Simulation of an experiment on crystal extraction of 900-GeV protons

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A Fermilab experiment [R. A. Carrigan, Jr. *et al.* (unpublished)] on crystal extraction of 900-GeV protons from the Tevatron has been simulated with the Monte Carlo code CATCH [Biryukov, CATCH 1.4 User's Guide, CERN SL Report No. 93-74, 1993 (unpublished); Phys. Rev. E **51**, 3522 (1995)] tested earlier in the CERN-SPS crystal extraction experiment [Akbari *et al.*, Phys. Lett. B **313**, 491 (1993)]. Detailed predictions for the extraction efficiency, angular scans, and extracted beam profiles are presented. Furthermore, several ideas are proposed and tested by the simulation on how to get key information about the extraction experiment: the "septum width" of the crystal and the dependence of the extraction efficiency on it, the impact parameters of the incident protons, and the contribution of the first and multipasses to the extraction. With the use of simulation, we analyze ways to optimize the Fermilab experiment.

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I. INTRODUCTION

Recent experiments [1] have shown impressive progress in high-efficiency steering of a particle beam by means of bent-crystal channeling [2]. The experiments on crystal-assisted extraction of protons from the CERN-SPS [3] and the Fermilab Tevatron [4] accelerators are of particular interest. These studies have in view possible application of channeling for beam extraction from a multi-TeV machine [4–6], where an extracted beam would open up very interesting possibilities for beauty quark physics.

The extraction technique (first demonstrated at lower energies by Avdeichikov *et al.* [7], Bavizhev *et al.*, and Asseev *et al.* [8]) employs a bent crystal, placed inside the accelerator vacuum chamber at the periphery of the circulating beam, in order to intercept the protons diffusing from the beam core to the halo and to deflect the trapped protons at a small angle required for the extraction. Beam deflection by a crystal is due to the trapping of some particles (parallel to the crystallographic plane within the Lindhard angle, also called the critical angle) in the potential well formed by the field of atomic planes, where the particles then follow the direction of (are *channeling* in) the atomic planes [9–11]. The channeling effect persists in a bent crystal until the ratio of the beam momentum p to the bending radius R becomes as high as the maximal field (~6 GeV/cm in silicon). However, the crystal bend reduces the phase space available for channeling, thus decreasing the fraction of particles channeled. The scattering processes in the crystal may also cause the trapped particle to come to a free state (dechanneling).

The primary impact parameters and divergence of the particles intercepted by the crystal are defined by the transverse speed of the amplitude growth for the particle betatron motion in the halo region of the circulating beam. The halo particles move to the periphery either because of the natural processes of scattering and nonlinearities of the accelerator fields, or due to an excitation (transverse or longitudinal) by means of noise applied to the circulating particles (see, e.g., Ref. [12]). The particles hit a crystal very close to its edge, with impact parameter b in the range from angstroms to micrometers. With noise applied, the impact parameters are controllable; however, they are still in the micrometer range. Such low values of b call for near perfection of the crystal edge. Alternatively to a perfect edge, one should investigate how the crystal extracts particles in the multipass mode, which involves many turns in the accelerator and several scatterings in the crystal of the circulating particles.

As the extraction process includes many passes and turns, there is no easy way to extrapolate the experimental results to a higher energy. This makes essential a detailed comparison of the measurements with the predictions of computer simulations. Such an analysis with use of Monte Carlo simulation was made [13] in the course of an experiment on the crystal extraction of 120-GeV protons from the CERN SPS. It showed good qualitative agreement of the theory with measurements [3]. The major outcome of the analysis was a prediction of the edge imperfection of the crystals used for extraction at SPS. The new SPS experiment, employing a crystal with an amorphous edge layer to test the above idea, has indeed shown the same efficiency [14]. The prediction for another SPS crystal with a new geometry ("U shaped") [13], namely, much the same efficiency but narrower [70 μrad full width at half maximum (FWHM)] angular scan, has also been confirmed (see comparison in Ref. [15]).

Making use of the same simulation code [16] tested at the SPS, here we model the crystal extraction of protons from the Tevatron beam halo with parameters matching the Fermilab E853 experiment [4,17].

The main objective of the present work, besides pre-

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dicting the results of the Fermilab experiment, is to propose several ideas on how to measure key parameters of the extraction experiment. Since a realistic crystal has a nonvanishing irregularity of its surface, this irregularity defines some range of inefficient impact parameters at the edge ("septum width") where channeling is disrupted. In view of the very small impact parameters, the multipass mode of extraction may well be the only feasible one. This makes a septum width and related things to be a central point. The following information is essential for understanding the crystal extraction process: (1) efficiency, and contributions to it from the first and secondary passes; (2) distribution of the primary and secondary impact parameters on the crystal; (3) septum width; and (4) dependence of efficiency on the septum width.

