$1/f$ noise in a two-lane highway traffic model

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A cellular automaton model of traffic current on a two-lane highway is proposed. Clustering and hopping mechanisms are introduced into the interactions between individual cars by the traffic rules. The power spectrum of the traffic current may show good $1/f$ behavior in certain parameter regions. The $1/f$ noise is the result of many-body interactions of clustering and hopping. It is found that a state with $1/f$ noise has a remarkable advantage allowing effective transporting.

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$1/f$ noise is one of the great mysteries of physics in recent years. It is observed not only in electrical measurements of condensed matter systems [1,2], but also in diverse systems such as freeway traffic [3], water flow in a river [4], music [5], etc. In these cases the power spectra of signals may show power-law decay with an exponent close to -1 over many frequency decades. The widespread occurrence of the $1/f$ behavior suggests that an underlying general explanation might exist. To this end, Bak, Tang, and Wiesenfeld introduced the concept of self-organized criticality (SOC) and used the cellular automaton (CA) to study discrete dynamics [6]. However, a later numerical analysis by Jensen, Christensen, and Fogdby [7] showed that the actual power spectrum of the model was in fact $1/f^2$, and this was supported by experiments [8,9]. Following the SOC idea, Jensen and coworkers [10,11] and Christensen, Olami, and Bak [12] proposed models with $1/f$ behavior. Nevertheless, to date, few models showing $1/f$ behavior have been offered.

Traditionally, traffic problems are simulated by various hydrodynamic models [13,14]. By using CA models, traffic problems are simulated in both a one dimension highway [15] and a two dimensional system (the whole city traffic) [16]. Lately, Ben-Naim, Krapiusky, and Redner studied the clustering behavior in a one-lane highway by a "ballistic model" [17]. In Ref. [16(c)], Nagatani numerically studied the Biham traffic model [16(a)], and found a $1/f$ noise of waiting time near the traffic-jam threshold. The Biham model intends to simulate an entire city traffic problem in which every road is strongly influenced by other roads, traffic lights, etc. This is not the system examined in Ref. [3]. To our knowledge a good reasonable model which directly simulates the mystical $1/f$ behavior in freeway traffic flow has never been offered up to now.

In this paper, we will study a discrete-time–continuous-space CA model simulating the two-lane highway traffic flow. The model is designed under the following two assumptions: (1) Every car wants to go as fast as it can. (2) Every car should abide by the traffic rules. It is the traffic rules that determine the interaction of individual cars. If there are only a few cars on road, then these cars may run freely, and will not interact with one another, so that every car can go as fast as it can. However, if there are too many cars on the road, and every car has a different velocity, then these cars will interact with one another surely, since every highway has only a few lanes.

We assume the highway has two lanes, one is slow, the other fast.

(1) On the left edge of the highway, cars can enter into the two lanes with a total probability of ρ (probability ρ means ρ cars enter onto the highway in every unit time). Every car has its fixed velocity v which is uniformly and randomly distributed in the range from $v_0-\Delta v/2$ to $v_0+\Delta v/2$. The length of the way is defined as $v_0\times l$. Here v_0, l, and Δv are control parameters. In the following, we fix $v_0=5$ unchanged.

(2) If a fast car (car 2) is going to collide with a slow car (car 1) (or a cluster of cars), then car 2 slows down to the velocity of car 1 (or the leading velocity of the cluster of cars), and keeps the distance ds from car 1 (or from the front car). Then car 1 and car 2 form a new cluster (or bigger cluster) of cars.

(3) In order to go fast, a car can hop between the two lanes. The rules of hopping are defined as follows. On the slow lane, when a fast car (car 2) is blocked by a slow car (car 1), then, in order to go fast again, car 2 can hop to the fast lane at the same place. There is one condition on which the "hopping step" is forbidden: if there is another car (car 3) in the fast lane which is behind car 2 initially, and is going to collide with car 2 if car 2 hops to the fast lane. When the "hopping step" is forbidden, car 2 can go ahead in the slow lane behind car 1 waiting for

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the chance to hop. The same holds in the fast lane; if a car is going to collide with another car, we let the fast car slow down, and because they are in the fast lane it is more reasonable to let the slow car hop to the slow lane. The hopping rule is the same as that given in the slow lane.

