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Surface growth of two kinds of particle deposition models

Wei Wang
National Laboratory of Solid State Microstructure, Institute for Solid State Physics and Physics Department, Nanjing University, Nanjing 210093, People's Republic of China and Center for Advanced Studies in Science and Technology of Microstructures, Nanjing 210093, People's Republic of China

Hilda A. Cerdeira
The International Center for Theoretical Physics, P.O. Box 586, 34100 Trieste, Italy
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The surface growth of two kinds of particles (A and C) depositing on a $(1+1)$-dimensional substrate has been studied. For two different models, the randomlike deposition and the ballisticlike deposition, the scaling behavior of the surface width W against the time t is obtained for different values of the deposition probability P of particle C. We found that the scaling behavior of the randomlike deposition model falls in the Edwards-Wilkinson class, while that of the ballisticlike deposition model belongs to the Kardar-Parisi-Zhang class. We also found that the saturated surface widths have a nonmonotonic relationship dependent on the probability P for both models.

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I. INTRODUCTION

Recently, there has been considerable interest in the study of the morphology of growing surfaces or interfaces, not only because of its potential technological importance, but also due to its manifestation of interesting nonequilibrium statistical physics at fundamental levels [1]. Most of these studies contain rough surfaces and stochastically growing interfaces in the context of ballistic deposition [2], the Eden model [3,4], and the solid on solid model [5,6], as well as the molecular-beam deposition [7,8] and the continuum stochastic equation of Kardar, Parisi, and Zhang (KPZ) [9].

The diffusion-limited aggregation model produces a self-similar fractal structure [10]. The Eden model and the ballistic deposition model as well as some other surface growing models give rough surfaces that are self-affine but not self-similar. Various growing models that have been studied for such surface phenomena only concern the growing of one kind of particle [1]. However, in the growing of real materials one may take into consideration that different kinds of particles are deposited on these structures such as alloys and impurities in materials. Thus, in the growing system, there may exist different interactions for different particles. The growing mechanism may also be changed.

In this work we report on results of two surface growth models, namely, the randomlike deposition (RLD) and the ballisticlike deposition (BLD), which are more complicated than that which we have studied in Ref. [11]. We describe the kinetic growth of the deposition of two kinds of particles A and C (particle A with probability $1-P$ and particle C with probability P) on a d-dimensional substrate, using the above models. The dynamical scaling behavior is characterized by the scaling of surface width with respect to the time t and the system size L for various probability P of particle C. The saturated surface width varying with the probability P is considered, and the morphologic structures for various values of P are also discussed. The arrangement of this paper is as follows. In Sec. II, we present the models and the physical motivations. In Sec. III, the dynamical scaling behavior of the surface width is studied. Summarizing remarks and conclusions are given in Sec. IV.

II. RANDOMLIKE AND BALLISTICLIKE DEPOSITION MODELS

Two different kinds of particles, particle A (the active particle) with a probability $1-P$, and particle C (the nonactive particle) with a probability P, are deposited on a $(1+1)$-dimensional substrate. The particles are allowed to fall straight down randomly, one at a time, onto a growing surface and stick where they land or diffuse to another position according to the models described below. In both models, first a site is chosen randomly, and then with probability $1-P$ (or P) a particle A (or particle C) is deposited on the surface of the aggregation depending on the following conditions.

Randomlike deposition model (RLD model): First, the deposition occurs when the particle on the top of the chosen site is particle A, or if the particle on the top of the chosen site is particle C and one of the nearest neighbors (one unit higher than the top of the chosen site) is particle A. Second, after this deposition, if the deposited particle is particle C, it is allowed to diffuse to its neighbors until it is located at a minimal height. However, there is no such diffusion for particle A.

Ballisticlike deposition model (BLD model): The deposition occurs once the dropping particle first encounters a particle A wherever it is on the top or in one of the two neighbor columns of the chosen site. For instance, if the dropping particle falls down along column i, and it first meets a particle A which is in the column $i+1$ (no matter whether this particle A is covered by a particle C or not), this dropping particle will stick to this particle A and the falling stops. Obviously, if the dropping particle first meets a particle A which is just on the top of column i, it stops and deposits there. The rules are depicted in Fig. 1 for both models.