We propose ways to get this information in the framework of the Fermilab experiment. With use of simulation, we also analyze ways to optimize the Fermilab experiment in order to get the highest efficiency and/or the best conditions for measuring the key parameters of the experiment. The important issue is how one can extrapolate the results.

II. QUALITATIVE DISCUSSION OF THE EXTRACTION

The essential difference of the Fermilab experiment (E853) at the Tevatron from its analog at the CERN SPS (RD22), besides a much higher energy (900 vs 120 GeV), is a "vertical" extraction scheme instead of a "horizontal" scheme. We call the extraction scheme with the crystal atomic planes *perpendicular* to the crystal face touching the beam "vertical." In both schemes, CERN and Fermilab, the crystal is offset horizontally from the beam; as a result, in both schemes the beam is diffused or kicked in the horizontal plane to reach the crystal. However, in the CERN experiment the protons trapped by a crystal are channeled and bent in the same horizontal plane, while in E853 the channeling and bending occurs in the vertical plane.

In the CERN scheme one has to align the crystal only in the horizontal plane, with an accuracy of the Lindhard angle (14 μrad at 120 GeV), and with no care on the vertical plane. In the Fermilab scheme one should align the crystal in both planes: in the channeling plane (vertical) with the same accuracy as the Lindhard angle (6 μrad at 900 GeV), and in the horizontal plane in order to keep the crystal face parallel to the incident protons (Fig. 1).

At first glance, this necessity to tune two angles is a serious inconvenience. Here we show that, in fact, this extra degree of freedom presents an excellent possibility to study thoroughly the process of crystal extraction in many details. *This* is why one can measure the septum width of a crystal and the dependence of the extraction efficiency on it, the distribution of the primary and secondary impact parameters at the crystal, and the contributions to the efficiency from the first and secondary passes in the crystal.

Let us see what happens if the crystal is misaligned

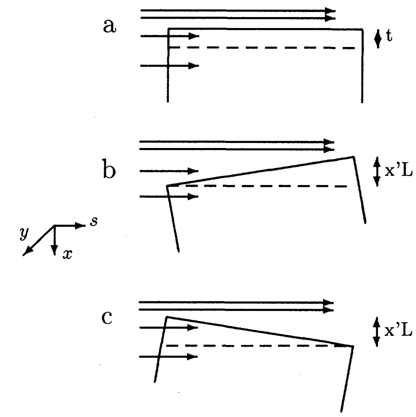

FIG. 1. Sketch of the proton interaction with an edge of the horizontally tilted crystal. Direction of proton motion is shown with arrows. The vertical axis is normal to the figure plane. (a) Perfectly aligned crystal. The septum width t is shown. (b) Tilt with $x' > 0$. (c) Tilt with $x' < 0$. The range of inefficient impact parameters ($x'L$) is shown.

with respect to the beam particles. With a vertical tilt y', the protons are misaligned with the crystal planes at first incidence. After scattering in one or a few inefficient passes, and several turns in the accelerator, some protons come parallel to the crystal planes and hence get trapped in the channeling mode. The width of the vertical angular scan thus reflects the divergence of the incident beam, angle of scattering in the crystal, and the angular acceptance of the crystal planes (Lindhard angle).

With a horizontal tilt x', the atomic planes' orientation y' with respect to the beam does not change. Depending on the sign of x', either the upstream end of the crystal approaches the beam (we define $x' < 0$ in this case), or the downstream one approaches the beam ($x' > 0$); see Fig. 1. Because of the x' tilt, some range of the inefficient impact parameters (*septum width*), as thick as $t = |x'|L$, occurs at the crystal edge; L=4 cm is the crystal length. The protons incident in this range, $0 < b < t$, do not traverse the full length of the crystal (Fig. 1). The result of the x' tilt depends dramatically on the sign of x'.

In the case of $x' > 0$, the protons traverse the downstream edge. This edge is misaligned by ~0.64 mrad (the crystal bending angle) with respect to the beam particles. Therefore these protons pass through the crystal edge as through an amorphous substance. This case imitates the crystal with an amorphous near-surface layer (septum width) as wide as $t \approx x'L$. Measuring the extraction efficiency F as a function of x' for $x' > 0$, one measures the dependence of F on the septum width t. From the theory (see also Ref. [18]) one expects a very weak $F(t)$ dependence at high energies. The confirmation would be quite encouraging for a multi-TeV application of crystal extraction. Notice that the minimal step of the septum width scan could be very fine: with $\delta x'$=2.5 μrad (the smallest step possible with the Fermilab goniometer) and L=4 cm one has δt= 0.1 μm.

In the case of $x' < 0$, the protons traverse the upstream edge. The upstream face is aligned with respect to the beam. Therefore many particles are trapped in the channeling mode. However, those incident in the range $0 < b < x'L$ traverse a reduced (<4 cm) length, thus getting a reduced (<0.64 mrad) deflection. These particles, deflected at a fraction of 0.64 mrad, are mostly lost, as they do not fit into the acceptance of either the accelerator or the extraction beam line.