(4) All cars which do not hop to another lane proceed at their fixed velocities until a collision occurs.

The process of the simulation is (1)-(2)-(3)-(4)-(1), updating to all cars on the road. We define these rules according to the true traffic ones. These rules result in the following two conflict evolutions: (a) Because of the difference in the fixed velocities of different cars, rule (2) will generate clusters due to collisions. Every cluster will begin with a leading slow car, and the cluster will become larger with the increase of time when the road is infinitely long. (b) According to rule (3), the clusters will be reduced or destroyed: either a larger cluster becomes many smaller clusters (when some cars belonging to the cluster hop to the other lane), or the leading velocity changes (when the leading car hops to the other lane).

To our knowledge, this paper is the first to suggest such a two-lane traffic model to simulate realistic freeway transport. In this traffic model, there are two kinds of interactions: clustering caused by collisions, which slows down the traffic current; and hopping due to the two-lane structure, which speeds up the current. The competition of these two sides makes the model dynamics interesting. The hopping mechanism is the essential difference of the present model from all one-lane models previously investigated, and it will be found that this two-lane mechanism is the key point to produce $1/f$ noise.

The parameters given above are reasonable in simulating the real highway flow. As in Ref. [15(a)], the distance of adjacent cars on the most condensed way is 7.5 m, this is the value of ds here. Thus the average velocity $v_0=5$ means that the real velocity is $v_0=(7.5/ds)\times 5=37.5/ds$ (m/unit time), which is close to the real free velocity of cars on the freeway (about 33 m/s). Thus in our model $ds=1$ means that one unit time corresponds to 1 s in the real system.

Now we specify the physical quantities measured in our CA model. On the right edge of the road, we record the sum of cars $n(t)$ going out of the way on the two lanes in every unit time t. At the same time, the distribution function $p(n)$ of the signal strength $n(t)$ is also recorded. We begin to record $n(t)$ when thousands of cars have gone out of the way, supposing that the system has reached an asymptotic state and the dynamics of the system will not then be influenced by the initial condition. In Fig. 1 we show a typical measurement of $n(t)$. In Fig. 1(b) we show a magnification of a section of Fig. 1(a) in order to exhibit the self-similar structure of the $n(t)$ signal.

The power spectrum is obtained by a direct Fourier transformation of $n(t)$. In order to reduce fluctuation we averaged many power spectra of successive time sequences. Figure 2(a) presents the power spectra for $\Delta v=4$, $ds=1$, $\rho=1.9$, and $l=1000$, 2000, and 4000, respectively. It is remarkable that in all cases the spectra clearly show $1/f$ behavior in the scaling segments. We can also see that there are some deviations in both low frequency and high frequency sides. The high frequency deviation is caused by the nonzero time step and other reasons (for example, alias). The low frequency side has a small white-noise segment, which is well known to be caused by the finite length of the road. Increasing the road length definitely reduce the white-noise spectrum segment, as can be seen in Fig. 2(a) (from I to II to III, the power-law scaling is improved successively by increasing l).

