Enhanced optical klystron gain with a multiharmonic wiggler

Michael G. Kong
Department of Electrical Engineering and Electronics, University of Liverpool, Liverpool L69 3BX, England
(Received 17 April 1995)

A detailed study of the electron beam bunching problem is presented for optical klystron devices. It is found that the conventional optical klystron configuration is intrinsically inefficient in optimizing the electron beam bunching and thus undermines the interaction gain that can be achieved. To overcome this problem, a multiharmonic wiggler is proposed as the first wiggler section of an optical klystron. By optimizing the electron beam bunching, this arrangement is capable of producing a gain enhancement at the fundamental frequency of up to 75% over what is attainable in a conventional optical klystron. More importantly, its beam quality requirement is found to be comparable to that of conventional optical klystrons. This modified optical klystron configuration should be particularly useful for short wavelength generation for which both a sizable gain and an affordable beam quality requirement are important.

PACS number(s): 41.60.Cr, 07.77.−n, 41.85.Lc, 52.75.Ms

I. INTRODUCTION

Free electron laser (FEL) devices based on wiggler magnets of single period are capable of producing very powerful coherent radiations in the spectrum from microwave up to ultraviolet. To improve their performance even further, a number of nonconventional wiggler configurations have been proposed recently. For instance, it was suggested to use an auxiliary harmonic wiggler for a powerful generation of higher harmonic when the field strength of the main wiggler is only modest [1,2]. A magnet system consisting of two wigglers of similar periods was also conceived as an alternative method to taper the magnetic field, and this scheme was shown to be particularly effective in controlling the FEL spectrum in the high gain Compton regime [3]. In addition, it was found useful to employ a wiggler system of a similar double-period structure for mode selection purpose in low gain waveguide FELs [4].

The most commonly used nonconventional wiggler configuration is an optical klystron which employs a drift section between two essentially identical wiggler magnets to enhance the electron beam bunching and thus increases the small signal gain [5]. To gain a fuller insight into the bunching mechanism and hence its possible improvement in optical klystron devices, the electron bunching problem is investigated in this study by considering the electron dynamics in the bunching direction. It is found that with this conventional optical klystron configuration neither the electron beam bunching nor the small signal gain is optimized. To overcome this problem, we propose a multiharmonic wiggler as the first wiggler section of an optical klystron. By optimizing the electron beam bunching, this new configuration is capable of producing a significant gain enhancement at the fundamental frequency over what is attainable in a conventional optical klystron. In addition, an analysis of the energy spread effect shows that its beam quality requirement is in principle comparable to that of conventional optical klystrons. This modified optical klystron arrangement should be particularly useful for short wavelength generation for which both a sizable gain and an affordable beam quality requirement are essential.

II. BEAM BUNCHING IN OPTICAL KLYSTRON DEVICES

To illustrate the physics underlying the derivation of this multiharmonic wiggler concept, we first study the electron bunching problem in a conventional optical klystron. For the sake of simplicity, we assume the two wiggler sections of the optical klystron are identical and of planar structure. In the one-dimensional limit, the field of either wiggler may be approximated by

$$\mathbf{B}_w = (0, B_{w0}\cos k_w z, 0) \ . \tag{1}$$

Furthermore, we assume a laser beam is present and its field has the following running wave expression:

$$\mathbf{E}_s = (-E_0\cos(\omega t - kz + \phi), 0, 0) \ , \tag{2a}$$

$$\mathbf{B}_s = (0, -B_0\cos(\omega t - kz + \phi), 0) \ , \tag{2b}$$

where $E_0 = (\omega/k)B_0$. When an electron transverses down a wiggler magnet, its velocity can be solved analytically from its equations of motion with the above field specification. In the small signal limit, the formulation of the transverse velocity is straightforward and this leads to

$$\beta_x \approx \frac{a_w}{\gamma_0}\sin k_w z + \frac{a_s}{\gamma_0}\sin(\omega t - kz + \phi) \ , \tag{3}$$

where

$$a_w = \frac{eB_{w0}}{mck_w}, \quad a_s = \frac{eE_0}{mc\omega}$$

are the dimensionless wiggler and laser fields, respectively. In the classical FEL analysis [6], the above expression is used to solve for the electron's energy variation, $\Delta\gamma$, from which the spontaneous spectrum and the small signal gain are formulated. This approach is mathematically efficient because it does not require the more complex formulation of β_z. Nevertheless the most direct understanding of the electron bunching mechanism comes from a thorough study of the beam dynamics in the longitudinal direction along which the bunching occurs. For this reason, β_z needs to be formulated.