The difference between the two cases $x'>0$ and $x'<0$ causes a strong asymmetry of the $F(x')$ dependence. The efficiency difference, $\Delta F = F(x') - F(-x')$, is proportional to the number of protons incident (on the primary or secondary passes) in the range $0 < b < x'L$. Varying x' and observing ΔF, one investigates the distribution function of the incident protons over the impact parameters b at the crystal, with an accuracy of up to $\delta b=\delta x'L=0.1$ μm.

To understand this better, suppose that an ideal crystal is perfectly aligned to the beam. Of all the *extracted* protons, many (suppose one-half) are extracted on the first pass; others are extracted on the secondary passes. Now we misalign the crystal at $x' = -b_{max}/L$, where b_{max} is the maximal impact parameter at the first incidence of protons. Still, the same number of protons is trapped in the first pass; but, these trapped protons are now *lost*, because they are bent by only part of the 0.64 mrad. The other protons are scattered and come at later turns with the secondary impact parameters $b \gg b_{max}$; some of them are extracted (in much the old manner). We see that the overall number of extracted protons has been reduced by at least a factor of 2 over the x' change from 0 to $-b_{max}/L$. Such a "step" in the $F(x')$ function can be easily observed and interpreted. The "step" width $\Delta x'$ is related to $b_{max} \approx \Delta x'L$. The "step" height ΔF is the contribution of the first-pass protons to the overall extraction efficiency.

Now suppose the crystal is imperfect: it has a quasi-amorphous layer of thickness $t > b_{max}$ just at the edge (a septum width). This crystal is insensitive to $b{\sim}1$ μm; however, in just the same way one measures the particle distribution over the *secondary* impact parameters (in a broad range $b > t$).

Notice that for an imperfect crystal angling within $-t/L < x' < t/L$ the x' sign is not important. Both edges, upstream and downstream, are amorphouslike and cannot trap particles. Only for bigger tilts, $x' < -t/L$, can the upstream edge trap and lose protons, in the way discussed above. Therefore, for a septum width $t \neq 0$, we expect the $F(x')$ function to be symmetric within $-t/L < x' < t/L$, but asymmetric for $|x'| > t/L$. The threshold x'_{th}, where the asymmetry of the $F(x')$ scan appears, is a measure of the crystal septum width: $t \approx x'_{th}L$.

The above picture is complicated by another interesting phenomenon. The protons incident on a perfectly aligned imperfect crystal with $b_{max} < t$ have to traverse the full length of the crystal. The respective scattering angle, $\theta_s=10$ μrad, and probability of nuclear interaction, $\approx$0.1, over 4 cm of silicon are sizable. Suppose this crystal is misaligned so that $b_{max}/x'L \approx 0.1$. Then, at first incidence the protons traverse only the crystal edge, with the length $\leq$0.1 that of the crystal. The respective scattering, $\theta_s \leq 3$ μrad, and probability of nuclear interaction, $\approx$0.01, over 0.4 cm of silicon are much smaller. In this case, with lower scattering and less absorption, the protons retain better chances for successful extraction with the later passes than in the nominal case of the perfect alignment. The secondary impact parameters of the scattered protons are still sufficiently large, $\approx$30 μm $\gg x'L$, so the "gap" $x'L$ is not dangerous.

We come to the conclusion, then, that a peak efficiency with an imperfect crystal is achieved at some tilt $x' \neq 0$, i.e., not at perfect alignment. Interestingly, since in the real experiment one scans x' while searching the peak, one comes at the above situation *automatically.* We used the case $b_{max}/x'L = 0.1$ as an illustration; the optimal x' will be found automatically in the scan. Further on, we refer to this case as the "prescatter" case, when protons first gently prescatter in the crystal edge to return later with low divergence but high impact parameters.

Understandably, with an imperfect crystal the prescatter case may appear also for a small negative tilt, $x' < 0$. Then, the $F(x')$ function may have *two* peaks, with a *dip* at $x'=0$. The width of the dip at $x' \approx 0$ may also be an indicator for the b_{max} value.

We explain the above ideas again (quantitatively) in Sec. IV in the context of the simulation results, and illustrate them with realistic x' scans for different crystals in Figs. 2–4.

Notice that all the experimental data obtained so far indicate that the edges of real crystals have a poor quality. Direct measurement (with photoemulsion and 70-GeV channeled beam) by Chesnokov [19] for several crystals Si and Ge, (110) and (111), gives a "septum width" in the range 40–60 μm. The CERN H8 experiment shows an unexpected structure at a crystal depth of up to ~0.1 mm (see Ref. [13]). The CERN-SPS experiment shows that a first-pass contribution to the extraction is not seen [14].

