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Comment on "Oscillation mode and 'nonlinear' radiation of the double sine-Gordon 2π kink"

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It is shown that recent results on the stability of a double sine-Gordon soliton and its oscillation mode obtained by E. Majerníková [Phys. Rev. E **49**, 3360 (1994)] include some essential errors. In fact, static and moving kink-soliton solutions of this equation are stable for any positive value of the characteristic parameter λ. The oscillation mode for this kink does not show any instability for large λ.

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Recently Majerníková [1] has considered the problem of kink solitons for the double sine-Gordon (DSG) model described by the equation

$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} + \lambda \sin\phi + \sin 2\phi = 0, \tag{1}$$

where the parameter $\lambda > 0$. The paper presents investigations of the "approximate solution" of (1) in the form

$$\phi_k(x,t) = 2\arctan[\exp(\theta+\Delta)] \\ +2\arctan[\exp(\theta-\Delta)], \tag{2}$$

where $\theta = (2+\lambda)^{1/2}(x-ut-x_0)/(1-u^2)^{1/2}$, $u^2<1$, $\cosh^2\Delta = 1+2/\lambda$, as well as of the oscillation mode of this kink in the "static and dynamic cases." The author reports that a condition for the linear stability of the system has been found, given the interaction parameter $\lambda < 0.774$. The analysis is based on the collective coordinate method and includes the effects of external constant perturbations.

In the past the DSG model has been the subject of many papers, e.g., [2–8], and we think that several comments are necessary concerning at least those results of [1] which refer to the properties of the DSG equation without perturbations.

(a) It is not true that the kink solution (2) representing a sum of π kinks is approximated. In fact one can directly check that the solution (2) is exact for *any* positive value of parameter λ including the limit $\lambda \to \infty$ where the π kinks cannot be distinguished.

(b) Since the DSG model (1) belongs to the well-known family of nonlinear Klein-Gordon systems [9], the stability of its kink solution (2) is guaranteed by *topological* reasons [10] for all positive values of λ. The only instability the exists for 2π kinks (2) appears for $\lambda \to 0$, which is the limit opposite to the one considered in [1]. Such an instability means a dissociation of the 2π kink into a pair of uncoupled π kinks [3–5] and is connected with experimentally observed soliton-induced phase transition in quasi-one-dimensional spin systems [5, 11–13].

(c) Because of the above reasons there is no place for any instability of the oscillation mode of the kink (2) except the well understood limit $\lambda \to 0$. Using the general properties of nonlinear Klein-Gordon models [9] one finds immediately that the frequency of the oscillation mode for the kink (2) must *always* be in the range between zero and the frequency of long wave phonons $\omega(k=0) = \sqrt{\lambda+2}$. This fact can also easily be seen from approximated analytical expressions for the shape and frequency of the oscillation mode, which have been obtained by means of supersymmetry [7] and are in very good agreement with numerical results [8] for various values of the parameter λ.

(d) Because of the Lorentz invariance of (1) the shape of the moving kink solution and its oscillation mode follow simply from the *Lorentz transformation.* Therefore the results presented in [1] for a velocity dependent plateau between two coupled π kinks and for a velocity dependent frequency of the oscillation mode cannot be correct.

In conclusion, we state that part of the results obtained by Majerníková [1] on the stability of the DSG kink solution and its oscillation mode are in disagreement with well-established properties of the DSG equation.

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