In order to compare with the experimental measure-

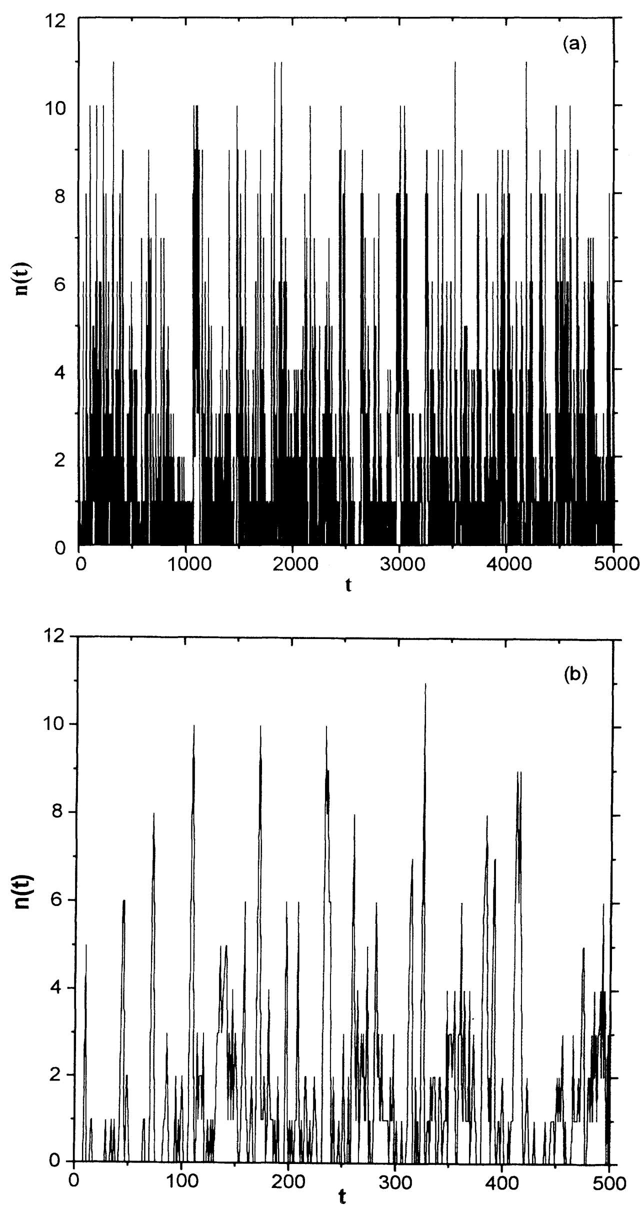

FIG. 1. (a) A time sequence of the number of cars going out of the highway in unit time. The parameters are $l=2000$, $\Delta v=4$, $\rho=1.4$, and $ds=1$. (b) A section of (a).

ment result in Ref. [3], we have also recorded the flux function

$$f(t)=\sum_i \delta(t-t_i) \tag{1}$$

in which t_i indicates the time the ith car goes out of the right edge of the road. The power spectrum is defined as [3,16(c)]

$$s(w)=\lim_{T\to\infty}\frac{1}{2T}\left|\int_{-T}^{T}f(t)e^{iwt}dt\right|^2 . \tag{2}$$

