Reversible axial segregation of rotating granular media

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Experimental measurements of axial segregation of binary mixtures of granular media combined in a horizontal cylinder and rotated like a drum mixer are reported. While in the traditional axial segregation effect the mixture of two different sizes of granular media will separate into bands of relatively pure single concentrations along the axis of rotation, in special cases the homogeneous mixed state can be restored simply by decreasing the speed of rotation. Systematic variation of the relative diameters of the components reveal three classes of behavior: no segregation at any rotation speed, a nonreversible axial segregation, and an axial segregation at high speeds that reverses back into the mixed state at low speeds. Measurements of the dynamic angle of repose of the mixed and segregated phases as a function of rotation speed support a model for the axial segregation effect that involves a diffusion equation with an effective axial diffusion coefficient that can be negative under the conditions for which segregation occurs.

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I. INTRODUCTION

A significant problem for industrial processes involving particle transport, mixing, and storage is the tendency for mixtures to segregate by size, density, shape, and other particle properties [1–3]. Two methods used to mix materials, vertical shaking and horizontal rotation, have been shown to actually lead to segregation in a number of granular systems. In 1929, Smith, Hardy, and Gard reported their observation of vertical size and density segregation during their preparation of powder samples [4]. In 1939, Brown reported vertical segregation of particles in coal transport [5] which depended not only on the relative size and density but on the coefficient of friction, surface roughness, resilience of particles, and other properties inherent to the media. Also in 1939, Oyama first reported axial segregation [6]. If a cylinder partially filled with a binary mixture of granular media is tipped on its side so that its axis of symmetry is horizontal and rotated about that axis, the individual species will segregate into alternating bands of relatively pure single concentrations along the axis of rotation. It is this phenomenon with which this paper is concerned.

Many other reports of axial segregation followed Oyama's. In 1962, Donald and Roseman focused on the "demixing" of rotated mixtures, rather than the problems of imperfect mixing [7]. They reported observing initially a radial segregation (of smaller particles toward the axis of rotation), followed by an end-longitudinal segregation, that is, radial segregation together with segregated bands of particles near the ends of the tube, and, finally, full axial segregation of single concentration bands of particles extending over the entire length of the tube. To account for these observations, Donald and Roseman presented a theory based on the velocity gradients of the beads along the length of the mixer. The argument relied on the difference between the static angles of repose of the components of the mixture, where the static angle of repose is the maximum angle at which the bulk of the material can support the surface of the material above the horizontal [8,9].

The axial segregation effect has been confirmed by many laboratories [1,2,10–16]. We have recently reported that certain mixtures of spherical glass beads will not only segregate when rotated at high speeds, but will also return to a homogeneous mixed state when rotated at lower speeds [17,18]. The argument proposed by Donald and Roseman would not be appropriate for this situation, as their model implied that the segregation behavior of each mixture would be independent of speed. Alternatively, Bridgwater, Sharpe, and Stocker have suggested that the axial segregation effect depends on the difference between the dynamic angles of repose of the mixed and segregated phases [10]. The dynamic angle of repose is defined as the angle that the moving surface of rotating granular material makes with the horizontal, as shown schematically in Fig. 1. (Though the expression "dynamic angle of repose" is in some sense contradictory, it is commonly used when discussing rotating granular media, and we will follow this convention here.) If the mixed phase has a higher dynamic angle of repose than the

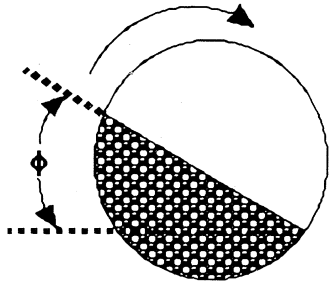

FIG. 1. Schematic drawing of the side view of the cylinder while rotating, indicating the dynamic angle of repose ϕ. Cylinder is rotating clockwise.

segregated phases, then the time evolution of the concentration of the separate granular materials can be described by a diffusion equation with an effective diffusion coefficient whose sign depends on this difference [14].

This paper is organized as follows. The observations of a reversible axial segregation effect are described in Sec. II, along with the results of systematic bead size variation and the time dependence of the band evolution. In Sec. III the adaptation of a model involving an effective diffusion coefficient for the reversible phenomenon will be presented. Section IV presents tests of this model using measurements of the dynamic angle of repose ϕ as a function of the rotation speed for mixtures exhibiting different segregation behaviors. The results are summarized in Sec. V.

II. EXPERIMENTAL TECHNIQUES AND RESULTS

The experimental setup is identical to that of Fauve, Laroche, and Douady [13]. This reference in fact provided the initial motivation for our studies of axial segregation. The cylinder used is a Plexiglas tube 2 ft long and 5 in. in diameter with Plexiglas end plates bolted onto both ends. Similar results are obtained using a cylinder 3 ft long. A bolt is attached to the outside of each end plate, coaxial to the cylinder, and each bolt rests on a ball bearing mount. One of these bolts has an additional flange at its other end connected via a belt to a $\frac{1}{17}$ hp motor (rotational speeds of 0–15 rpm, for a maximum centripetal acceleration of the contents of 0.31 m/s^2, small compared to gravitational acceleration).