For the randomlike deposition model, the effect of the diffusion for particle C is that particle C is always deposited at a local minimal height. Such kind of diffusion has been used in many deposition models and it mimics the actual process of the growing of the film or some other growth models (see Ref. [1] and references therein). Obviously, for $P=0$, the deposition process of the randomlike deposition model is just the same as the random model [12], which is a trivial surface growth model in which a particle simply falls until it reaches the top of a column. Since there are no correlations between the columns, these grow independently; however, the surface is rough. When $P\neq 0$ once a particle C is deposited on a column, its growth will strongly depend on the local structure. This introduces a correlation between the different columns. For the ballisticlike deposition model, the deposition happens not only on the top of a column, but also beside a column. When the falling particle first meets a particle C on a neighboring column, it can fall continuously until it meets a particle A if it has not reached the top of the on-site column. This process might be considered as a kind of diffusion. When $P=0$, our ballisticlike deposition model is reduced to the usual ballistic model with only one kind of particle involved, and it has been studied extensively [2]. When $P\neq 0$, the deposition process is more interesting and will be affected by the existence of particle C.

The physical motivations of our two models might be

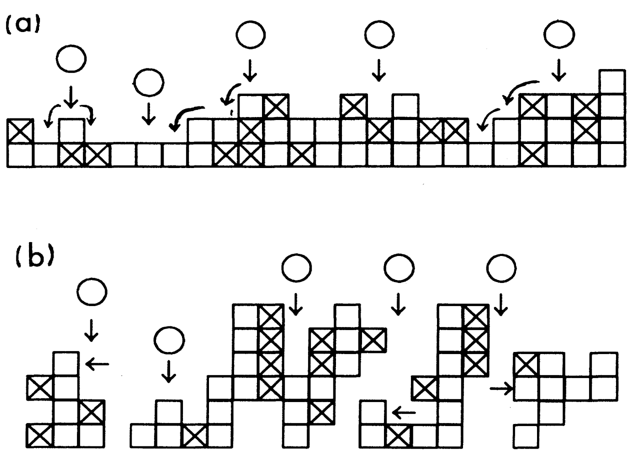

FIG. 1. Two deposition models: (a) The randomlike deposition model; (b) the ballisticlike deposition model. The circles represent the falling particles (particle A or particle C), the squares represent particle A's, and the squares with a cross denote particle C's. The down arrows show the falling and the right and left arrows show the positions where the falling particle will stick.

threefold. First, they may represent the chemical reactions which take place in the growing surface of materials. For example, we model the following reaction process: $A+B=C$. Particle A and particle B are active, and once particle A is touched by a particle B the combination produces a reactant C, which is no longer active. The particle A is chosen with a probability $1-P$, and the particle B with P. That is, the reactant C is produced with the probability P when P is small. Thus, in these systems, some of the surface sites continue to react while some sites do not. We introduce naturally the diffusion only for particle C's since they have less interacting "bonds" with other particles and they can move more freely to a place with lower height, just like most of the diffusion processes studied by other authors (see Ref. [1] and references therein). The initial flat substrate placed with particle A is for the reactions. Second, these models can also represent a growing interface of a material with low concentration of impurities. Our models mimic the role of the impurity atoms as follows. An impurity atom (particle C) is introduced with a probability P while it has less active bonds for other atoms (particle A). Third, our models may describe the deposition of one kind of heavy particle and another kind of light particle. The heavy and light particles are supposed to have different attractive forces between themselves and each other. Thus, the light particles may diffuse more easily than the heavy ones.