In any FEL system, an electron's longitudinal velocity may be expressed in the following general form:

$$\beta_z=\beta_{zw}+\tilde{\beta}_z \ , \tag{4}$$

where β_{zw} represents the electron's axial movement in the wiggler field only and $\tilde{\beta}_z$ is a correction term resulted from the presence of the laser field. Under the condition of $a_w \ll 1$, β_{zw} may be approximated as

$$\beta_{zw}\approx\beta_{z0}-\frac{a_w^2}{4\gamma_0^2\beta_{z0}}(1-\cos 2k_w z) \ ,$$

where β_{z0} is the electron's initial velocity in the longitudinal direction. On the other hand, a lengthy but otherwise straightforward derivation shows that, to the first order of the laser field,

$$\tilde{\beta}_z(z,\phi)\approx-\frac{a_w a_s}{\gamma_0^2\beta_{z0}}\left[\frac{(k+k_w)c-\omega\beta_{z0}}{\omega-\beta_{z0}(k+k_w)c}\right] \times\sin(\Delta k z/2)\sin(\omega t_0+\Delta k z/2) \ . \tag{5}$$

Here $\Delta k=\omega/\beta_{z0}c-(k+k_w)$ is the FEL detuning wave number and ϕ has been replaced by the electron's entry phase at the wiggler magnet, ωt_0.

It is of interest to examine the dependence of $\tilde{\beta}_z$ upon the electron's entry time, t_0, near the synchronism condition $\Delta k=0$ under which Eq. (5) reduces to

$$\tilde{\beta}_z(z,t_0)=-\frac{a_w a_s}{\gamma_0^2\gamma_{z0}^2\beta_{z0}^2}\left[\frac{\omega z}{2c}\right]\sin(\omega t_0) \ . \tag{6}$$

From the above equation, it is clear that electrons entering the wiggler magnet at different instants obtain different velocity variations. This leads to a velocity modulation.

If the electrons are subsequently led into a drift section, they may then catch up each other because of the different velocities at which they leave the wiggler magnet. This can be illustrated by an electron's departure time from the drift section. To this end, we first formulate the electron's transit time through a drift section of length D. Assuming the laser field in the wiggler magnet is small such that $\tilde{\beta}_z \ll \beta_{z0}$, we may approximate the transit time by

$$\Delta t\equiv\frac{D}{\beta_z c}\approx\frac{D}{c\beta_{z0}}[1+M\sin(\omega t_0+\Delta k L/2)] \ ,$$

where $L=N\lambda_w$ is the length of the wiggler magnet and

$$M=\frac{a_w a_s L}{2\gamma_0^2\beta_{z0}}\left[(k+k_w)-\frac{\omega}{c}\beta_{z0}\right]\left[\frac{\sin(\Delta k L/2)}{(\Delta k L/2)}\right] \tag{7}$$

represents a measure of the modulation strength and is referred to as the modulation parameter. Consequently the electron's departure time from the drift section $t_d=t_0+\Delta t$ can be expressed by its entry time at the wiggler magnet as follows:

$$\omega t_d=\omega t_0+\theta_0+M\theta_0\sin[\omega t_0+(\Delta k L/2)] \ , \tag{8}$$

where $\theta_0=\omega D/c\beta_{z0}$ is the electron transit angle through the drift section. This suggests that an electron's departure time depends on both its entry time and the velocity modulation. Therefore with an appropriate combination of the entry time and the amount of the velocity modulation, it is possible for some electrons to leave the drift section simultaneously. In other words, an electron beam bunching is formed and the velocity modulation is converted into a density modulation.

