Electron collisionless layers near evaporating plasma-heated anodes

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Electron collisionless (Knudsen) layers near evaporating plasma-heated anodes are analyzed, and a corresponding general model is formulated. Since the ratio of ion to electron mean free paths is small, the layer is subdivided into ion continuum and ion collisionless layers. Distribution functions for charged species are derived and auxiliary non-problem-specific relationships that are required to complete the model are given. The principal results are as follows: (i) the electron distribution function is little affected by evaporation; f_e is essentially identical to the distribution found near nonevaporating anodes. (ii) Neutral evaporating particles are heated to the plasma heavy particle temperature well within the electron Knudsen layer. (iii) An analytical expression for the heavy particle temperature distribution across the electron Knudsen layer is derived.

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Plasma sheaths form near solid and liquid boundaries that are subjected to plasma heating. Due to an unbalanced space charge, relatively large electric fields can exist within the sheath, significantly influencing the energy flux to the anode. Although significant anode evaporation can occur, particularly at high energy densities, evaporation effects on the near-anode region have not received thorough theoretical attention (see, e.g., [1–7]) and presently lie outside the realm of direct measurement [8]. This paper's objectives are to first develop a realistic physical picture of the near-anode region, taking into account anode vaporization, and then to formulate a self-consistent mathematical model of the region. The hybrid model developed is applicable to the electron collisionless (Knudsen) layer, and in contrast with earlier formulations [2,4], accounts for heavy particle collisions and energy transfer within this layer. The model applies to anode heating by steady, high intensity (current ≥ 50 A), atmospheric pressure plasmas, assumes that the anode liquid is at or near its boiling point, and allows for any number of singly and multi-ionized species. Thus, the model is relevant to a number of plasma-based materials processing applications, e.g., gas tungsten arc welding.

In order to develop the model, we first consider the relative magnitudes of the Debye shielding length λ_D and the electron mean free path λ_e. In particular, when $\lambda_e/\lambda_D \ll 1$ the sheath can be modeled as a continuum, whereas a kinetic formulation is required for $\lambda_e/\lambda_D \gg 1$. Preliminary estimates of λ_e and λ_D follow by considering an atmospheric argon plasma over the approximate range of electron temperatures expected near a *nonevaporating* anode [2]: $8000 \text{ K} \leq T_e \leq 12\,000$ K. λ_e can be estimated using $\lambda_e = \overline{C}_e/\nu_e$, while $\lambda_D = (\varepsilon_0 k T_e/n_0 e^2)^{1/2}$. In these expressions, ε_0 is the permittivity constant, k is Boltzmann's constant, $n_0 = n_i = n_e$ is the charged particle

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number density within the bulk plasma, T_e is the electron temperature, $\overline{C}_e$ is the mean electron speed, and ν_e is the average electron momentum-transfer collision frequency. Considering the last quantity first, $\nu_e = \sum_{s=1}^{N} \overline{C}_e \overline{Q}_{es} n_s (m_e/m_{es})^{1/2}$, where $\overline{Q}_{es}$ is the energy averaged cross section for collisions between electrons and species of type s, n_s is the number density of s, m_e is the electron mass, and $m_{es} = m_e m_s/(m_e + m_s)$ is the reduced mass. At $T_e = 12\,000$ K, energy averaged cross sections for collisions between electrons and argon atoms are on the order of 10^{-20} m^2 [9]. Employing the relation [9] $\overline{Q}_{ei} = 6\pi b_0^2 (\ln\Lambda - 1.37)$, where the collision and plasma parameters are given by $b_0 = Ze^2/(12\pi k T_e)$ and $\Lambda = \lambda_D/b_0$, we find that energy averaged cross sections for collisions between Ar^{+1} ions and electrons, $\overline{Q}_{ei}$, and between electrons, $\overline{Q}_{ee}$, are both on the order of 10^{-17} m^2. Thus, since number densities for charged and neutral species within the bulk plasma are approximately equal ($n_e = n_i \approx n_n \approx 10^{23}$ m^{-3}, as estimated by the Saha equation), charged particle collisions predominate and $\lambda_e \approx 10^{-6}$ m. Note that this estimate is consistent with the estimate reported by Dinulescu and Pfender [1]. At $T_e = 8000$ K, $n_e \approx 1 \times 10^{22}$ m^{-3} [2], $n_n \approx 10^{24}$ m^{-3} [3], and electron–neutral species collisions become as prominent as electron–electron collisions; in this case, $\lambda_e \approx 1 \times 10^{-5}$ m. Considering the associated Debye lengths, λ_D is on the order of 10^{-8} m when $n_0 = 10^{23}$ m^{-3} and $T_e = 12\,000$ K, and on the order of 10^{-7} m when $T_e = 8000$ K and $\overline{n}_0 = 1 \times 10^{22}$ m^{-3}. Thus, since the ratio λ_e/λ_D is on the order of 10^2 over the range of electron temperatures and densities expected near nonevaporating anodes, the sheath in this case lies well within the electron collisionless layer.