III. DYNAMICAL SCALING BEHAVIOR AND DISCUSSION

Now let us report on the results of a (1+1)-dimensional case. The substrate is a strip with the width L from $L=60$ to 1100 in the X direction. The aggregation is in the Y direction. At the beginning, all sites are occupied by particle A for $Y\leq 0$. A periodic boundary condition is used in the L direction (in the X direction). The dynamical scaling behavior is characterized by scaling the surface width

$$W^2(L,t)=\sum_r L^{1-d}[h(r,t)-\overline{h(t)}]^2 , \quad (1)$$

where $h(r,t)$ is the height of the surface at position r and time, $t,\overline{h(t)}$ is the average height at time t, and $d'=d-1$ is the substrate dimension in d-dimensional space.

Figure 2 shows a plot of the surface width W as a function of time (the numbers of the deposited particles) for different deposition probabilities of particle C for the randomlike deposition model. Statistic average is found by averaging over 100–1000 runs. Curve (a) shows the case of the random deposition of only one kind of particles (particle A), i.e., with $P=0$ for particle C. As there are no correlations between the columns, the height of the columns follows a Poisson distribution, and the width of the surface is proportional to the square root of time t, $W\sim t^{\beta}$, i.e., $\beta=\frac{1}{2}$, independent of the dimension [12]. At the same time, there is no saturated width, that is, no steady state. While $P\neq 0$, the scaling behavior is changed. From curves (b)–(e) shown in Fig. 2 for different deposition probabilities of the particle C, we see

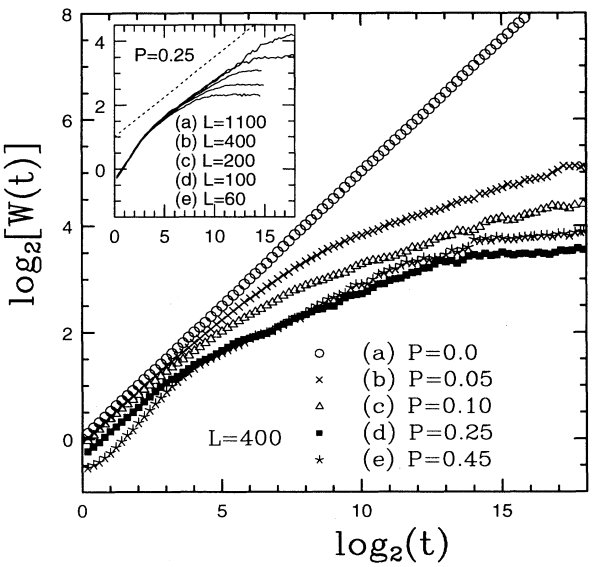

FIG. 2. $\log_2 W(t)$ versus $\log_2 t$ for the randomlike deposition model with all the parameters shown in the figure. The slope of the dashed line (in the inset) gives $\beta=0.25\pm0.01$.

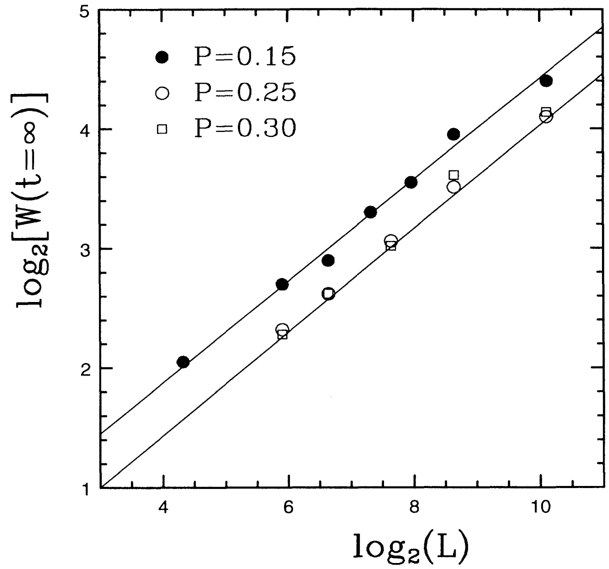

FIG. 3. Log-log plots of the saturated surface width $W(t=\infty)$ against the system size L.