For each run the cylinder was filled one third by volume with either a binary mixture of spherical glass beads of two different diameters, each approximately uniform in size, or one of glass beads of one size and sand. The glass beads ranged from about 0.7 to 4.8 in diameter, and the sand was very irregularly shaped with an average distance across each grain of about 0.4 mm. The mixture used in Figs. 2(a)–(d) is one of glass beads of diameters 3.1 and .7 mm. The smaller beads were dyed black to im-

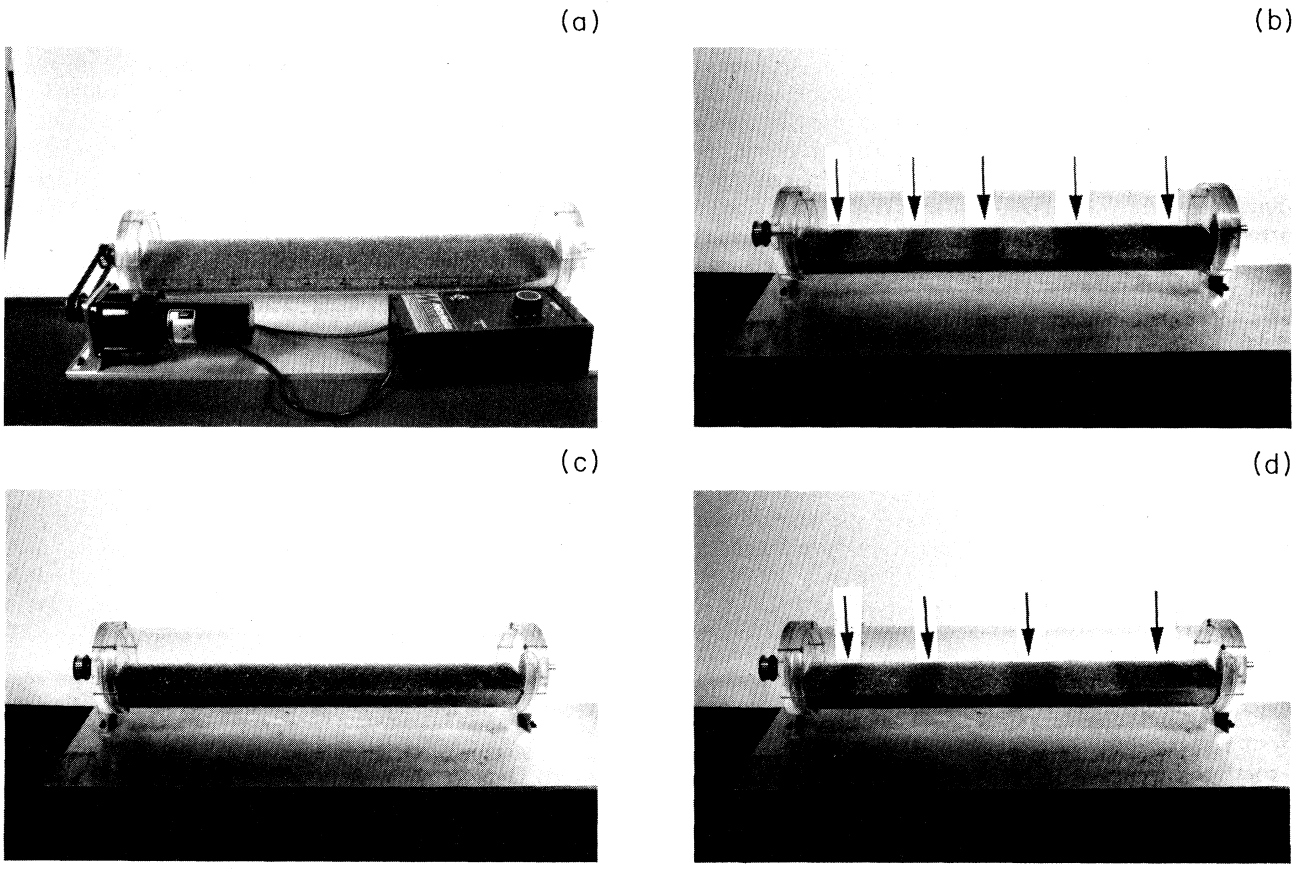

FIG. 2. Photographs of the horizontal rotating cylinder described in the paper. The cylinder is $\frac{1}{3}$ filled with a 50%-50% mixture of large ($d \sim 3$ mm) and "small" ($d \sim 0.7$ mm) glass beads. The initial homogeneous mixed state is shown in (a). After rotation about the long axis for approximately 15 min at 15 rpm, the large and small glass beads have segregated out into alternating bands, as shown in (b). The arrows indicate the locations of the bands of the smaller beads. After axial segregation has occurred, as shown in (b), the cylinder continues to rotate, only now at 5 rpm. After rotation for approximately 1 h at this slower speed, the homogeneous mixed state is restored, as shown in (c). (d) shows the effect of additional rotation at 15 rpm for approximately 15 min, which restores the axial segregated state, though with the segregated bands in different locations and having different widths.

prove the contrast in the photographs. The dye does not affect the results. The cylinder was leveled using shims, though the results do not depend on precise leveling. As shown in Fig. 2(a), the two components are initially homogeneously mixed by first manually mixing the two components and then, after being poured into the cylinder, by shaking and rocking the cylinder. After rotation about the horizontal axis at about 15 rpm for approximately 15 min, unmistakable bands of segregated material form, as shown in Fig. 2(b). The arrows indicate the locations of the bands of the smaller beads. The motor speed was then decreased to 5 rpm. After approximately one hour, the bands had disappeared, and the system had returned to its homogeneously mixed state as shown in Fig. 2(c). When the speed of the motor was again increased to 15 rpm for another 15 min, the segregated bands returned, though the precise pattern, that is, the width and location of the bands, differed as shown in Fig. 2(d). In this way the axial segregation state can be considered reversible, as in a ferromagnetic-paramagnetic phase transition or in spinodal decomposition, with the rotation speed playing the same role as temperature in a thermodynamic phase transition.