It is of particular interest to further investigate the sinusoidal velocity modulation of Eq. (5) near the synchronism condition $\Delta k=0$. Suppose we consider a portion of electrons which distribute uniformly over an initial phase bucket of $0\le\omega t_0\le 2\pi$. For the first half of these electrons ($0<\omega t_0<\pi$), they receive a velocity reduction in the wiggler magnet [see Eq. (6)] and hence are pulled towards the central electron of $\omega t_0=\pi$ in the drift section. On the other hand, electrons of the second half ($\pi<\omega t_0<2\pi$) gain a velocity increment in the wiggler magnet and therefore are pushed towards this central electron in the drift section. As a result, an electron bunching is formed around the central electron. It should be noted that with a sinusoidal velocity modulation, the biggest amount of velocity increment is not received by the extreme electron of $\omega t_0=2\pi$ which has the biggest initial phase delay from the central electron. Thus by the time this extreme electron catches the central electron, electrons of $\pi<\omega t_0<2\pi$ may have already passed it. In fact, electrons of $3\pi/2<\omega t_0<2\pi$ always reach the central electron at a later time than those of $\pi<\omega t_0<3\pi/2$. The same mistiming can be found for electrons of the first half, whose phases fall into $0<\omega t_0<\pi$. As a result, there are always some unbunched electrons wavering between bunches. The physical implication of the above observation is that with the sinusoidal velocity modulation it is intrinsically impossible to sweep all electrons into bunches.

Very often, it is desirable to obtain a perfect bunching of the electron beam so that all electrons can enter a second wiggler section together. This allows every electron to interact with the laser field there in an identical manner and thus permits the possibility of the maximum laser amplification. To produce such an idealized beam bunching, each electron has to be given a velocity increment or reduction so that its initial phase delay or advance from a reference electron (the central electron in the above discussion) can be completely compensated upon the exit of the drift section. In other words, the required amount of velocity variation for an electron must be proportional to its initial distance from the reference electron. Thus for an electron beam with an uniform initial phase distribution, a perfect beam bunching requires a linear velocity modulation rather than a sinusoidal one of Eq. (5). In mathematical terms, the velocity modulation in the first wiggler section has to be such that it results in a density modulation of $(\alpha_0-\omega t_0)$ in the drift section with α_0 being a constant. This allows each electron to have an identical departure time of $t_d=\alpha_0/\omega$.

If an electron beam is velocity modulation linearly such that its front and end electrons receive the greatest velocity reduction and increment, respectively, all electrons in the beam are then forced to form a big single bunch in the drift section. This can induce an enormous amount of space charges, and the electrons may then start to repel each other strongly before a good beam bunching is achieved. However, to produce a laser amplification of a given amount, a sequence of small electron bunches is as effective as a big single bunch provided the separation distance between any adjacent small bunches equals to the laser wavelength. The advantage of using small electron bunches is that a higher degree of bunching can be achieved without inducing a significant amount of space charges. It is therefore desirable to use a sequence of small electron bunches. This may be produced with a sawtooth velocity modulation at the frequency of the laser signal.

III. REALIZATION OF THE OPTIMIZED BEAM BUNCHING

It is perhaps not straightforward to conceive how to realize a sawtooth velocity modulation in a FEL as the combination of a constant-period wiggler magnet and a monochromatic laser field leads to a sinusoidal modulation. However, Fourier analysis shows that a sawtooth signal $(\pi-\omega t_0)$ can be expanded to

$$\pi-\omega t_0=2[\sin\omega t_0+\tfrac{1}{2}\sin 2\omega t_0+\tfrac{1}{3}\sin 3\omega t_0+\cdots] \tag{9}$$

over a phase period of $[0,2\pi]$. This suggests that if all necessary harmonic sinusoidal modulations are generated in addition to the fundamental one of Eq. (5), a sawtooth modulation can be achieved.