that the growth of the aggregation is divided into three stages. First, at the early time (for $\log_2 t<5$), the width grows with a power of time $W\sim t^{1/2}$, the same as the random deposition process. Second, the width grows with a power of time $W\sim t^{\beta}$ with $\beta\simeq0.25$. Finally, there is a saturated value of the width. The exponent β is calculated for each probability P by using the linear parts of the $\log_2 W-\log_2 t$ plots of various values of L ($60\leq L\leq1100$). The inset of Fig. 2 shows one example for the probability $P=0.25$, and the slope of the dashed line gives the value of β. We believe that the finite-size effects are minimal since the linear parts are over several decades of time. In Fig. 3 we have plotted the values of saturated width $W(t=\infty)$ against the system size L. The exponent α in the scaling $W(t=\infty)\sim L^{\alpha}$ is found to be $\alpha\simeq0.44$, which means that our randomlike deposition model belongs to the Edwards-Wilkinson (EW) class [13,14], although, on one hand, there is a big deviation from the EW prediction for the exponent $\alpha=0.5$. On the other hand, for our randomlike deposition model, the growth dynamics is basically linear, since most of the growth processes of columns are independent of its local geometry. The growing mainly depends on whether the chosen position is crowned by an A- or C-type particle. Moreover, the correlations in the distribution of the particles by geometry might conceivably introduce a weak nonlinearity, but it should be very small. Therefore, the width shown grow as the $\frac{1}{4}$ power of time, i.e., $\beta\simeq0.25$, as indeed it does in the simulation. Thus, together with the exponent $\alpha\simeq0.44$ obtained from our simulation which is close to $\frac{1}{2}$, we conclude that our model belongs to the EW class. Actually, as we have seen in our simulation, for large probability P ($P>0.25$), the exponents β and α become slightly bigger than that of the small probability P. As we will discuss below, for small P the diffusion is limited, while for large P the diffusion introduces a weak nonlinear diffusive term into the Edwards-Wilkinson kinetic equation [13]. As a result, the weak nonlinearity enables the exponents α and β to be different from the exact values of $\alpha=0.5$ and $\beta=0.25$ of the EW theory. This might be the reason why the value of α deviates from 0.5 more and more as the probability P increases in our case, and especially for $P=0.3$ the points do not fall on the scaling line very well (see Fig. 3). However, we can still expect that our randomlike deposition model is Edwards-Wilkinson-like.

In addition, an interesting result is that the saturated width varying with the probability P shows a nonmonotonic relationship (cf. Fig. 4), that is, $W(t=\infty)$ first decreasing and then increasing as the probability P increases. The value of the probability of the minimal point is around 0.25, i.e., $P_m=0.25$. Furthermore, in order to verify the above, we have also counted the concentration N_c of particle C on the surface. We found that N_c has a linear relationship varying with the probability P but with different slopes before and after the minimum. However, the change of the slopes is not too big, which enables us hardly to pronounce that there exists a phase transition, but rather we conclude that there is only a change of the morphologic structures (see following discussion). Finally, in order to make sure that our results are not due to the finite-size effects, we have carried out several simulations on system size L for $L=100$, 400, and 1100. All of the results indicate the same behavior.

Now we answer the question of why the diffusion is limited or weak during the surface growing, and explain how the morphologic structures change as the probability

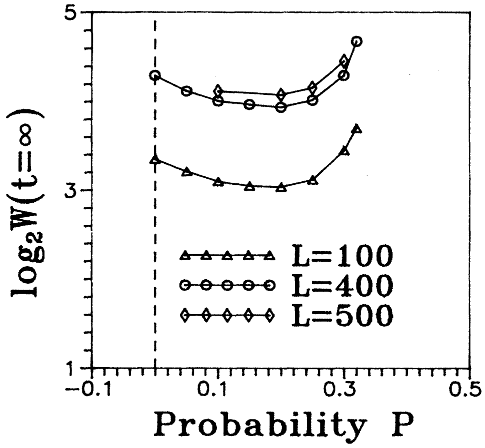

FIG. 4. Plot of $\log_2 W(t=\infty)$ versus the probability P of particle C for the randomlike deposition model.