Figure 3(a) shows a close-up photograph of three segregated bands for the system shown in Fig. 2. The interfaces between the segregated bands are quite sharp, varying by no more than a few bead diameters. Figure 3(b) shows a further close-up of two segregated bands. In this photograph it is evident that the segregation is not complete, and there are clearly small beads in the large bead-rich band as well as large beads in the small bead-rich band. In addition, not visible in the photograph, the bands are not of uniform width. Rather, they may be on the order of a centimeter larger at the top of the avalanching surface than at the bottom. The determining factor for the variable bandwidth is not understood.

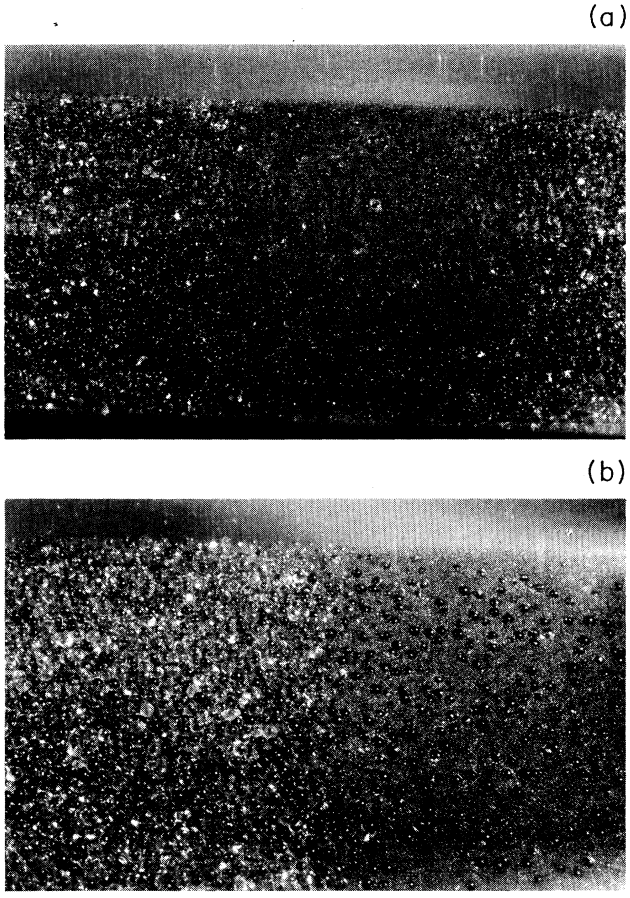

FIG. 3. Close-up photographs, showing three segregated bands (a) and two segregated bands (b) for the same mixture as in Fig. 2 after 15 min rotation at 15 rpm.

The axial segregation experiment was repeated when the diameters of the glass heads were systematically varied. The behavior of the different mixtures is summarized in Table I. Note that, due to the experimental difficulty of the material involved, two of the possible combinations were not tested. Beads of very similar diameter were very hard to completely separate once mixed. Moreover, we noticed that extended use of the sand tended to scratch the walls of the cylinder, changing the friction coefficient between the granular media and the cylinder; consequently, tests involving mixtures with sand were discontinued. Depending on the components, the mixtures displayed three different behaviors when the speed of rotation was varied from 0 to 15 rpm. Some mixtures exhibited the reversible axial segregation as described above and illustrated in Fig. 2. That is, at high speeds the components segregated into bands of relatively pure concentrations, while at low speeds they remixed. Other mixtures exhibited a nonreversible segregating behavior, such that at all speeds segregation occurred and the homogeneous mixture did not return at slower speeds. The last group of mixtures remained mixed regardless of the speed or length of time of rotation. Since most mixtures that segregated showed signs of initial segregation within the first few minutes of rotation, it was concluded that those mixtures that had not shown any sign of segregation within the first few hours of rotation would not segregate. Similarly, since it took on the order of a few hours for the mixtures, once segregated, to completely remix, the cylinder was rotated for over 24 h at the lower speeds before we concluded that those mixtures that still had bands would not remix. No clear relationship was found between the ratio of the diameter of the components and the behavior of the mixture.

As indicated in Table I, the phenomenon varies with size ratio, but not monotonically. The behavior appears to depend on the absolute size of the beads relative to the cylinder as well. Other factors such as surface roughness of the beads or friction between the cylinder and the beads may also influence the segregation effect. The nature of granular media is highly sensitive to the boundary conditions of its container. Thus it is not surprising that, for horizontally rotated mixtures of granular media, small changes in system parameters often result in large changes in segregating behavior.

While the location and size of the axially segregated bands varies from run to run and mixture to mixture, for those mixtures that exhibit a segregation effect there are reproducible features at the beginning and end of each

TABLE I. Segregating or nonsegregating behaviors of different binary mixtures of granular media. δ is the diameter ratio of components.