To illustrate how to produce a harmonic modulation, we consider a simple case where the first wiggler section of the optical klystron consists of two different sets of periodic magnet arrays, one at the fundamental and the other at the second harmonic. The total magnetic field is assumed to have the following one-dimensional expression:

$$B_w=(0,B_{w1}\cos k_w z+B_{w2}\cos 2k_w z,0)\ . \tag{10}$$

We further assume that the laser beam contains both the fundamental and the second harmonic signals and its field is given by

$$\mathbf{E}_s=(-E_1\cos\Phi_1-E_2\cos\Phi_2,0,0)\ , \tag{11a}$$

$$\mathbf{B}_s=(0,-B_1\cos\Phi_1-B_2\cos\Phi_2,0)\ , \tag{11b}$$

where $E_n=(n\omega/nk)B_n$ and $\Phi_n=n\omega t-nkz+\phi_n(n=1,2)$.

Similar to the case of a single-period wiggler magnet, the transverse velocity of an electron can be readily derived from its equations of motion in the small signal regime. To the first order of the laser field, this leads to

$$\beta_x=\sum_{n=1}^{2}\left[\frac{a_{wn}}{\gamma_0}\sin nk_w z+\frac{a_{sn}}{\gamma_0}\sin\Phi_n\right]\ , \tag{12}$$

where the dimensionless field strength parameters a_{wn} and a_{sn} are defined as

$$a_{wn}=\frac{eB_{wn}}{mc(nk_w)},\quad a_{sn}=\frac{eE_n}{mc(n\omega)}\quad (n=1,2)\ .$$

Equation (12) may be substituted into the equations of motion to solve for the longitudinal velocity of the electron. After a lengthy derivation, we have

$$\begin{aligned}\frac{d(\gamma\beta_z)}{dt}=&-\frac{k_w c}{\gamma}\{-a_{w1}a_{w2}\sin k_w z+a_{w1}^2\sin 2k_w z+3a_{w1}a_{w2}\sin 3k_w z+a_{w2}^2\sin 4k_w z\}\\&-\frac{a_{w1}a_{s1}c}{2\gamma}\{(k_w+k)\sin(\Phi_1-k_w z)+(k_w-k)\sin(\Phi_1+k_w z)\}\\&-\frac{a_{w2}a_{s2}c}{2\gamma}\{(2k_w+2k)\sin(\Phi_2-2k_w z)+(2k_w-2k)\sin(\Phi_2+2k_w z)\}\\&-\frac{a_{w2}a_{s1}c}{2\gamma}\{(2k_w+k)\sin(\Phi_1-2k_w z)+(2k_w-k)\sin(\Phi_1+2k_w z)\}\\&-\frac{a_{w1}a_{s2}c}{2\gamma}\{(k_w+2k)\sin(\Phi_2-k_w z)+(k_w-2k)\sin(\Phi_2+k_w z)\}\ .\end{aligned} \tag{13}$$

On the right-hand side of the above equation, the first bracketed term depends on the wiggler magnet only whereas others are a collection of small correction terms resulted from the presence of the laser field. Therefore, like that in Eq. (4), we may write $\beta_z=\beta_{zw}+\tilde{\beta}_z$. If the optical klystron is operated near the synchronism condition, $\Delta k=0$, most of these correction terms in $\tilde{\beta}_z$ become fast oscillating functions of the axial distance and hence their net contributions to $\tilde{\beta}_z$ are small. The only significant net contribution comes from the $\sin(\Phi_1-k_w z)$ and $\sin(\Phi_2-2k_w z)$ terms which vary slowly as the electron travels down the wiggler magnet. As a result, Eq. (13) is approximated to

$$\frac{d(\gamma\tilde{\beta}_z)}{dt} = -\frac{k_w+k}{2\gamma}c\sum_{n=1}^{2} na_{wn}a_{sn}\sin(n\Delta kz+\phi_n) . \quad (14)$$