III. SIMULATION PROCEDURE

In this simulation we have tracked 900-GeV protons through the curved crystal lattice with small (~1 μm)

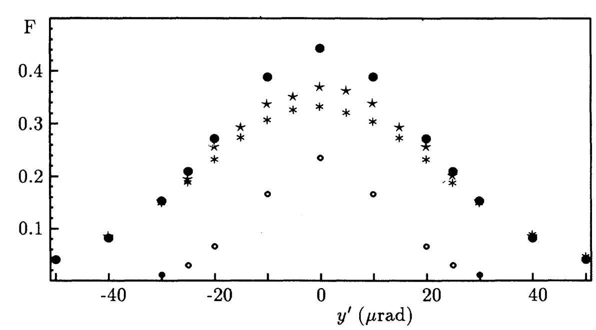

FIG. 2. Vertical angular scan of the efficiency for the perfect horizontal alignment, $x'=0$. Ideal crystal: (o) is the first-pass efficiency, and (•) is the overall efficiency. Imperfect crystal: ($\star$) is the overall efficiency with $t=1$ μm, ($*$) is the same with $t=50$ μm.

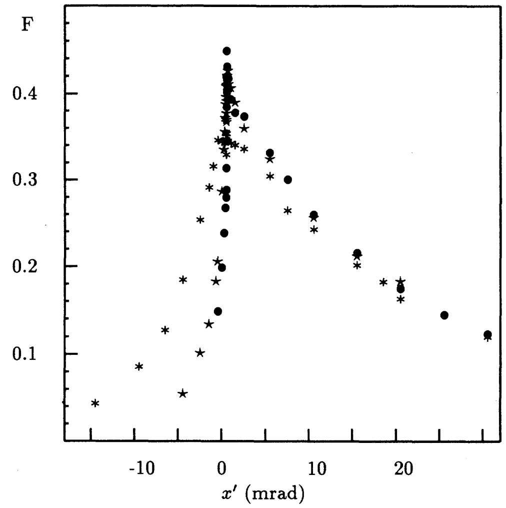

FIG. 3. Horizontal angular scan of the overall efficiency for the perfect vertical alignment, $y'=0$. For ideal (•) and imperfect crystals: (⋆) is $t=1$ μm, (∗) is $t=50$ μm. See details of the peak in Fig. 4.

steps applying the Monte Carlo code CATCH [16]. This code uses Lindhard's continuous-potential approach to the field of atomic planes, and takes the processes of both single and multiple scattering on electrons and nuclei into account. Further details on this code may be found in Ref. [16]. For most of the simulation the crystal was a Si(110) slab 40 mm long by 3 mm thick by 3 mm wide bent 0.64 mrad. Possible effects from the variation of the crystal size, bending, and atomic planes [(111) instead of (110)] will also be discussed. We assumed the crystal to have a perfect lattice and be curved with a constant longitudinal curvature to deflect protons in vertical direction. As an option, we model also an amorphous layer at the crystal edge, and/or irregularities of the surface. Possible effects of the crystal lattice dislocations on the bending efficiency at high energies have been studied by

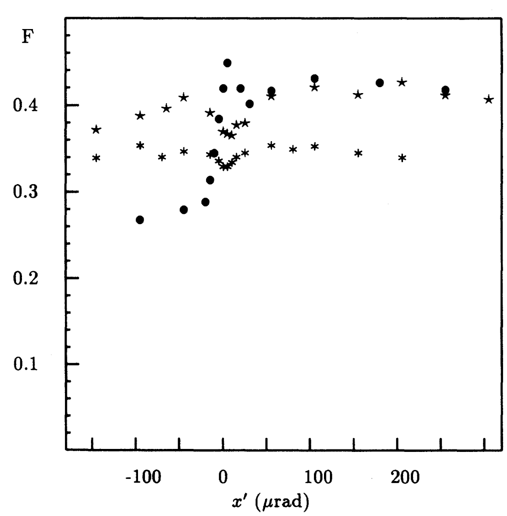

FIG. 4. The same as in Fig. 3 near the peak.

simulation in Ref. [20].

The crystal was located 61 m upstream of the C0 point of the Tevatron lattice, with the edge at the horizontal distance of $X=1.75$ mm from the beam axis. At the crystal location, the machine parameters were $\beta_x=105.7$ m, $\alpha_x=0.109$ (horizontally), and $\beta_y=21.5$ m, $\alpha_y=0.148$ (vertically); tune values $Q_x=20.5853$ and $Q_y=20.5744$. The beam invariant emittance was 2.5 mm mrad (1σ), which corresponds to vertical divergence $\sigma_{y'}=11.5$ μrad and width $\sigma_y=0.24$ mm at the crystal location. We assumed a Gaussian distribution of the incident particles over y, y' with the above σ's.