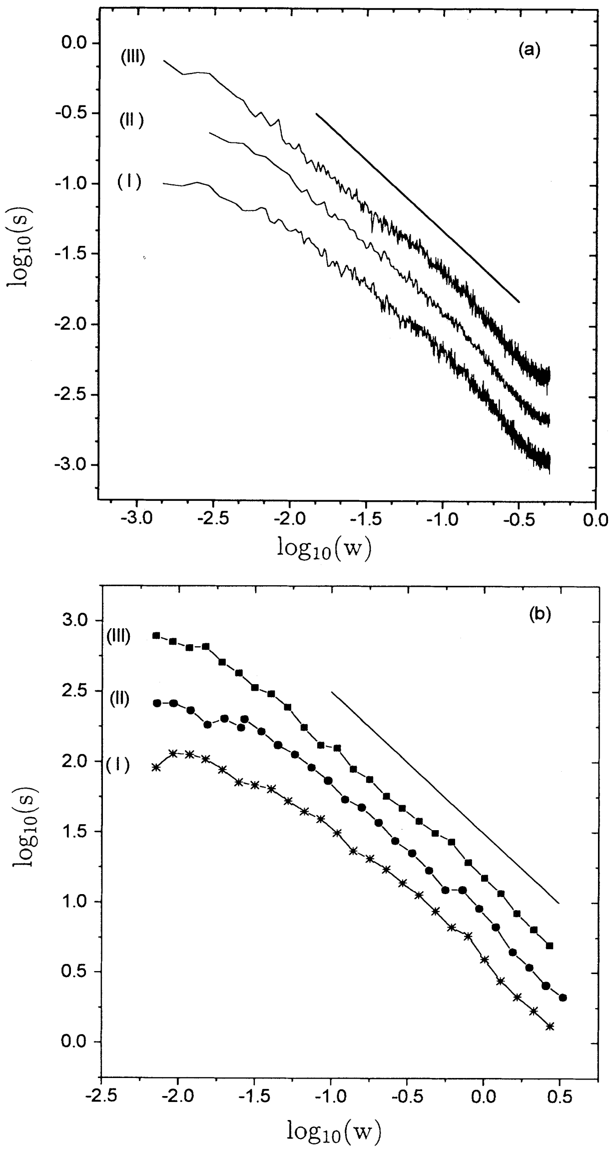

FIG. 2. (a) The power spectra of the systems. We fix $\Delta v=4$, $\rho=1.9$, and $ds=1$, and take different lengths as $l=1000$ (I), 2000 (II), and 4000 (III). The low frequency cutoff scales with the system size. $1/f$ behavior is identified in the scaling parts. For larger l we have a smaller cutoff on the low frequency side. The exponent of the power spectrum in the scaling segment changes weakly with the system size l. (b) The power spectra of the time series Eq. (1). All the parameters are the same as in (a). In both (a) and (b) the data have been multiplied by factors 1, 2, and 4, respectively from bottom to top in order to keep the curves apart. (The straight line is $1/f$. The fluctuations of the curves are caused by the limit of our CPU time.)

In Fig. 2(b) we plot the spectrum of $f(t)$ by taking the same parameters as in Fig. 2(a). It is clear that the flux function $f(t)$ also has $1/f$ behavior. The essential features are completely the same as those in Fig. 2(a). Moreover, the $1/f$ noise revealed in the experiment of Ref. [3] is realized in our model.

We have also analyzed the influence of parameter Δv on the power spectrum. The general tendency is the same as that in Fig. 2. When the distribution range of velocity Δv is wider, the $1/f$ power spectrum covers larger frequency range. ds also influences the frequency range of the power-law behavior. The result is also like Fig. 2. With larger ds, the $1/f$ power spectrum may extend to the lower frequency range. However, it is found that all three parameters l, ds, and Δv do not considerably influence the exponent of the power spectrum in the scaling segment. So the $1/f$ behavior of our model is robust for their changes.

The above observation can be understood heuristically, based on the interaction rules. In our model, cars enter onto the way randomly; this is a white noise. If the road is short, the collision frequency is small; then the dissipation caused by the interactions of individual cars lasts only for a short time, and the clusters are also small. The time correlation leading to the $1/f$ noise can be identified only for a short time scale, which shifts the $1/f$ segment to a higher frequency and leaves the low frequency behaving like the white-noise spectrum. When the highway is long, the two conflicting facts, clustering and hopping, will lead the system to a definite state where the interactions of cars play a key role in causing dissipation, producing large clusters and frequent hopping, and introducing a long time correlation of signals. This is characterized by a better $1/f$ behavior at low frequency.

For the same reason, it is easy to understand why the low frequency cutoff can be influenced by Δv and ds: If Δv is broader, there are more collisions and more hopping occurring leading to a longer correlation of individual cars. Then we have a better $1/f$ spectrum. When the interaction distance ds is long, the system will have a long spacious correlation, which appears as a long time correlation of the signals. Therefore, the low frequency cutoff of the power-law behavior will also be reduced by increasing ds.