The same argument of significant net contribution can also be used to formulate $(d\gamma/dt)$ from the energy conservation law and this reduces Eq. (14) to

$$\frac{d\tilde{\beta}_z}{dt} = -\frac{(k_w+k)c-\beta_z\omega}{2\gamma^2}\sum_{n=1}^{2} na_{wn}a_{sn}\sin(n\Delta kz+\phi_n) .$$

Hence at the exit of the first wiggler section ($z=L$), we have

$$\beta_z=\beta_{z0}-\frac{1}{\gamma_0^2\beta_{z0}}\left[\frac{(k_w+k)c-\beta_{z0}\omega}{\omega-\beta_{z0}(k_w+k)c}\right] \times\sum_{n=1}^{2} a_{wn}a_{sn}\sin\left[\frac{n\Delta kL}{2}\right]\sin\left[\phi_n+\frac{n\Delta kL}{2}\right] , \quad (15)$$

where $\beta_{zw}(L)=\beta_{z0}$ has been used. It is worth noting that if $a_{w2}=0$ is assumed, $\tilde{\beta}_z$ in the above equation becomes that given in Eq. (5).

Similar to the derivation from Eq. (5) to Eq. (8), the above equation can be used to formulate the electron's departure time from the drift section. If the laser field is small such that $\tilde{\beta}_z \ll \beta_{z0}$, this leads to

$$\omega t_d=\omega t_0+\theta_0+\sum_{n=1}^{2} M_n\theta_0\sin(\phi_n+n\Delta kL/2) , \quad (16)$$

where M_n is the nth harmonic modulation parameter given by

$$M_n=\frac{na_{wn}a_{sn}L}{2\gamma_0^2\beta_{z0}}\left[(k_w+k)-\frac{\omega}{c}\beta_{z0}\right]\left[\frac{\sin(n\Delta kL/2)}{n\Delta kL/2}\right] .$$

If we assume that $M_1=2M_2$ is attainable by means of, for instance, a field adjustment of the wiggler magnet at the second harmonic, Eq. (16) is reduced to

$$\omega t_d=\omega t_0+\theta_0+M_1\theta_0(\sin\omega t_0+\tfrac{1}{2}\sin2\omega t_0) . \quad (17)$$

Here a simple phase relationship of $\phi_2=2\phi_1=2\omega t_0$ has been used since during the time over which the fundamental signal advances 2π in phase, the second harmonic signal advances 4π in phase. Equation (17) shows that the combination of a harmonic wiggler magnet with a laser beam of the same harmonic results in a sinusoidal density modulation at this particular harmonic. Thus if the wiggler magnet and the laser beam contain all the necessary harmonics, a perfect sawtooth modulation can be obtained.

IV. SMALL SIGNAL GAIN ENHANCEMENT

To demonstrate the gain enhancement with the multiharmonic modulation of Eq. (17), we consider the beam-wave interaction in the second wiggler section of an optical klystron configuration. Suppose this second section consists of a single-period wiggler magnet and a monochromatic laser beam both at the fundamental harmonic. An injected electron exchanges energy with the laser field in a manner governed by the energy conservation law. Its energy change calculated can then be used to formulate the net energy change of the beam, $\langle\Delta\gamma\rangle$, obtained by averaging over the electron's phase at the entrance of the first wiggler section. To the first order of the laser field, this leads to

$$\langle\Delta\gamma\rangle\approx-\frac{a_wa_s}{\gamma_0\beta_{z0}}\left[\frac{\omega L}{2c}\right]\left[\frac{\sin(\Delta kL/2)}{\Delta kL/2}\right] \times\sin(\theta_0+\Delta kL/2)f(\omega t_d) , \quad (18)$$

where $f(\omega t_d)$ is the gain function defined as

$$f(\omega t_d)=\frac{\frac{1}{2\pi}\int_0^{2\pi}\sin(\omega t_d+\Delta kL/2)d(\omega t_0)}{\sin(\theta_0+\Delta kL/2)} . \quad (19)$$