We can show that our preliminary λ_e and λ_D estimates remain valid near evaporating anodes. To prove this, we first consider the average number of collisions n_{ah} required to heat evaporating particles (having temperature T_a) to the plasma's heavy particle temperature T_h. The quantity n_{ah} is given by $n_{ah} = 3k(T_h - T_a)/(2\Delta\overline{\varepsilon}_{ah})$

$=(m_a+m_h)/(2m_{ah})$, where $\Delta\bar{\varepsilon}_{ah}$ is the average energy transferred per elastic collision between heavy and evaporate particles [9]. The corresponding thermalization time scale is $\tau_{ah}=n_{ah}/v_{ah}^{(1)}$, where $v_{ah}^{(1)}=g_{ah}Q_{ah}^{(1)}n_h$ is the momentum transfer collision frequency and g_{ah} is the relative speed between heavy and evaporate particles. A conservative estimate of τ_{ah} follows by assuming a relatively small cross-section of $Q_{ah}^{(1)}=10^{-19}$ m^2 [9], and by setting $T_h=5000$ K [1] and $n_h(=n_i+n_n)=1.4\times10^{24}$ m^{-3} [3]. Approximating g_{ah} as the heavy particle thermal speed within the plasma (≈1600 m/s), we obtain a *minimum* collision frequency of $v_{ah}^{(1)}\approx2\times10^8$ s^{-1}. Since heavy plasma particles and evaporate particles have masses of the same order of magnitude, $n_{ah}\approx1$. Thus, $\tau_{ah}\approx1\times10^{-8}$ s. (Due to the small mass of electrons, electron-evaporate energy transfer is much less effective, with the shortest realistic time scale for energy transfer being on the order of 10^{-6} s.) Since the speed of evaporate particles v_{1v} is on the order of 10^2 m/s [10], the *maximum* distance required to heat evaporate to T_h (given by $v_{1v}\tau_{ah}$) is 1×10^{-6} m. In the case of iron or aluminum alloys heated by argon plasmas, cross sections for collisions between cool Fe or Al atoms and neutral and ionized Ar and hot neutral and ionized evaporating particles are estimated to be on the order of 10^{-18} m^{-2} or more [9]. In this case, the thermalization distance will be at least an order of magnitude smaller than the above estimate. Thus, in many practical circumstances, evaporating particles reach T_h well within the electron collisionless layer. Since evaporate and native heavy particle masses are of the same order of magnitude, electron cooling within the bulk plasma clearly does not occur, proving that T_e, and thus λ_e and λ_D, are unaffected by evaporation.

Considering now the ion mean free path, we find that over the range 8000 K $\leq T_e\leq$ 12 000 K, λ_i is two orders of magnitude smaller than λ_e, in agreement with Dinulescu and Pfender [1]. Thus, in analogy with the conditions determining a collision-dominated electron gas [9], ions over most of the electron Knudsen layer are collision dominated and can be considered a continuum (see also Ref. [4]). We thus model ions within the electron Knudsen layer as residing in either a continuum ion layer or in a very thin collisionless layer abutting the anode (Fig. 1).

Having established a physical picture of the electron Knudsen layer, we now develop a suitable mathematical model. Within the collisionless electron and ion layers, the following form of Boltzmann's equation applies:

$$w_1 f_{s,x_1}-eZ\phi_{,x_1}f_{s,w_1}/m_s=0 \ , \tag{1}$$

where partial derivatives are denoted with a comma, Z is the net number of charges on particle s, m_s is the mass of s, $\phi=\phi(x_1)$ is the electric potential, and x_1 is the coordinate direction normal to the anode surface. The solution to (1) is given by

$$f_s(\mathbf{x},\mathbf{w})=a_s\exp[b_s(m_s w_1^2/2+eZ\phi)]$$
$$\times\exp[-m_s(w_2^2+w_3^2)/(2kT_s)] \ , \tag{2}$$

where T_s is the local temperature of s, w_2 and w_3 are orthogonal absolute particle velocities in planes parallel to
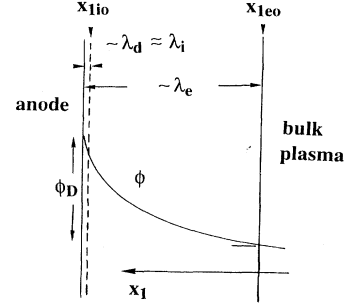

FIG. 1. Schematic of the electron Knudsen layer. Ions reside in a collision-dominated continuum over most of the layer but are collisionless immediately adjacent to the anode.

the anode, and w_1 is the corresponding x_1 component.