Type size (mm)	Sand 0.42±0.08	Glass beads 0.70±0.06	Glass beads 0.97±0.07	Glass beads 1.2±0.1	Glass beads 3.1±0.2
Glass beads 0.70±0.06	nonreversibly segregated $\delta=1.67$	single phase			
Glass beads 0.97±0.07	not performed $\delta=2.3$	not performed $\delta=1.4$	single phase		
Glass beads 1.2±0.01	nonreversibly segregated $\delta=2.9$	nonreversibly segregated $\delta=1.7$	nonreversibly segregated $\delta=1.2$	single phase	
Glass beads 3.1±0.2	did not segregate $\delta=7.4$	reversibly segregated $\delta=4.4$	nonreversibly segregated $\delta=3.2$	reversibly segregated $\delta=2.6$	single phase
Glass beads 4.8±0.1	did not segregate $\delta=11.4$	reversibly segregated $\delta=6.9$	reversibly segregated $\delta=4.9$	reversibly segregated $\delta=4.0$	did not segregate $\delta=1.5$

run. Upon beginning rotation of the mixture, some of the small beads move toward the axis of rotation almost immediately to form a radially segregated mode. This has been observed previously by many others and is attributed to the variation in compactivity extending from the axis of rotation to the freely flowing top surface [19,20]. The radial segregation effect was not extensively investigated here. Recent research on this effect is described in Refs. [21] and [22] for two dimensional systems. Another feature consistently observed is that generally the first two axially segregated bands form at the two ends of the cylinder, while the remainder of the segregated bands (typically two or three for the cylinder and mixture shown in Fig. 2) appear in no particular order, their location depending on the axial concentration fluctuations, as discussed below. This initial segregation at the end plates is almost certainly a boundary effect, and is also observed in a cylinder which is 3 ft long.

Finally, the initial banding pattern as indicated in Fig. 2(b) or 2(d) is neither reproducible nor stable, but when the mixture is rotated for an extended period of time at 15 rpm the bands evolve almost invariably to a final pattern of one central band of large bands with bands of small beads adjacent on either side and narrow bands of large beads at both end plates. The time dependent evolution of the segregated bands is complicated, and the mechanism that motivates it remains poorly understood. Savage reported observing the banding pattern changing in time, with the bands moving in the axial direction, merging, and occasionally dividing [14]. Nakagawa also reported observing the merging of axially segregated bands for extended rotation periods [16]. Similarly, band movement and merging has been seen for the binary mixtures of granular media described here. The arrangement of bands changes from one moderately stable pattern to the next, each with fewer bands than the last. After the initial pattern forms, on the order of 10–20 min pass while the bands may be moving very slowly axially without merging. The bands then evolve into the next configuration, in general, with one fewer of each band (large and small beads, each), as two bands of one type merge and the one between them disappears.

When two bands merge, they do not, in general, move closer toward each other until they meet. Rather, in the case of the merging of two small bead bands, for example, it is observed that the area between them becomes increasingly more concentrated with small beads, while the small bead concentration with the bands themselves appears to decrease slightly, until the concentration of all three regions is roughly uniform. Finally, the now single band of small beads "sharpens." The band itself narrows, while the region within it becomes more highly concentrated with small beads. The pattern is then stable again for a period with only minor axial movement of the bands until the next relatively sudden merging event occurs. The temporary stability of each pattern is suggestive of a system progressing through various metastable configurations. However, additional studies and quantitative analysis are needed before further conclusions can be drawn.

III. MODEL FOR AXIAL SEGREGATION

We now describe a model for the reversible axial segregation effect that is a compilation of arguments proposed by Savage [14], Bridgwater, Sharpe, and Stocker [10], Donald and Roseman [7], and Das Gupta, Khakhar, and Bhatia [12]. These authors addressed the observation of axial segregation. Their model has been adapted here to account for the reversible effect as well.

An understanding of the process of axial segregation requires a discussion of the factors that influence the value of the dynamic angle of repose ϕ as defined in the Introduction and shown in Fig. 1. Clearly, this angle,

which represents the height at which the granular material loses contact with the back wall of the cylinder and begins flowing down the top surface, will increase when the speed of rotation is increased. This angle also depends, among other factors, on the interparticle friction and the friction between the granular media and the walls of the container, as well as the relative concentration of the components within the mixture. A mixture of particles of different sizes can achieve a denser packing than monodisperse spheres [23]. With a higher void fraction, the mondisperse phase is less able to sustain a shear flow than a mixture, and will therefore have a lower dynamic angle of repose than the mixed phase. As a result, as shown schematically in Fig. 4, if the mixed phase has a greater dynamic angle of repose than a monodisperse phase, any concentration fluctuation will result in height variations of the mixture along the axis. Specifically, the lower dynamic angle of repose of large beads, compared to that of a mixture, results in a lower height for a region rich in large beads.

At the same time, the radial segregation of small beads beneath the flowing surface described in Sec. II causes the mixed phase in the top flowing layer to become enriched with large beads. The beads in the flowing layer move to the lowest point on the surface, which, due to an axial height variation arising from a concentration fluctuation, includes motion along the horizontal axis of rotation. In other words, there will be axial drift, due to gravity, of large beads from the mixed phase to the sections rich in large beads, which enhances rather than decreases an initial fluctuation of large beads.

This axial motion of large particles can be expressed in terms of a drift current j_{drift} of the large beads out of the mixed phase. Such a current only exists at the interface between a mixed phase region and a region of segregated large beads and hence is proportional to the axial gradient of the larger particle concentration C_L:

$$j_{\text{drift}} = \beta \partial C_L / \partial x \ , \tag{1}$$

where the constant of proportionality β will vary monotonically with the bead height and hence with $\Delta\phi$, where $\Delta\phi = \phi_M - \phi_L$, the difference between the dynamic angle of repose of the mixed phase and the large bead phase. Particularly, when $\Delta\phi = 0$, then $\beta = 0$ as well. If $\Delta\phi$ increases, so will j_{drift} and the tendency for segregation.