At the synchronism condition under which the strongest beam-wave interaction occurs, the above equation can be reduced to

$$f(\omega t_d)=\frac{1}{2\pi}\int_0^{2\pi}\cos\left[\omega t_0+M_1\theta_0\sum_{n=1}^{2}\frac{\sin(n\omega t_0)}{n}\right]d(\omega t_0) .$$

When the density modulation of the electron beam is weak such that $M_1\theta_0\ll1$, both the gain function and the net energy change of the electron beam are very small. Since Eq. (18) is formulated to the first order of the laser field, any strong beam-wave interaction is only possible at higher orders. This is similar to the case of FEL where $\langle\Delta\gamma\rangle$ has to be formulated to the second order of the laser field [6]. Nevertheless when the electron density modulation is significant, $f(\omega t_d)$ becomes finite and a considerable amount of energy exchange occurs at the first order of the laser field. This can be illustrated analytically by the single harmonic modulation of Eq. (8), with which $f(\omega t_d)$ is in fact the first order Bessel function, $J_1(M_1\theta_0)$. Thus if we define the fundamental bunching parameter as $\chi_1=M_1\theta_0$, Eq. (18) becomes

$$\langle\Delta\gamma\rangle=-\frac{a_wa_s}{\gamma_0\beta_{z0}}\frac{\omega L}{c}J_1(\chi_1)\sin(\theta_0) \quad (20)$$

suggesting a strong beam-wave interaction at the first order of the laser field. A similar gain formula was reported in a study on high gain optical klystrons [7].

If the density modulation contains more than one harmonic, however, it becomes difficult to calculate $f(\omega t_d)$ analytically and a numerical integration becomes appropriate. Figure 1 plots the gain function against the fundamental bunching parameter at the synchronism condition. It is shown that with the addition of the second harmonic modulation, the maximum of the gain function and hence the maximum small signal gain are increased by almost 30% from that with the fundamental modulation only (the conventional optical klystron case). If higher harmonic contents are added into Eq. (17), numerical calculation shows further gain enhancement. The greatest gain enhancement is about 75% obtained with a sawtooth modulation. It is worth noting that because of the perfect bunching reached with the sawtooth modulation, the optical klystron gain with this 75%

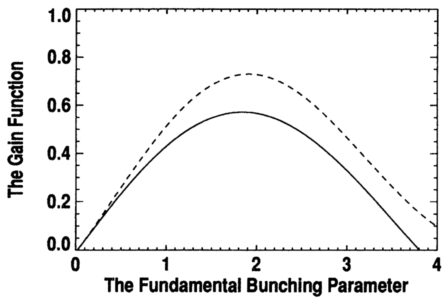

FIG. 1. The gain function curve with the fundamental harmonic modulation (solid line) and with both the fundamental and the second harmonic modulations (dashed line).

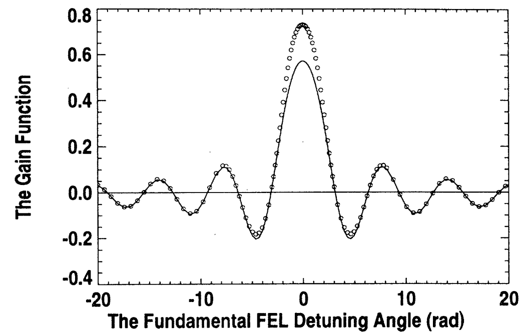

FIG. 2. The comparison of the energy acceptance between a conventional optical klystron (solid line) and a two-harmonic wiggler optical klystron (circles).

enhancement in fact represents an upper limit of the small signal gain attainable in optical klystron devices.

It is also of interest to observe from Fig. 1 that with the fundamental bunching parameter around $\chi_1=1.9$, the maximum gain is reached for both optical klystron configurations. This suggests that the laser field and the wiggler magnet parameters of the fundamental part of the multiharmonic wiggler optical klystron are about the same as that of a conventional optical klystron. Therefore the criterion for designing a conventional optical klystron [7] should be applicable to the design of the fundamental part of a multiharmonic wiggler optical klystron. For the harmonic part however, the parameter choice is determined by the condition of $M_2=0.5M_1$ or

$$2a_{s2}B_{w2}=a_{s1}B_{w1} \ . \tag{21}$$

The numerical integral of $f(\omega t_d)$ indicates that Eq. (21) needs to be satisfied only approximately since the 30% gain enhancement can be maintained within a reasonably wide parameter range around the above condition.