P increases. For small P ($P<P_m=0.25$), the depositions of particle A occur more frequently and these depositions form many barriers. As a result, the surface becomes locally rougher. This rough surface limits the diffusion of particle C, and diffusion happens only within a small range. However, for large P, particle C diffuses more easily and may travel a large distance, which makes the surface smooth. Thus, morphologically, in the region $P<P_m$ the surface appears rough down to short length scales. For $P=P_m$ the surface is smoother than that of $P<P_m$, and when $P>P_m$, the surface is dominated mainly by relatively large terraces. We have shown two examples with respect to these two cases of $P>P_m$ and $P<P_m$ in Fig. 5. We see that indeed the surface structure is rougher for $P=0.15$ than for $P=0.35$ In our recent simulation for higher dimension, we have found also a similar change of the morphologic structures.

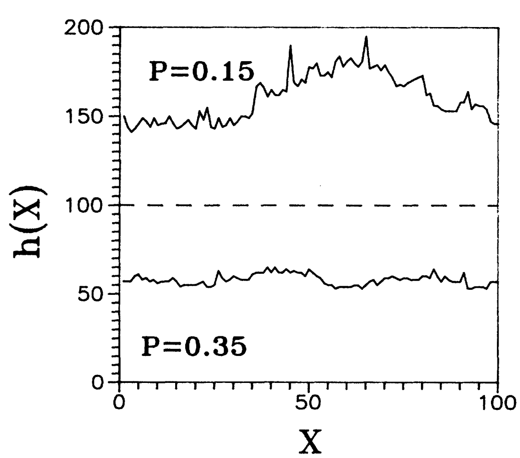

FIG. 5. Surface structure in the steady state of the randomlike deposition model for the system size $L=100$: curve (a) for $P=0.15$ and curve (b) for $P=0.35$. Only the region close to the surface is shown.

We now turn to the ballisticlike deposition model. Figure 6 shows the plots of the surface width as a function of time for various values of probability P. For each curve the statistical average is obtained over 400 runs. It can be seen that there are also three different regions in the plot just like the situation for the randomlike deposition model: at the early time ($\log_2 t \leq 3$) of the deposition the surface width can be scaled as $W \sim t^{1/2}$, while $W \sim t^{\beta}$ for the intermediate time. Then follows a saturation. For $P=0$, the result is exactly like the case of standard ballistic deposition with only one kind of particle (particle A), which gives $\beta=\frac{1}{3}$ and $\alpha=\frac{1}{2}$ (without any finite-size effects) [15]. When $P\neq 0$, the value of β is calculated by using the same method for the randomlike deposition model, and is found to be 0.30, i.e., $\beta\simeq 0.30$, as shown by the slope of the dashed line plotted in the inset of Fig. 6. From the logarithmic plot of $W(t=\infty)$ versus system size L, a scaling $W(t=\infty)\sim L^{\alpha}$ with $\alpha\simeq 0.46$ is obtained (see Fig. 7). The exponents α and β satisfy the scaling law $\alpha+Z=2$ with $Z=\alpha/\beta$. Therefore, our ballisticlike deposition model falls in the KPZ universality class [9], which implies that this model can be well described by the KPZ equation

$$\frac{\partial h}{\partial t}=\nabla^2 h+\frac{\lambda(P)}{2}(\nabla h)^2+\eta(x,t) , \tag{2}$$

but with the diffusion term related to the probability P. In Fig. 8, we have plotted the saturation width $W(t=\infty)$ versus the probability P. One can see that there is also a minimum: The minimal point is also about $P_m=0.25$. As the same reason argued for the randomlike deposition model, this nonmonotonic relationship re-