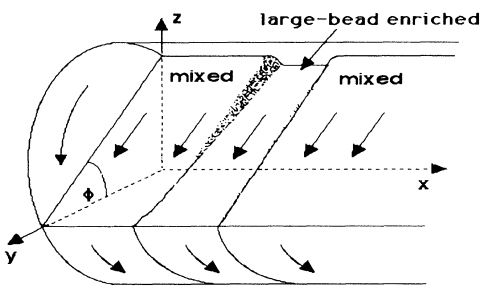

FIG. 4. Schematic drawing of the surface of rotating beads, indicating bead height variation with concentration.

In addition to the tendency to segregate due to the lowering of potential energy reflected by j_{drift}, the random collisions that lead to mixing must also be considered. This is expressed in terms of the normal Fickian diffusion current which is also proportional to the concentration gradient at a band interface:

$$j_{\text{diff}} = -D \, \partial C_L / \partial x \ , \tag{2}$$

where D is the axial diffusion coefficient for the larger component. The net total current is given by the sum of the diffusion and drift currents: $j = j_{\text{diff}} + j_{\text{drift}}$. Thus the rate of growth of the segregated phase C_L is given by the continuity equation

$$\partial C_L / \partial t = -\partial j / \partial x = \partial [(D - \beta) \partial C_L / \partial x] / \partial x \ . \tag{3}$$

This is of the form of the traditional diffusion equation, if the normal Fickian diffusion coefficient is replaced by an effective diffusion coefficient $D_{\text{eff}} = D - \beta$. If $\Delta\phi = 0$ then $\beta = 0$ and the effective diffusion coefficient is simply the normal Fickian axial diffusion coefficient, that is, $D_{\text{eff}} = D$, and the effect of random collisions will simply be to remix any segregated bands which may occur due to random fluctuations. However, if $\Delta\phi$ is large enough so that $\beta > D$ then $D_{\text{eff}} < 0$, which would be interpreted as leading to the enhancement of concentration fluctuations, and ultimately segregated band formation. Hence a necessary condition in order to observe a reversible axial segregation effect, based upon the above argument, would be that $\Delta\phi = \phi_M - \phi_l$ be large and positive at high rotation speeds (so that $D_{\text{eff}} = D - \beta < 0$) but $\Delta\phi$ be nearly zero at low but finite rotation speeds (so that $D_{\text{eff}} \sim D > 0$). For a nonreversible axial segregation effect $\Delta\phi$ should be large and positive at all rotation speeds, while for those binary mixtures of granular material which do not segregate, $\Delta\phi \sim 0$ at all rotation speeds.

IV. ANALYSIS

To test the model for axial segregation described above, the dynamic angle of repose of the different mixtures and each homogeneous state at different rotation speeds was measured from photographs taken along the end of the cylinder down the axis. The angle that the free-flowing surface made with a fixed horizontal marker, namely, the top of the support post, was measured first for homogeneous binary mixtures of two components and then each of the components alone. In addition to the error associated with measuring a moving surface, the avalanching surface does not form a perfect plane, due to centrifugal forces and wall friction which give the flowing surface an s-shaped contour at high rotation speeds [24]. This distortion was small for the speeds at which the data for this paper were taken and is discussed in more detail at the end of this section. To account for any error, the values of ϕ presented are the averaged values of multiple ($\sim 5-10$) separate measurements.

Representative results of these measurements are shown in Figs. 5–7. Each is a graph of the dynamic angles of repose for both the mixed phase and the monodisperse phase of the larger component against rotation

speed for six speeds ranging from about 3 to 15 rpm. Figure 5 is a graph of the dynamic angles of repose for the beads of diameter 0.7 and 3.1 mm which segregated at higher speeds and remixed at lower speeds (the system pictured in Fig. 2). The measurements of ϕ for the mixed phase were made during the first few minutes of rotation, prior to the development of axial segregation. In agreement with the above model there is a large difference between the dynamic angle of repose of the mixture and the monodisperse large bead phase at high speeds where the axial segregation is observed. Since $\Delta\phi$ is large at this speed β is as well, consistent with a negative effective diffusion coefficient in Eq. (3). At lower speeds, where the segregated bands remixed back into a homogeneous state, there is little or no difference between the two dynamic angles of repose. Hence $\beta=0$, and consequently $D_{\text{eff}}=D>0$, so random collisions will only lead to remixing, with no tendency to segregate, also in accord with the observations.

Further support for the above model comes from considering mixtures which exhibit either a nonreversible axial segregation or no segregation at all. Figure 6 is a graph of the angles of repose for a mixture of beads of 1.2 mm in diameter and sand (grain width ≈ 0.4 mm) which segregates at all speeds and does not remix at any speed (nonreversible axial segregation). The D_{eff} model would require that the dynamic angle of repose for the mixed phase be larger than that of the monodisperse phase at all rotation speeds, which is indeed demonstrated in Fig. 6. Therefore at all rotation speeds it is consistent that $D_{\text{eff}}=D-\beta<0$, and random fluctuations in concentration will grow in time. Figure 7 is a graph of the angles of repose for a mixture of 4.8 and 3.1 mm beads which do not segregate at any speed. As shown in Fig. 7, $\Delta\phi=0$ for all rotation speeds, and $D_{\text{eff}}=D>0$, so random collisions will dissipate any concentration fluctuations.