So far, our discussion is restricted to the comparison of the peak interaction gain attainable in the small signal regime. It is also important to compare the gain dependence on the electron energy since this provides an indication of energy spread effect. To this end, we consider a two-harmonic wiggler optical klystron which satisfies

$$\chi_1=1.9\left[\frac{\sin(\Delta kL/2)}{(\Delta kL/2)}\right], \quad \theta_0=\frac{\pi}{2} \ .$$

Of the above two conditions, the first is chosen because it leads to the maximum peak gain at the synchronism condition of $\Delta kL=0$. The second is concerned with the choice of the beam transit angle through the drift section θ_0, which is a slow function of the electron beam energy. For a highly relativistic electron beam however, θ_0 may be approximated as a constant provided the initial energy spread is reasonably small. With the above assumptions, the gain function is plotted in Fig. 2 against $\Delta kL \approx 4N\pi(\gamma-\gamma_0)/\gamma_0$, the fundamental FEL detuning angle, for both types of optical klystron configurations. It is shown in Fig. 2 that the energy acceptance [8] is practically the same for both cases. This seems surprising because the greater interaction gain of conventional optical klystrons is achieved at the expense of their smaller energy acceptance [9]. Thus it appears a more stringent beam quality requirement has to be satisfied before the further enhanced gain of a multiharmonic wiggler optical klystron can be achieved.

To arrive at a fuller interpretation of Fig. 2, we consider the gain enhancement mechanism in a multiharmonic wiggler optical klystron which is basically derived from the fact that there is a sizable amount of unbunched electrons wavering between bunches in a sinusoidally modulated electron beam. By using additional harmonic modulations, those unbunched electrons are swept into bunches. This increases the number of electrons per bunch and hence leads to more electrons contributing constructively to the laser amplification when electron bunches are favorably phased at the entrance of the second wiggler section. The optimum electron phase is that at which an electron enters the second wiggler section to experience the greatest energy loss to the laser field there. An optical klystron system is usually designed such that at the nominal energy the electron beam arrives at the second wiggler section with each bunch centered at the optimum phase in sequence. If the electron energy deviates from the nominal energy however, each electron bunch becomes shifted away from the optimum phase into a less favorable entry phase leading to a gain degradation. As such an energy deviation increases, there arrives a critical electron energy at which the phase of each electron bunch is displaced from the optimum phase onto the zeros of the FEL ponderomotive wave in the second wiggler section. As a result, there is hardly any energy exchange between the electron beam and the laser field and the interaction gain becomes negligibly small. The amount of this critical electron energy depends on the parameters of many system components actually used in the optical klystron, for instance, that of the wiggler magnets, the drift section, and the laser field. However, with the space charge effect neglected it should not be dependent on the number of electrons per bunch that undergo the aforementioned phase displacement. This is because if ballistic electrons are confined into a sufficiently narrow phase bucket of (such as in a bunch) to have an approximately identical phase at the entrance of the second wiggler section, they are to experience the same wiggler and laser fields there as a single electron would had there were only one electron in the phase bucket. Therefore no matter how many electrons per bunch undergo the energy deviation process, the value of the critical electron energy obtained is the same. This is shown in Fig. 2 where the interaction gain vanishes at an identical set of electron energies for both optical klystron configurations which merely differ from each other in the number of electrons per bunch. Of these energies, the two that are closest to the gain peak (satisfying $4\pi N\sigma_\epsilon \approx 3.0$ in Fig. 2 with $\sigma_\epsilon = |\Delta\gamma|/\gamma_0$ being the initial energy spread) are of particular interest since their difference represents the energy acceptance [8]. The physical implication of the very similar energy acceptance for the two different optical klystron configurations in Fig. 2 is that their beam quality requirements should be at least comparable if not the same. On the other hand, it is also worth noting from Fig. 2 that the usual perception that a better beam quality leads to a stronger interaction in conventional optical klystrons is still applicable to multiharmonic wiggler optical klystrons. However, it is not correct to treat the multiharmonic wiggler optical klystron as a mere extension of the conventional optical klystron since this would lead to the fallacy that the beam quality requirement of the former is more stringent than that of the latter.