The parameters a_s and b_s in (2) are determined by taking the first two moments of the distribution function. Assuming that the anode is perfectly catalytic and that the electric field at the outer edge of the electron Knudsen layer (located at x_{1eo}) is small, only electrons with zero or positive normal velocities ($w_1\geq0$) pass into the electron Knudsen layer. In this case, the minimum absolute normal velocity for incident electrons is $w_{1e}^{min}(x_1)=\sqrt{2\phi(x_1)/m_e}$ [where, for convenience, we set $\phi(x_{1eo})=0$]. Similarly, within the ion collisionless layer, the minimum ion velocity $w_{1i}^{min}(x_1)=-\sqrt{2eZ(\phi_D-\phi(x_1))/m_i}$ corresponds to ions that nearly reach the anode and then accelerate backward. Here, ϕ_D is the potential at the anode.

Using the above limiting electron velocity, we obtain expressions for a_e and b_e in the electron distribution function f_e,

$$a_e=n_{e0}/D(m_e/2\pi kT_e)^{3/2}, \quad b_e=1/(kT_eD^2) \ , \tag{3}$$

where $D=[1+\zeta]^{1/2}$, $\zeta=\varepsilon\{16/\pi-4+32/\pi[\varepsilon(1/\pi-4)+4]^{1/2}\}$ $\varepsilon=m_e v_1^2/(kT_e)$, $n_{e0}=n_e(x_{1eo})$, and v_1 is the mass average velocity in the x_1 direction. Neglecting a small contribution due to electrons, v_1 can be obtained using $v_1=(\rho_v v_{1v}+\sum_{i=1}^{N_I}\rho_i v_{1i})/\rho$, where ρ_v and v_{1v} are the evaporating mass density and normal velocity, ρ is the sum of all mass densities, N_I is the number of ionic species, v_{1i} is ith ionic specie's mean normal velocity, and $\rho_i=n_i m_i$. Since the anode material is at or near its boiling point, evaporating and native neutral number densities within the near-anode region are at least an order of magnitude larger than charged particle number densities [1,3]. Thus, v_1 essentially equals v_{1v}, so that $\zeta=O[m_e v_{1v}^2/(kT_e)]\ll1$. Thus, the electron distribution is virtually identical to that found near *nonevaporating* anodes [11].

Using the above minimum ion velocity within the ion collisionless zone leads to two relationships for a_i and b_i,

$$n_{io}=a_i(2\pi^3 k^2 T_{io}^2/b_i m_i^3)^{1/2}$$
$$\times\exp(-b_i Ze\phi_o)\text{erfc}[\sqrt{b_i Ze(\phi_D-\phi_o)}] \tag{4}$$

and

$(kT_{i0}-1/b_i-m_i v_1^2)\mathrm{erfc}[\sqrt{b_i Ze(\phi_D-\phi_0)}]/2$

$+\exp[-b_i Ze(\phi_D-\phi_0)](m_i/b_i\pi)^{1/2}$

$\times\{[Ze(\phi_D-\phi_0)/m_i]^{1/2}+\sqrt{2}v_1\}=0$, $\qquad$ (5)

where x_{1i0} is the nominal boundary of the ion collisionless zone, and where $n_{io}=n_i(x_{1io})$ and $T_{io}=T_i(x_{1io})$.

Within the ion continuum zone, we assume that ions are in equilibrium with respect to the mass average velocity v_1. This assumption requires that mean ion velocities equal v_1, a condition that should be approached due to the short mean free path for momentum transfer between heavy species. The corresponding displaced Maxwellian distributions are described by

$a_i^0=n_i^0(m_i/2\pi kT_i)^{3/2}$, $\quad b_i^0=1/kT_i$. $\qquad$ (6)

Although the problem of describing the electron Knudsen layer in the presence of evaporation is, in principle, complete, auxiliary relationships are needed to close the problem. In particular, relationships describing $\phi=\phi(x_1)$, $\quad n_e=n_e(x_1)$, $\quad n_i=n_i(x_1)$, $\quad T_i=T_i(x_1)$, $v_1=v_1(x_1)$, ϕ_D, ϕ_0, and T_e are needed. Maintaining a non-problem-specific formulation, we briefly outline these relationships. The potential $\phi(x_1)$ follows from the Gauss law

$$\frac{d^2\phi}{dx_1^2}=-\frac{e}{\varepsilon_0}\left[\sum_{i=1}^{N_i}n_i-n_e\right] ,\qquad (7)$$

where thermionic emission has been neglected. Note that based on Freund's data [12] (which shows negligible electron impact ionization of singly ionized Fe, Al, Cu, Si, Mg, Pb, and Ga at $T_e<5$ eV), the density of multi-ionized anode particles is typically small. The number density of charged species is obtained by integrating corresponding distribution functions over the velocity ranges determined above. Thus, for electrons