Although the measurements of the dynamic angles of repose support the explanation for the observed segregation behaviors involving an effective diffusion coefficient whose sign depends on $\Delta\phi=\phi_M-\phi_L$, the measurements

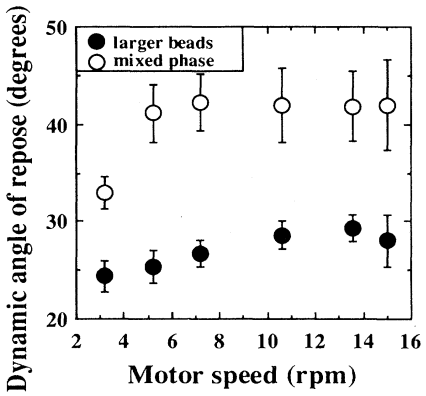

FIG. 6. Dynamic angles of repose for the mixed phase and the larger monodisperse phase against rotation speed for glass beads of diameter 1.2 mm and sand; mixture segregates at all speeds examined.

of the angles of repose were obtained from photographs taken of the cross section of the avalanching surface at the end of the cylinder. This raises the issue as to whether or not the conditions for the beads at the end plates are the same in the center of the cylinder, and, if not, how any difference affects the measured values of the dynamic angle of repose. In particular, the frictional force between the end plates and the beads directly adjacent to the end plates could drag those beads to a greater height, which would lead to a higher perceived dynamic angle of repose when measured from the end. To test whether this additional drag influences the validity of the measurements of the dynamic angle of repose, the height of the granular material was measured at different points along the back wall of the cylinder. The height of the beads adjacent to the end plates was found to be approximately 1.5 mm greater than the average bead height further from the ends along the length of the tube. However, this height shift was roughly constant for all phases

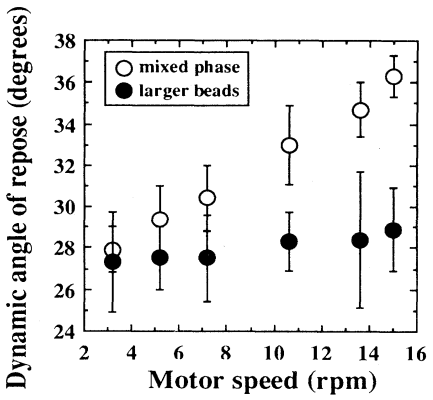

FIG. 5. Dynamic angles of repose for the mixed phase and the larger monodisperse phase against rotation speed for glass beads of diameters 0.7 and 3.1 mm; mixture segregates at higher speeds and remixes at lower speeds.

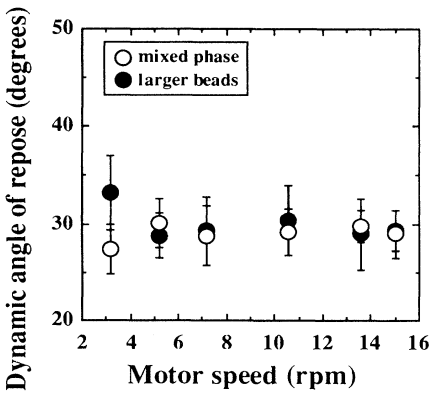

FIG. 7. Dynamic angles of repose for the mixed phase and the larger monodisperse phase against rotation speed for glass beads of diameters 3.1 and 4.8 mm; mixture does not segregate at any speed examined.

considered, mixed and monodisperse, at all rotation speeds. Therefore, although the precise values for the angles of repose (Figs. 5–7) presented may not be accurate to within $\sim 1-2°$, the qualitative results for the variation of $\Delta\phi$ with rotation speed are still valid.

Another complication can arise due to the detailed shape of the avalanching surface away from the end plates. If either the surface of the flowing beads forms an s-shaped contour (see, for example, Refs. [15,24]) or the height variation due to $\Delta\phi$ is symmetric about the axis of rotation so that a large height variation at the top is balanced by an equivalent reversal at the bottom, then it is not obvious that any segregation occurring at the top of the flowing surface (of large beads out of the mixed phase) will not be reversed at the bottom. In order to investigate the contour of the flowing surface, the dynamic angle was measured at three locations as indicated in Fig. 8 from photographs taken at the end plate as before. The highest angle, ϕ_1, was presented as the dynamic angle of repose for the measurements of Figs. 5–7.

In general, for all phases measured, the values of the highest and middle angles, ϕ_1 and ϕ_2, were found to increase with speed, while the angle measured at the lowest position, ϕ_3, falls, implying more of a curved surface, similar to the slight s-shaped curve suggested in Refs. [15,24]. For the monodisperse phases investigated, the highest angle ϕ_1 was found to increase by about the same amount that the lowest angle ϕ_3 decreases when the speed is increased from 3 to 15 rpm. However, when the motor speed is increased for a mixture of small and large beads, ϕ_1 rises significantly more than ϕ_3 decreases. For example, if $\Delta\phi_1$ and $\Delta\phi_3$ represent the difference between the dynamic angle of repose of the mixed phase and the large bead phase for the highest and lowest position on the slope, respectively, for the 3.1 and 0.7 mm glass beads, $\Delta\phi_1$ was approximately 10° at high rotation speeds while $\Delta\phi_3$ was only $-4°$. This indicates sufficient asymmetry in height variation to yield a net tendency toward segregation. As mentioned above, the denser mixed phase will have a different dynamic angle of repose for a given rotation speed than will the separate monodisperse granular materials. This sensitivity of the dynamic angle of repose to the relative concentration of the granular media is apparently also reflected in the detailed shape of the avalanching surface. This may account for the observed nonuniform width of the segregated bands as described in Sec. III, as well as how the geometry of the surface shape relates to segregation. The relationship between the curvature of the avalanching surface and the segregation phenomenon remains unclear and requires further study.