In practice, the suggestion of Fig. 2 that the optical klystron gain can be increased without inducing a more stringent beam quality requirement is significant. This is not only because it can be used to enhance the beam-wave interaction using the same electron beam of a given quality, but also because it can be used to relax the beam quality requirement to achieve a given amount of small signal gain. The latter possibility is particularly attractive because it allows the generation of short wavelengths using some existing accelerator facilities which would otherwise require substantial upgrading.

V. CONCLUSIONS

The beam bunching problem in optical klystron devices has been analyzed to the first order of the laser field. We have established that the beam-wave interaction is not maximized with the sinusoidal velocity modulation produced in conventional optical klystrons. Instead its maximization may be realized with a sawtooth velocity modulation. To produce such a modulation, a multiharmonic wiggler has been proposed as the first wiggler section of an optical klystron. A small signal analysis has been presented for optical klystron devices of this type. It was shown that with the new arrangement, a significant gain enhancement of up to 75% can be achieved over what is attainable with a conventional optical klystron configuration. More importantly, it was concluded that a higher beam quality requirement is not necessary to guarantee such a greater interaction gain that of conventional optical klystrons.

Generating a sawtooth modulation may involve some demanding practical problems, such as the design and fabrication of a series of wiggler arrays to cover all necessary harmonics. However it should be fairly straightforward to design a magnet system which consists of only two wigglers, one at the fundamental and the other at the second harmonic [1,2]. This version of the multiharmonic wiggler is capable of producing an optical klystron gain enhancement of about 30% without imposing a more stringent requirement for the electron beam quality than a conventional optical klystron. On the other hand, for producing an optical klystron gain of a given amount, this feature can be used to allow for the usage of a lower quality electron beam, an attractive option for short wavelength generation using some existing accelerators which would otherwise require some extensive upgrading.

For further gain enhancement or/and lower beam quality requirement, the problem of producing the necessary higher harmonic wiggler fields may be overcome by using some nonconventional wiggler configurations. For instance, the standard design for permanent wiggler magnets employs four identical squared magnet bars per period to produce a perfect sinusoidal magnetic field on the axis [10]. By altering both the relative width and height of these magnet bars, it should be possible to increase the harmonic contents of the on-axis magnetic field and hence facilitate the possibility of designing a single magnet array that can produce an on-axis wiggler field with many necessary higher harmonics.

[1] M. J. Schmitt and C. J. Elliott, IEEE J. Quantum Electron. **QE-23**, 1552 (1987).

[2] M. Asakawa, K. Mima, K. Imasaki, and C. Yamanaka, Nucl. Instrum. Methods A **318**, 538 (1992).

[3] D. Iracane and P. Bamas, Phys. Rev. Lett. **67**, 3086 (1991).

[4] G. Kong and R. A. Stuart, Nucl. Instrum. Methods A **318**, 518 (1992).

[5] N. A. Vinokurov and A. N. Skrinsky, Novosibirsk Institute of Nuclear Physics Report No. INP 78-88, 1978.

[6] W. B. Colson, Phys. Quantum Electron. **5**, 157 (1978).

[7] R. Bonifacio, R. Corsini, and P. Pierini, Phys. Rev. A **45**, 4091 (1992).

[8] T. I. Smith, J. M. J. Madey, L. R. Elias, and D. A. G. Deacon, J. Appl. Phys. **50**, 4580 (1979).

[9] P. Elleaume, Nucl. Instrum. Methods A **250**, 220 (1986).

[10] G. Brown, K. Halbach, J. Harris, and H. Winick, Nucl. Instrum. Methods A **208**, 65 (1983).