As mentioned, the horizontal parameters x, x' of incident particles are defined by the mechanism of diffusion. Simulation of this diffusion, to predict the primary impact parameters and intensity of incident protons, is an especially complicated problem. Luckily, these two processes, diffusion and crystal extraction, are perfectly unfolded in the E853 scheme. Beam parameters in the channeling plane (vertical) are not disturbed by this diffusion. The horizontal impact parameters are quite close to the crystal edge X, being distributed between X and $X + b_{max}$. Irrespective of the diffusion mechanism, one has $b_{max} \ll X$, and x' is much smaller than the rms angle of scattering over the crystal length, $\theta_s \approx 10$ μrad. Therefore the distribution of the secondary passes in the horizontal plane is also practically unfolded from the primary x, x'. The exact value of b_{max} may have meaning only with respect to the (unknown) "septum width" t. Since t is unknown for the real crystal, we can postulate $b_{max}=1$ μm, and then model the crystals with different t.

For b_{max} comparable to (or even much lower than) the septum width, multiple passes are essential. In any inefficient pass the proton is scattered by some angle θ_s, leading to an increase in the amplitude of betatron oscillation which is $(X^2 + \beta_x^2\theta^2)^{1/2}$. At some later turn this proton hits the crystal with the impact parameter b increased by $\Delta b \approx (X^2 + \beta_x^2\theta_s^2)^{1/2} - X \approx \beta_x^2\theta_s^2/2X$. In our case $\Delta b \simeq 0.3$ mm $\gg b_{max}$. This means almost no sensitivity of crystal extraction to the primary parameters of protons. This also means that a septum width of even $\sim$100 μm should not be dangerous for the multipasses.

Because of the absorption (nuclear reactions) and substantial scattering in the crystal, any particle may traverse it only several times before the eventual loss. This corresponds typically to some dozen turns in the accelerator. For such a short period we may assume a linear dynamics of the protons in the accelerator described by the transfer matrices.

The accelerator aperture was found to make a minor effect on the crystal extraction, since the crystal traps only the small-angle (i.e., small amplitude) protons. The aperture variations affect the lifetime of particles with large amplitudes only, with almost no contribution to channeling. The practical horizontal aperture was set by the crystal. Particles scattered in it ended up soon either on the vacuum chamber or in interactions with the crystal nuclei. We can roughly estimate when the accelerator aperture may have an influence on the multipass extraction. The maximal angle of scattering over many passes in the crystal is set by the interaction (L_N) and radiation (L_R) lengths:

$$\theta_N = \frac{14\text{ MeV}}{pv}\sqrt{\frac{L_N}{L_R}} \approx 30\ \mu\text{rad} \qquad (1)$$

at ~1 TeV. We use L_N=30 cm for silicon to include also the elastic nuclear scattering which corresponds to angles much larger than the Lindhard angle; these elastically scattered protons are lost for channeling. For a particle with the angle θ_N and coordinate X, the amplitude of the betatron oscillation is

$$\sqrt{X^2+\beta_x^2\theta_N^2} \simeq 4\text{ mm}. \qquad (2)$$

Any element may affect extraction only if it is closer to the beam than Eq. (2), which is (3–4)σ_x horizontally. The vertical limit is $\beta_y\theta_N \approx$0.6 mm $\approx 3\sigma_y$. In the simulation we see no effect from the horizontal amplitude constraint at the crystal location, until it is as low as ~5 mm.

We find that the dynamics of the longitudinal momentum p is not important for multipass crystal extraction in the Fermilab experiment. Because of the energy loss in crystal, Δpc, the orbit of the circulating proton shifts by $D\Delta p/p$, where $D \simeq$2 m is the dispersion function at the crystal location. The corresponding variation of the impact parameter Δb_D is $(D\Delta p/p)^2/2X$. This should be compared to the variation due to the β function and the angle of scattering in crystal, $b \approx (\beta_x\theta_s)^2/2X$. Their ratio is

$$\Delta b_D/b \simeq \frac{D^2LL_R}{(14\text{ MeV})^2\beta_x^2}\frac{dE}{ds}, \qquad (3)$$

with dE/ds being the energy loss per unit length. Even over many passes, with $L \simeq L_N$, this ratio is of order 10^{-3} in the Fermilab experiment. Hence, the analysis may be restricted to the two transverse dimensions.

IV. RESULTS

Figure 2 shows the vertical (y') angular scan of the multipass overall efficiency of extraction for the crystal perfectly aligned in the horizontal angle, x'=0. For an ideal crystal both the multipass efficiency and the contribution from the first pass of protons are shown. The peak efficiency of an ideal crystal is $\simeq$44%. The same figure shows the angular scans for a crystal with a septum width t=1 μm (i.e., $t=b_{max}$) and t=50 μm, where the efficiency at perfect alignment $y'=x'=0$ comes down to about 36% and 32%, respectively. However, for an imperfect crystal the real peak was not found at x'=0 (see Fig. 4 and Sec. II). By optimizing x', the peak efficiency has increased to 42% and 35% for t=1 μm and t=50 μm, respectively. Notably, the efficiencies and angular scans are quite weakly dependent on the thickness of the crystal imperfection. The width of the vertical angular scan was found to be 50–55 μrad full width at half maximum (FWHM) in the cases considered.