The influence of the probability ρ of cars entering onto the road is presented in Table I where we take the parameters $l=2000$, $ds=1$, and $\Delta v=4$. Figure 3(a) shows some power spectra of these systems. We can see that the exponent depends sensitively on ρ (contrary to the dependencies on l, Δv, and ds). For $\rho=0.3$ we find the exponent is -0.1, and when $\rho=1.9$ we obtain the exponent -1. Figure 4 shows the dependence of the exponent on ρ. It is noted that $1/f$ behavior occurs only when the density of cars on the road is sufficiently high to allow strong dissipation and interaction [18]. In fact, in Ref. [3] the authors observed that the average number $n(t)$

TABLE I. Measured exponent β of power spectrum $S(f) \propto f^{-\beta}$ of $n(t)$ for different ρ with $l=2000$, $ds=1$, and $\Delta v=4$.

Models	Exponents	Models	Exponents
$\rho=0.1$	$\beta=0.00$	$\rho=0.8$	$\beta=0.73$
$\rho=0.2$	$\beta=0.06$	$\rho=0.9$	$\beta=0.82$
$\rho=0.3$	$\beta=0.10$	$\rho=1.0$	$\beta=0.86$
$\rho=0.35$	$\beta=0.14$	$\rho=1.2$	$\beta=0.86$
$\rho=0.4$	$\beta=0.46$	$\rho=1.3$	$\beta=0.97$
$\rho=0.5$	$\beta=0.55$	$\rho=1.4$	$\beta=1.00$
$\rho=0.6$	$\beta=0.56$	$\rho=1.9$	$\beta=1.06$
$\rho=0.7$	$\beta=0.70$		

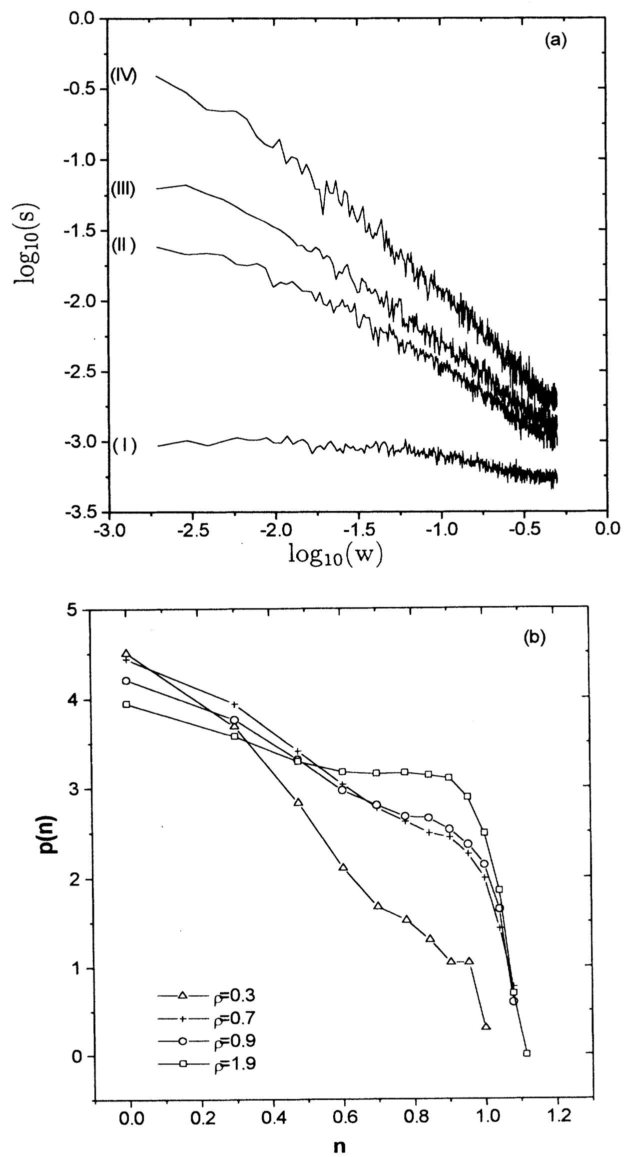

FIG. 3. $l=2000$, $\Delta v=4$, and $ds=1$, and $\rho=0.3(I)$, $0.7(II)$, $0.9(III)$, and $1.9(IV)$. (a) The power spectra of $n(t)$. (b) The distribution function $p(n)$ vs n.