$n_e(x_1)=n_{eo}/2\exp(e\phi/kT_eD^2)\mathrm{erfc}(\sqrt{e\phi/kT_eD^2})$, $\quad$ (8)

while within the ion collisionless layer

$n_i(x_1)=a_i(2\pi^3k^2T_i^2/b_im_i^3)^{1/2}\exp(-b_iZe\phi)$

$\times\mathrm{erfc}[\sqrt{b_iZe(\phi_D-\phi)}]$. $\qquad$ (9)

Within the continuum ion layer, the Boltzmann relation results are

$n_i^0(x_1)=n_{io}^o\exp(-e\phi/kT_i)$. $\qquad$ (10)

(Note that by setting $D=1$ in (8), corresponding to zero evaporation, we recover the expression obtained by Sutton and Sherman [11].)

The ion temperature distribution within the ion continuum layer follows from the heavy particle energy equation [13]

$(\frac{5}{2})k(n_hT_hv_1)_{,x_1}=(K_hT_h)_{,x_1}$ $\qquad$ (11)

where we have assumed that $T_i(x_1)=T_h(x_1)$ and that derivatives in the x_1 direction are dominant. In this equation, K_h is the heavy particle thermal conductivity [13] and $n_h=\sum_{i=1}^{N_i}n_i+n_n$. Note that there is no collisional energy transfer from electrons to ions. Also note that condensation of evaporate at the anode occurs; a method for estimating the condensate mass flux is given by Anisimov [14]. The solution to (11) reads

$T_h=c_0\exp[-\zeta(x_1)]\int\exp[\zeta(x_1)]K_h^{-1}(x_1)dx_1$

$+c_1\exp[-\zeta(x_1)]$, $\qquad$ (12)

where $\quad\zeta(x_1)=\int\{5kn_h(x_1)v_1(x_1)/[2K_h(x_1)]\}$, $\quad$ and where c_0 and c_1 are determined by problem-specific conditions at the boundaries to the ion continuum zone. With regard to this last point, due to the short mean free path for heavy species, one can set T_h equal to the bulk heavy particle temperature at the outer boundary and to the anode temperature T_a at the inner boundary. To complete evaluation of (12), the heavy particle number density $n_h(=n_v+\sum_{i=1}^{N_I}n_i)$ can be determined using expressions given above and by an appropriate equation of state for the evaporate (e.g., the ideal gas law). In addition, $v_1\approx v_{1v}$, which can be evaluated using $v_1v=KT_a^{-1/2}\exp(-B/T_a)$, where K and B are material-specific parameters [15]. Finally, the heavy particle thermal conductivity can be determined via the Chapman-Enskog approach [16].

The overall potential increase ϕ_D is obtained by calculating the total current density within the ion collisionless zone. In the case where two singly ionized species are present (i.e., ionized plasma gas and ionized anode atoms), the result is

$J_0/e=a_e(2\pi kT_e/b_em_e^2)$

$\quad-\sum_{j=1}^{2}a_j(2\pi kT_i/b_jm_j^2)\exp(-b_je\phi_D)$. $\qquad$ (13)

Notice that multi-ionized species can be readily accommodated in the calculation. The fact that every integral is independent of x_1 provides a useful check on the consistency of the distribution function derivations given above. The boundary of the ion collisionless zone, $x_1=x_{1io}$, and the corresponding potential, $\phi_0=\phi(x_{1io})$, are obtained in the course of solving the Gauss equation [2]. Similarly, the nominal boundary to the electron Knudsen layer, x_{1eo}, determined by solving the bulk plasma problem, corresponds to the location where spatial variations in n_e are comparable to the local magnitude of n_e [2]. Note that electric potential variations over distances greater than λ_e can be readily accommodated by replacing ϕ in all previous expressions with $\phi-\phi(x_{1eo})$ [where $\phi(x_{1eo})$ is determined when x_{1eo} is determined]. The remaining parameters in the problem, $n_{io}^o=n_i^o(x_1=x_{1eo})$, $\quad n_{eo}=n_e(x_1=x_{1eo})$, $\quad T_{io}=T_h(x_1=x_{1io})$, $T_e=T_e(x_1=x_{1eo})$, and J_o, are problem specific and are determined by conditions outside the electron Knudsen layer. Based on the preceding discussion, T_{io} can in many cases be taken as T_a.

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