The horizontal angular scan of efficiency $F(x')$, shown in Figs. 3 and 4, provides the most interesting information. First of all, the scan has double "peaks" or shoulders as expected from the qualitative discussion in Sec. II. The shoulder of a "prescatter" nature appears at $x' \approx$0.1–0.2 mrad for the ideal crystal, at $x' \approx$0.1–0.2 mrad and $x' \approx -0.05$ mrad for t=1 μm, and is subtle for t=50 μm. The depth of the dip at $x' \approx$0 (i.e., for the perfect alignment) is $\simeq$14% and $\simeq$7% with respect to the peak for t=1 μm and t=50 μm, respectively. The width $\Delta x'$ of the peculiarity (either peak or dip) near $x' \approx$0 is roughly b_{max}/L, which is 25 μrad in our simulation.

The efficiency is one-half of the maximum at $x' \simeq$14 mrad and -0.3 mrad for an ideal crystal (FWHM of the horizontal scan is $\simeq$14 mrad), at $x' \simeq$15 mrad and -1.2 mrad for the crystal with t=1 μm (FWHM $\simeq$16 mrad), and at $x' \simeq$18 mrad and -5 mrad for the crystal with t=50 μm (FWHM $\simeq$23 mrad).

The asymmetry of the scan, $F(x') \neq F(-x')$, is due to the loss of the protons trapped in channeling near the crystal edge (see Sec. II). With an ideal crystal, the asymmetry exists for any x'. With a septum width t, the asymmetry can be seen for an angling $\pm x'$ larger than t/L only. In our simulation with t=50 μm, the scan is symmetric indeed within $\pm$1.3 mrad but asymmetric outside this range of x'; note that 50 μm/40 mm =1.25 mrad. We therefore expect this x' threshold for an asymmetry to be a good measure of the septum width t. The magnitude of the asymmetry also depends on t; comparing the measured scan with those simulated, we can deduce t as well.

Furthermore, if one plots the magnitude of asymmetry, $F(x')-F(-x')$, as a function of $x'L$, one obtains a rough estimate of the beam distribution over the impact parameter b at the crystal. We noticed already that the minimal step $\delta b = \delta x'L$=0.1 μm is much finer than the precision of the coordinate detectors ($\approx$100 μm).

Notice the abrupt decrease in efficiency of the ideal crystal over the range of $x'L$ from 0 to $-b_{max}$: from 44% at x'=0 to 28% at $x' = -b_{max}/L$. This drop presents an excellent opportunity to measure the primary b_{max} with a precision of δb=0.1 μm. We point out that with an ideal crystal one can measure a distribution over the *primary* impact parameters (in the range of ~1 μm). An imperfect crystal ($t > b_{max}$) is insensitive to b in the range of ~1 μm; however, in just the same manner one measures the distribution over the *secondary* impact parameters (in the broad range from ~t to ~1 mm). Clearly, the same idea is applicable to the case of a kick mode, where the impact parameters are very high (say b_{max} ~0.1 mm). In the kick mode we hope any crystal seems ideal (i.e., $t \ll b_{max}$); then, if the kicks are well reproducible, one can measure the primary distribution over kick b.

Finally, the dependence $F(x')$ for x'>0 actually gives the dependence of the extraction efficiency on the septum width $t \simeq x'L$. Since the crystal angling with x'>0 is not exactly the same as the amorphous edge layer of thickness $t=x'L$, we compare in Fig. 5 the two functions $F(x'L)$ and $F(t)$; the latter was simulated for a perfectly aligned crystal with a septum width t.

Notice that even an ideal crystal would have in the

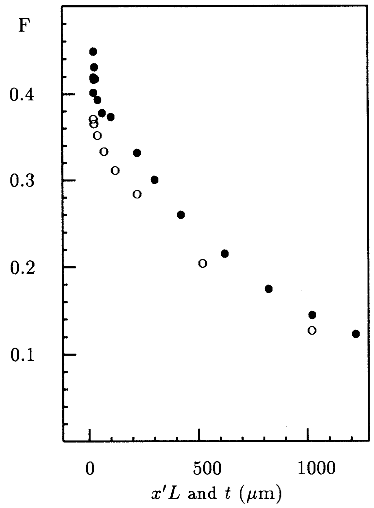

FIG. 5. Efficiency as a function of $x'L$ (•) for an ideal crystal, and as a function of septum width t (o) for an imperfect crystal.