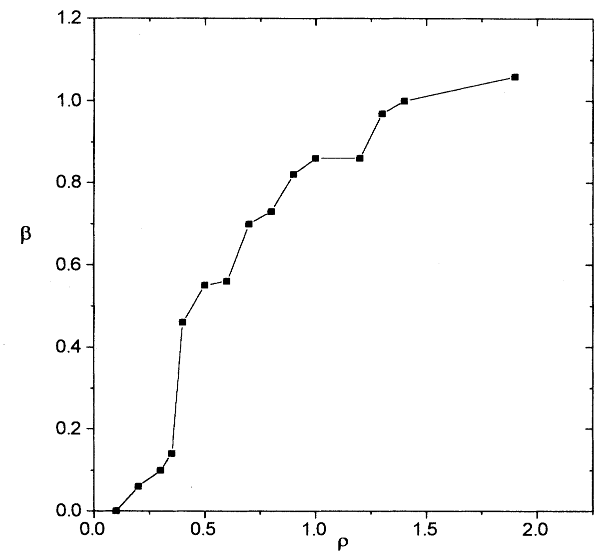

FIG. 4. The dependence of the exponent β [$S(f) \propto f^{-\beta}$;$S(f)$ is the power spectrum of $n(t)$] on the probability ρ.

was almost 1/s. This is in accordance with the ρ range corresponding to $1/f$ noise in our case.

It should be emphasized that $1/f$ behavior can occur only for intermediate ρ. Values of ρ both too small and too large may destroy $1/f$ property. In the small ρ limit, the clustering effect of cars will become very weak, so the free running of cars play a key role, which yields the white-noise spectrum. In the large ρ limit, hopping between the two lanes is almost forbidden, and the clustering effect of cars will become very strong and the model will go back to the high density one-lane model, where no $1/f$ can be observed. However, when there is an intermediate density of cars on the road (as we studied, $\rho=1.9$), the hopping and clustering mechanisms both become important, and they lead to the good $1/f$ behavior. (In our model the maximum ρ can only be up to $\rho=2$.)

In order to investigate how the motion of individual cars is influenced by others, we investigate the distribution function $p(n)$ of the signal $n(t)$. In Fig. 3(b) we show $p(n)$ vs n with the same parameters as given in Fig. 3(a). Comparing system $\rho=0.3$, for which the power exponent value is -0.1, with system $\rho=1.9$ in which the exponent reaches approximately -1, we can conclude that cars have the trend to bunch themselves in a good $1/f$ system. This result has also been obtained in Ref. [3]. In Fig. 3(b) stronger signals of $n(t)$ mean more cars output from the road in unit time. The implication of Fig. 3(b) is the following: When there are many cars on a long road, after inner interactions, the system tries to organize itself to a definite state favorable to effective transporting. The $1/f$ noise behavior is a typical feature of this definite state. However, this assumption is based on only a few observations, and should be confirmed by many more experiments. The intuitive concept is interesting, which is useful not only in this model, but also in other transporting phenomena, such as the flux flow in type-II superconductors, electrical measurement, water flow in a river, and so on.

In conclusion, we have presented a more reasonable discrete-time and continuous-space model of cars running on a freeway which abide by the traffic rules. The model offers a good $1/f$ behavior, which is in accordance with the real experiment result. From this model we can see that the $1/f$ noise phenomenon of the traffic current is just the result of the competition of both interactions of clustering and hopping between the cars which are defined by the traffic rules. The simple model sheds light on the understanding of the widespread $1/f$ noise phenomena. The intuitive idea drawn from the simple model is also useful to understand why in many systems $1/f$ noise behavior can be found only at some critical points or in certain control parameter regions [2,19].

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