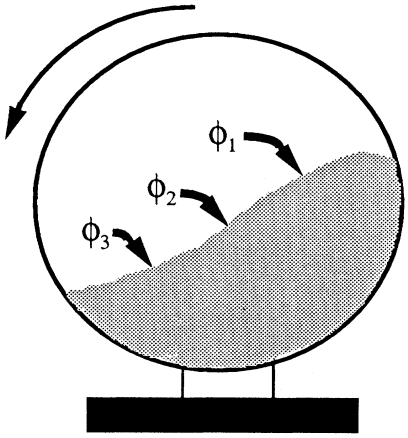

FIG. 8. Schematic of different positions on avalanching surface at which the dynamic angle was measured.

V. CONCLUSIONS AND COMMENTS

The main conclusions of this paper are as follows. Observations of a reversible axial segregation effect for horizontally rotated binary mixtures of glass beads were described. The behavior of the mixtures under systematic variation of bead diameter was found to fall into one of three different categories. Depending on the diameters of the components, the mixture would either exhibit reversible axial segregation, a segregation effect which is not reversible at slower rotation speeds, or no segregation effect at all. The behavior did not appear to vary systematically with particle size ratio. Various measurements were taken to determine if an explanation for the segregation effect involving the competition between axial drift and diffusion currents leading to a diffusion equation with an *effective* diffusion coefficient holds for all three behaviors. Measurements of the dynamic angle of repose of the different phases support this explanation. When $\Delta\phi > 0$ it was found that the mixture undergoes segregation. When $\Delta\phi \sim 0$ the components mixed. Measurements of the meniscus effect showed no effective contamination of the data due to edge effects, and the value of ϕ at different points on the avalanching surface displayed sufficient asymmetry for the D_{eff} argument to hold. Finally, observations of pattern development present a possible example of a system progressing through metastable configurations, though it is clear more work will have to be done before anything conclusive can be said about this aspect.

We conclude by commenting on the relationship between spontaneous pattern formation in granular media and that of other non-Newtonian fluid flow systems. One example is traffic flow, which uses a very similar diffusion equation with an effective diffusion coefficient to explain how small changes in driving conditions can lead from smooth traffic flow to traffic jams [17,25]. As described by Lighthill and Whitham, Richards, Newell, Chandler, Herman, and Montroll, and others, the simplest approximation to highway traffic involves a continuity equation $\partial\rho/\partial t + \partial(\rho v)/\partial x = 0$ for flow in the x direction, where ρ is the density per unit length of cars in a highway lane. The diffusion limit accounts for the driver's awareness of traffic conditions ahead by introducing the car density gradient into the current q of cars traveling down the lane $q = q_0 - \nu\,\partial\rho/\partial x$ where ν is a constant. Taking into consideration the finite response time τ of the drivers to any changes in traffic conditions, the continuity equation becomes, for small perturbations about the equilibrium auto density and velocity (i.e., $\rho = \rho_0 + r$, $v = v_0 + w$)

$$\partial r/\partial t + c_0 \partial r/\partial x = [\nu - (v_0 - c_0)^2 \tau] \partial^2 r/\partial x^2 = D_{\text{eff}} \partial^2 r/\partial x^2 ,$$

where c_0 is the kinematic wave speed. The similarity to Eq. (3) which describes axial segregation is apparent. When D_{eff} in the above equation is positive then stable wave solutions exist, while if $D_{\text{eff}} < 0$ then instabilities result and the flow decreases exponentially fast to zero. It therefore appears that the same conditions which lead to axial segregation in rotating sandpiles also lead to traffic jams that are intrinsic to the queuing system (as opposed to extrinsic causes such as accidents or road construction). The question as to whether this identification leads to any new insights into either granular media or traffic flow remains open.