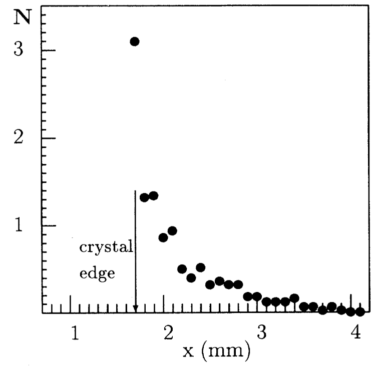

FIG. 6. The horizontal profile of the extracted protons at the crystal location. For a perfect crystal one should add a narrow (~1 μm) first-pass peak at the edge.

Fermilab experiment an effective septum width of order 0.1 μm defined by the finite minimal angular step of the horizontal goniometer, 2.5 μrad, and the crystal length of 4 cm.

Scattering of the channeled protons over the crystal length on the electrons and nuclei of the crystal causes a gradual dechanneling of the initially trapped protons. As the crystal is bent, the dechanneled protons are spread in the angle y' from ~0 to the crystal bending angle of 0.64 mrad with a roughly flat angular distribution downstream from the crystal. The number of dechanneled protons is a factor ~0.2 of the number of protons in the bent peak. The dechanneling loss caused by the scattering is commonly described with a dechanneling length L_D, along which the beam channeled fraction decreases by a factor of $1/e$. For a perfect Si(110) at 0.9 TeV, one expects $L_D \simeq 40$ cm in a straight or slightly bent crystal [21]. However, dechanneling follows the law $\sim\exp(-L/L_D)$ only for L comparable to or higher than L_D, while for $L \ll L_D$ the dechanneling rate is essentially higher (see discussion and simulations in Ref. [21]); in our case the "local" value of L_D [as derived from the data fit with $\exp(-L/L_D)$] is only ~4 cm/0.2 =20 cm, due to a rapid dechanneling of the particles with the highest amplitudes of channeling. Near the unbent peak the elastic scattering of the nonchanneled protons contributes to the background.

The profiles of the extracted beam are shown in Figs. 6 and 7. All the figures correspond to perfect alignment of the crystal. For understanding both the interplay of the crystal with the other accelerator elements (collimators) and the requirements for the crystal face perfection, the distribution of the extracted particles over the transverse coordinate x at the crystal face is essential. Figure 6 shows this distribution for the protons extracted with secondary passes. For an ideal crystal one should add a narrow (~1 μm) first-pass peak at the edge. From Fig. 6 we find that one-half of the extracted protons have penetrated into the crystal depth by >0.3 mm; another half had $b<0.3$ mm.

The vertical profile of the extracted protons at the crystal location was close to that of the primary incident protons, with a width of 0.60–0.65 mm FWHM roughly independent of t. The primary protons had a width of 0.57 mm FWHM with a Gaussian shape.

The vertical divergence of the extracted beam was defined by the channeling properties of the Si(110) crystal; its full width, $2\theta_c$, was ≈12.8 μrad ($\theta_c \simeq 6.4$ μrad is the Lindhard angle), and FWHM ≈9 μrad. The horizontal divergence was ≃5 μrad FWHM with the ideal crystal and ~12 μrad FWHM with t=1 μm. It was bigger than that of the incident beam due to scattering in inefficient passes before extraction.

It must be said that the extracted beam vertical divergence (and hence the profile downstream) can be influenced in the experiment by a variation of the bending angle (crystal twist). The horizontal divergence is mostly caused by scattering, thus giving information on the mean number of passes made in the crystal before extraction.

Downstream of the crystal the extracted beam passes

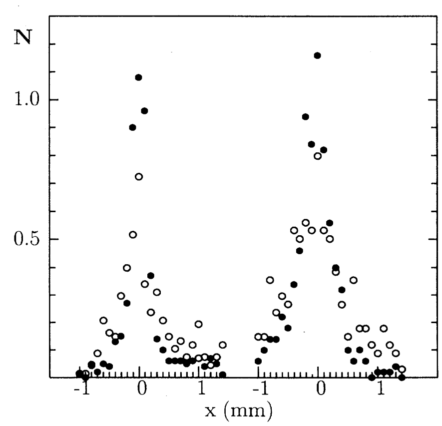

FIG. 7. The horizontal profile of the extracted protons at detectors 1 (left) and 2 (right); bin width is 0.1 mm. For ideal crystal (•) and imperfect one (o) with t=1 μm.

through a quadrupole doublet to the Lambertson-type magnets. Downstream of it, after a drift space, two detectors (hodoscopes with 0.1-mm bins) were placed at 80.5 m (detector 1) and 120.5 m (detector 2) from the crystal to measure the bent-beam profiles. The horizontal profiles of the bent beam at detectors are shown in Fig. 7. The width was $\simeq$ 0.3 and 0.4 mm FWHM for the ideal and t=1 μm crystals, respectively, at detector 1, and $\simeq$ 0.5 and 0.7–0.9 mm FWHM at detector 2.

