can provide the attractive wake potential. The mechanism is similar to that which is responsible for the Cooper pairing. We note that the physical idea of the Cooper-pairing effect is that the test electron polarizes the medium by attracting positive ions. The excess positive ions, in turn, attract the second electron. If this attractive interaction is strong enough to overcome the repulsive screened Coulomb interaction, the effective attraction between two electrons can be realized, and superconductivity results. For example, the jellium model [24] leads to an expression which is similar to our Eq. (5) where the first term is due to the screened Coulomb repulsion, whereas the second term corresponds to the attraction forces. For the effective attraction, the speed of *moving* charged particles (in the frame where the background medium is at rest) should exceed the sound velocity. However, the mechanism is the most effective when the particles' speed is quite close to the latter (in this case the resonant interaction with phonons is maximum). In our situation, with *static* charged dust particulates, the *moving* ions of the flow create the polarization necessary for the resulting attraction. Thus, in order for the latter to be operative it is required that one has continuous ion flow whose speed exceeds the ion-acoustic velocity. Note that this condition is usually observed in the sheath region of low-temperature laboratory plasmas. The test particulates can attract each other forming the quasi-lattice structures with the characteristic period of order $\lambda_D\sqrt{M^2-1}$ which is close to experimental data [8–12].

We note, that, in principle, any charged particles, under the corresponding conditions, can have similar near-field attractive potentials. However, the speed of electrons is usually too high (this corresponds to the condition $v_{Te} \gg v_s$) for the effective attraction to be realized. On the other hand, the ion velocity is too small. For the model considered here (with ions moving as a flow), we have to compare $v_{i0}$ with the characteristic velocity of collective perturbations in such type of plasma. Note that for ion-acoustic waves in the flow the (relative) speed of the ions is too small (which corresponds to the reference frame moving with $v_{i0}$). In principle, the ions of the flow can interact with other low-frequency perturbations in the plasma, for example, dust-acoustic waves which are supported by static dust particulates. However, the phase velocity of the dust-acoustic waves is quite small compared with $v_{i0}$ for the interaction to be effectively operative. Therefore we can conclude that the interaction of the static dust particulate with the ion oscillations in the flow provides the most effective contribution for the situation considered.

Finally, we note that the present investigation provides a qualitative possibility for the attraction of dust particulates in plasmas with finite flows. For quantitative comparison with results of the concrete experiments, other factors should be taken into account. In particular, the potential of ensemble (in contrast to the isolated test particle) of dust particulates might be calculated. This can be done by either adding the contributions of the isolated particulates (if their density is not high), or introducing their distribution function (when dust collective effects become important). Furthermore, contributions of other forces (e.g., gravitation) acting on the dust, as well as such factors as inhomogeneity of the flow, should be considered for the detailed picture. These are subjects of future investigations. Some of them are underway now, and the results will be reported elsewhere.

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