Reply to "Comment on 'Oscillation mode and "nonlinear" radiation of the double sine-Gordon 2π kink'"

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The 2π kink is by no means stable. The linear stability analysis of the collective coordinate Δ according to its definition characterizing the relative motion of two π kinks was intended to find conditions of linear stability of each of two π kinks when perturbed by the coupling with the other. The relativistic invariance of Δ and Ω is evident from the formulas (6) and (12) of the paper [E. Majerníková, Phys. Rev. E **49**, 3360 (1994)]. In Sec. III of that paper there are clearly stated conditions of the approximate analytic calculations, including also $\dot{x}_0^2 \ll 1$.

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(i) In the analysis of the double sine-Gordon equation

$$\phi_{tt} - \phi_{xx} + \lambda \sin\phi + \sin 2\phi = 0 \tag{1}$$

presented in the paper [1], I used a parametrized ansatz for the solution (solitary wave)

$$\phi = 2\arctan[\exp(\theta + \Delta)] + 2\arctan[\exp(\theta - \Delta)], \tag{2}$$

where $\cosh^2\Delta = 1 + 2/\lambda$ was obtained by substitution of (2) into (1). Therefore there cannot be any doubt that the solution (2) is exact. In the paper [1] it states "...it is reasonable to choose the *solution* to (1) in the form...." This means that in the analysis that followed I preferred the form (2) rather than the equivalent forms given by Eq. (4) of the paper [1]. However, full solution must include also oscillation modes. It is known that with the exception of $\lambda = 0$, Eq. (1) is not integrable and does not have soliton solutions.

(ii) In the paper [1] a collective coordinate Δ which represents relative motion of two π solitons has been introduced as a respective degree of freedom. The oscillations of Δ describe internal oscillations of the 2π kink. It has been shown that the oscillation mode is linearly unstable for $\lambda > 0.774$. For $\lambda < 0.774$ two π solitons oscillate around their center while for $\lambda > 0.774$ the mode becomes unstable and then after the collision the kinks radiate energy [2–4]. In this sense internal motion of the two coupled π kinks is unstable.

(iii) The relativistic invariance of Δ is evident from the formula (6) of the paper [1], where the argument of (2) has been generalized as

$$\theta \pm \Delta \to (2+\lambda)^{1/2}[x - x_0(t) \pm \Delta(t)]/(1-\dot{x}_0^2)^{1/2}.$$

As was noted at the beginning of Sec. III of Ref. [1], the result for the frequency Ω {Eq. (20) in Ref. [1]} has been looked for in a simplified case when $\ddot{x}_0 = 0$ and $\dot{x}_0^2 \ll 0$, so that the relativistic invariance, present in the exact Hamiltonian (12),

$$H = E_0 + \frac{m(\Delta_0)\dot{x}_0^2}{2(1-\dot{x}_0^2)}\left(1 + \left[1 + \frac{x_0\ddot{x}_0}{1-\dot{x}_0^2}\right]^2\right) + \frac{M(\Delta_0)}{2(1-\dot{x}_0^2)}\left[1 + \frac{x_0\ddot{x}_0}{1-\dot{x}_0^2}\right]^2 \dot{l}^2 + \frac{2(2+\lambda)^{1/2}}{(1-\dot{x}_0^2)} \times \sum_{n=1}^{\infty}\frac{1}{n!}\left(f_{1n}\dot{x}_0^2\left[1 + \frac{x_0\ddot{x}_0}{1-\dot{x}_0^2}\right]^2 + g_n\right) l^n,$$

was lost due to neglecting the respective terms.

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