The results are given for a crystal at room temperature. The crystal actual temperature in the Fermilab experiment may be somewhere between 293 K and 4 K. We have also simulated the extraction with the same crystal cooled to absolute zero. With this cooling, the extraction efficiency was increased by a factor of order 1.1. We do not expect, therefore, a high sensitivity of the simulation results (scans and profiles) to the crystal temperature.

The Fermilab program assumes the possible use of various crystals. The actual crystal used for the very recent (and very impressive) demonstration of 900-GeV crystal extraction [22] was Si(111) with size 40 mm (longitudinal) by 10 mm (horizontal) by 3 mm (vertical). The "four-point" bending was used, so the curvature was constant in the central 18 mm, then changed gradually from that constant to zero at the 24-mm length. Figure 6 shows that there are hardly any particles at x >3 mm from the crystal edge, so the change in horizontal size has no effect on our results. In order to check if there is any effect from the crystal plane change, (110) to (111), and from the variable curvature of the crystal, we have remade a part of the simulation with the crystal geometry as stated above. We observed only an insignificant decrease in the extraction efficiency, by the order of 1%. The results reported earlier remain valid for the case considered.

V. OPTIMIZATION

The efficiency of extraction can be increased with the use of a shorter crystal, and/or with a smaller angular divergence of the incident protons (higher β_y or smaller emittance). A shorter crystal disturbs the beam less. That means more attempts (passes) with smaller divergence (scattering) per proton on the average. Figure 8 shows the extraction efficiency dependence on the crystal length L, for uniform bending at 0.64 mrad. Two cases were studied: an ideal crystal, and the crystal with t=1 μm. The efficiency is maximal, near 70%, in the length range from 0.4 to 1.0 cm, irrespective of the crystal perfection and temperature.

The efficiency might be increased further with use of a "horizontal" (see Sec. II) scheme applied at CERN SPS, because the horizontal divergence of the incident particles is much smaller due to the small impact parameters. Even with the account of scattering in the inefficient first pass, this divergence is quite low and can easily fit the Lindhard angle at 900 GeV.

In contrast to efficiency, we find that a measurement of such parameters as b_{max} and t is easier when the relative contribution of the secondary passes is smaller. This is

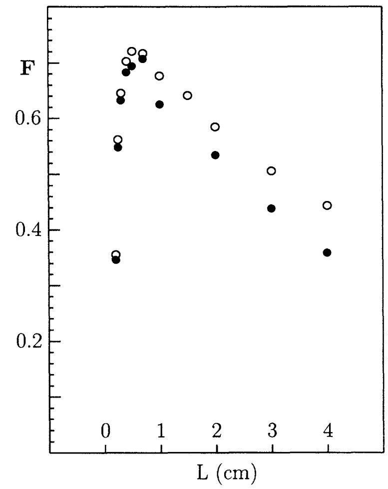

FIG. 8. Efficiency as a function of L for the ideal (o) and imperfect (•), t=1 μm, crystals.

the case if the crystal is long (the longer the better), and/or if the beam divergence is smaller; the present length L=4 cm is reasonably good for these measurements.

The position of the crystal edge X is not important for the extraction efficiency. The only effect of it is that the distribution of the secondary impact parameters shrinks as $b \sim 1/X$ with respect to the edge. This might be useful for investigation of the crystal edge structure, if any, since with $X \simeq$15 mm one has secondary $b \simeq$50 μm.

VI. CONCLUSIONS

We have shown that key information about the multipass crystal extraction process, namely, the septum width and dependence of efficiency on it, distribution of particles at the crystal, and contribution from the first or secondary pass, can be obtained from an analysis of a horizontal angular scan of efficiency. In a considered way, by use of a crystal extraction one can study the impact parameters of halo particles and/or the structure of the crystal edge with an accuracy as fine as 0.1 μm.

The extraction efficiency is expected as high as $\simeq$40% irrespective of the crystal septum width, and can be increased up to $\sim$70% with the use of a shorter ($\leq$1 cm) crystal.

The difference in efficiency between the ideal and imperfect crystals is very low, because of the predominance of the multi-passes in extraction at high energies, and partly because of the effect we found of a gentle "prescattering" in the edge of a crystal tilted horizontally. This quite small drop of efficiency, from 44% to 42%, suggests that the double-scattering scheme of extraction proposed in Ref. [23] would not assist the Fermilab setup. That scheme has suggested, in addition to a bent crystal, use of two scattering elements, one amorphous and one thin crystalline, in order to increase the impact parameters at the bent crystal; all three elements are to be mutually aligned in position and angle with a high precision (few μm and μrad). In contrast, the natural course of events studied in our work provides an elementary (and automatic) solution to the problem of a finite septum width and infinitesimal impact parameters.

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