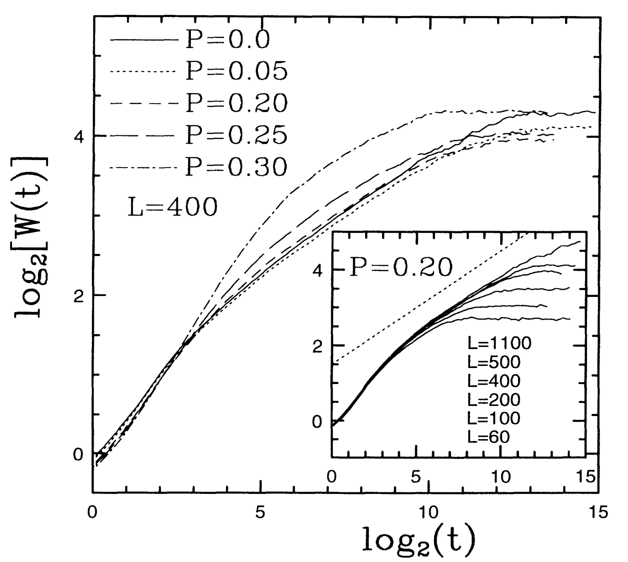

FIG. 6. $\log_2 W(t)$ versus $\log_2 t$ for the ballisticlike deposition model with all the parameters shown in the figure. The slope of the dashed line (in the inset) gives $\beta=0.30\pm 0.01$.

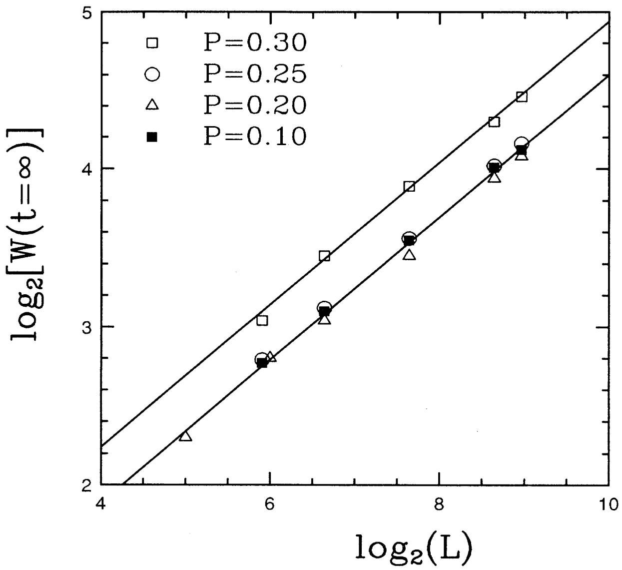

FIG. 7. Log-log plots of the saturated surface width $W(t=\infty)$ against the system size L for the ballisticlike deposition model.

sults from the diffusion process when the surface grows. The nonactive particles tend to aggregate together, and the morphologic structures are different when the deposition probability P is below or above $P_m=0.25$. From a figure (not shown here) similar to Fig. 5, we have seen that the surface is rough (or smooth) for small (or large) P, respectively. However, a complete understanding of the kinetic behavior is difficult. Physically, our ballisticlike deposition model is somewhat close to a model investigated by Pelligrini and Jullien [16]. We considered the active and nonactive particles in our model just like their mixture of sticky and sliding particles. Their model is also ballisticlike, and they have shown that there is a roughening transition in high dimensions ($d=3$ and 4). For our present results on the $(1+1)$-dimensional case, as the probability P only varies within a small range, it is hard to make a conclusion of the existence of phase transition. Moreover, the dimension of $d=2$ is a critical dimension and the situation is always very complex. To check whether there is a phase transition in our model or not, much more numerical work is needed for higher dimensional case. In a recent work, we have seen that there is an evidence for the phase transition in $d=3$ dimensional case [17], and the work is in progress.

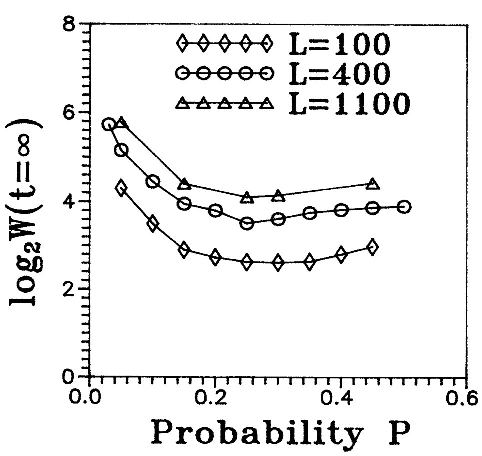

FIG. 8. Plot of $\log_2 W(t=\infty)$ versus the probability P of particle C for the ballisticlike deposition model.