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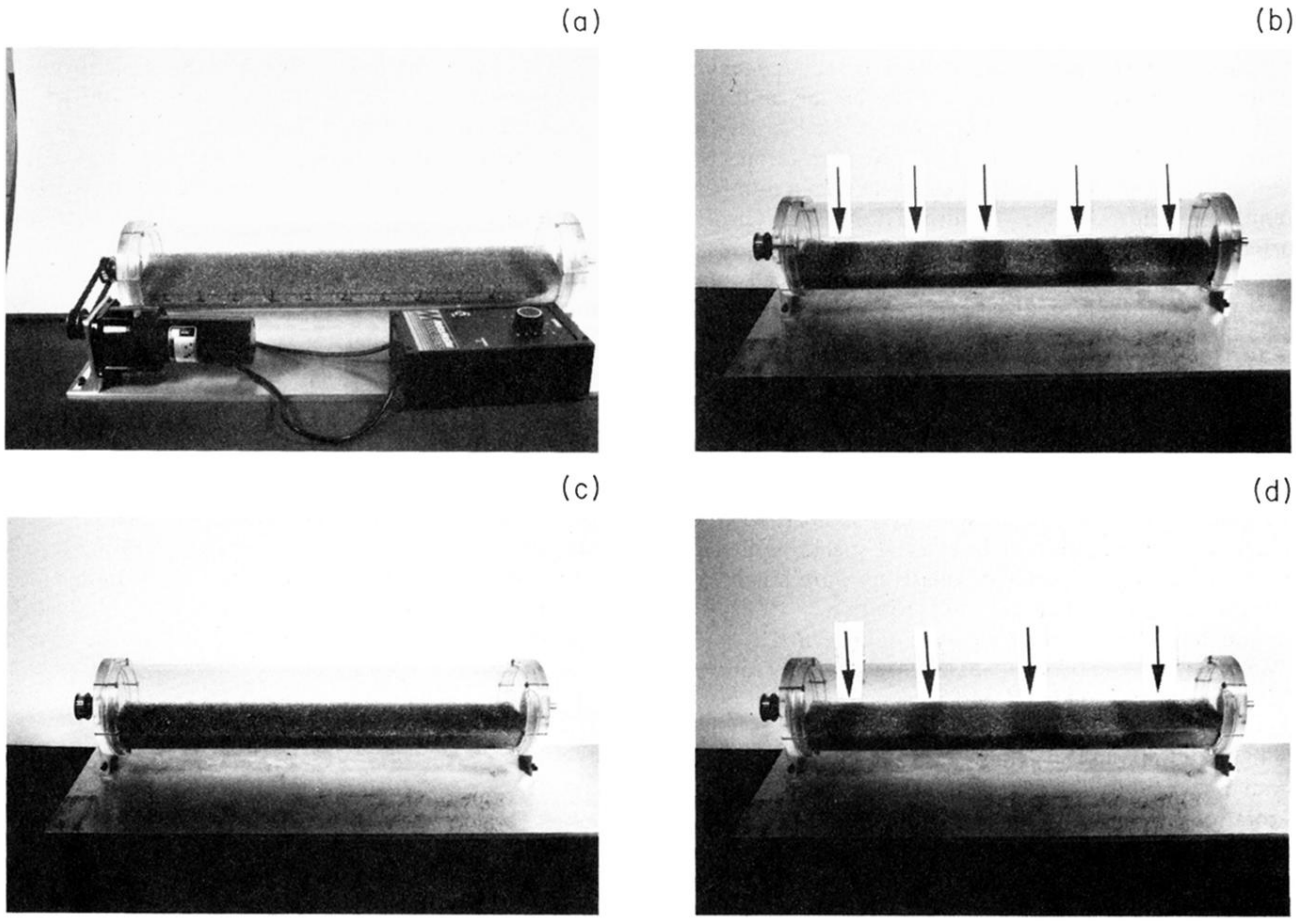
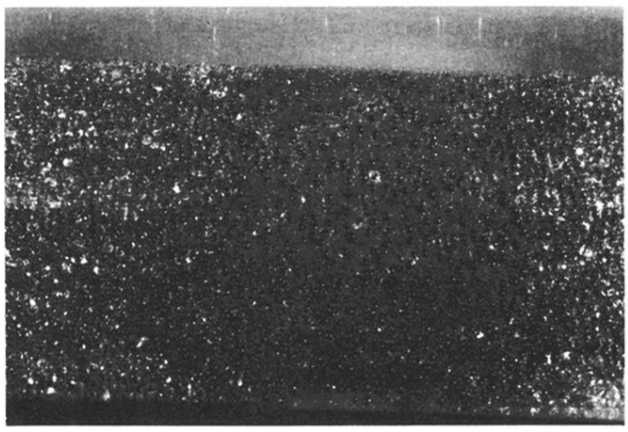


FIG. 2. Photographs of the horizontal rotating cylinder described in the paper. The cylinder is $\frac{1}{3}$ filled with a 50%-50% mixture of large ($d \sim 3$ mm) and "small" ($d \sim 0.7$ mm) glass beads. The initial homogeneous mixed state is shown in (a). After rotation about the long axis for approximately 15 min at 15 rpm, the large and small glass beads have segregated out into alternating bands, as shown in (b). The arrows indicate the locations of the bands of the smaller beads. After axial segregation has occurred, as shown in (b), the cylinder continues to rotate, only now at 5 rpm. After rotation for approximately 1 h at this slower speed, the homogeneous mixed state is restored, as shown in (c). (d) shows the effect of additional rotation at 15 rpm for approximately 15 min, which restores the axial segregated state, though with the segregated bands in different locations and having different widths.

(a)



(b)

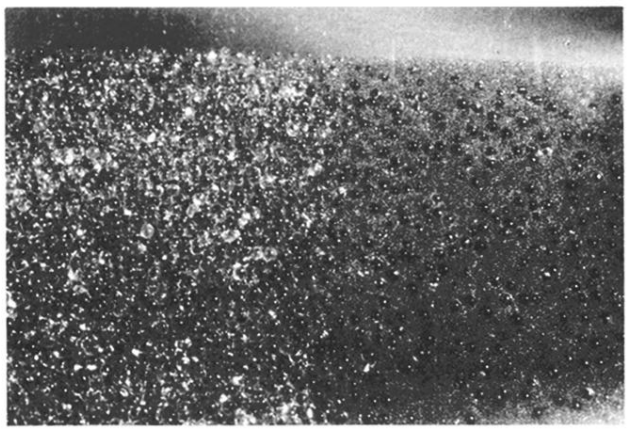


FIG. 3. Close-up photographs, showing three segregated bands (a) and two segregated bands (b) for the same mixture as in Fig. 2 after 15 min rotation at 15 rpm.