It is worth noting that in Fig. 8, we have included the values of the saturated widths for $P=0$. As we can see that these saturated widths are $\log_2 W(t=\infty)=3.35$ and 4.30 for $L=100$ and 400, respectively. They are just on the lines shown in Fig. 8. This strongly supports the monotonic decreasing of the saturated width with respect to the probability. Finally, from our results we have also found that the deposition will be stopped when the deposition probability $P \geq 0.35$ since the surface sites are all covered by nonactive particle C's. That is, disappearance of bonding sites may take place when the probability P becomes too large.

IV. REMARKS AND CONCLUSION

We would like to make a few relevant remarks on our results of both models. (1) In our study, the exponent β is defined by the slope of the linear parts in the log-log plot of the surface width versus the time for different system sizes. This reduces the finite-size effects and gives reliable results, although our system is not too big. (2) From the plot of $P=0.45$ for the randomlike deposition model (cf. Fig. 2) and $P=0.30$ for the ballisticlike deposition model (cf. Fig. 6), we have seen that there is a rapid increasing of the surface width and the system reaches the saturated state quickly. Physically, this rapid increasing is due to the forming of clusters of nonactive particle C. (3) For both models, we have observed a nonmonotonic relationship of the saturated width varying with the probability P. From the counting of the concentration N_c of particle C on the surface, we found that the variation of the relationship of N_c versus P is small before and after the minimal points, P_m, for both models. In order to verify that the increase of the saturated width above P_m is not due to the finite-size effects, we also did several runs for a larger system size $L=3000$. Our results show that at least there is an increase of $W(t=\infty)$ for the probability P above P_m. Comparing the present work with our previous one in Ref. [11], where we studied the same models but without considering the diffusion and found that there is no such minimum, the diffusion bears the responsibility for the nonmonotonic behavior: The nonactive particles C tend to aggregate together. Thus the morphologic structures are different for large and small probability P's since the diffusive processes of particles C are different from each other in these two regimes. The change of the morphologic structures for both models is somewhat like that found by Amar and Family in a restricted solid on solid surface growth [18], and it appears to be the nonequilibrium analog of the roughening transition. However, for our case, it is difficult to conclude whether there is a phase transition or not. This deserves to be studied in further work. (4) For our randomlike deposition model, we have implemented a physically realistic process for particle C to diffuse. Immediately after the deposition, the particle C moves via random walk along the surface. It stops when it reaches either a local minimum or when it has moved l_{cd} steps on the surface. That is, there is a diffusion length for particle C. In the present work, this length l_{cd} is limited to be 20 lattice spaces $l_{cd}=20$, which is rather large and may incorporate to the experimental situation with a high temperature. For small probability P, almost all the diffusion stops within l_{cd} since the morphologic structure is locally rough, while for large probability P the diffusion steps are big and may reach the value of l_{cd}. Recently, we have done some runs for different values of l_{cd}, i.e., different limitations for the range of diffusion; we found that the early-time scaling has some differences, but the asymptotic statistical scaling properties of the surface are basically the same. A more detailed study is in progress and will be presented elsewhere. Nevertheless, for the ballisticlike deposition model, the diffusion is not limited before the particle C sticks to a particle A.

In conclusion, we have proposed a surface kinetics of two kinds of particles (A and C) deposition on a $(1+1)$-dimensional substrate. The scaling behavior of the surface width W is obtained for different values of the deposition probability P of particle C and system sizes, for two models, the randomlike and ballisticlike deposition model, respectively. We found that the scaling behavior of the randomlike deposition model falls in the Edwards-Wilkinson class, while that of the ballisticlike deposition model belongs to the KPZs class. We also found that there is a nonmonotonic relationship of the saturation surface widths dependent on the probability P for both models.

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