Crisis-induced intermittency in a third-order electrical circuit

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We study the transition from a spiral to a double-scroll attractor in a canonical circuit realization of a third-order electrical circuit (Chua circuit) family as a control parameter p_c (capacitance, C_2) is varied. A sudden widening of the chaotic attractor is observed during this transition, meaning a crisis-induced intermittency. The dependence of the characteristic time τ on the system control parameter has the well-known exponential form $\tau \sim |p - p_c|^{-\gamma}$ for p close to the critical value p_c, the critical exponent γ having a value of 0.5016.

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I. INTRODUCTION

Electric circuits with a nonlinear resistor, which are characterized by a piecewise linear v-i characteristic, have emerged as a simple yet powerful experimental and analytical tool in studying chaotic behavior in nonlinear dynamics.

Among the piecewise linear circuits that have been studied, particularly the members of the Chua circuit family [1] have been investigated in depth. Each member of this family consists of linear resistors, three linear dynamic elements (capacitors and/or inductors), and a nonlinear resistor characterized by a piecewise linear v-i characteristic [2] with at least one segment having a negative slope. Spiral, double scroll, torus, and other interesting attractors among other dynamics phenomena have been observed at different members of this family [3–7].

Recently, Chua and Lin presented a new third-order electrical piecewise linear circuit [8] capable of realizing every member of the Chua circuit family. It is canonical in the sense that (i) it can exhibit all possible phenomena associated with any three-region symmetric piecewise linear continuous vector field, and (ii) it contains the minimum number (six) of circuit elements needed for such a circuit.

In this paper we study the transition from the spiral to the double-scroll attractor in a third-order electrical circuit realization of the Chua circuit family. During this transition there is a sudden widening of the chaotic attractor, a phenomenon called crisis [9]. The characteristic behavior associated with this phenomenon is an intermittent bursting out of the phase space region within which the attractor was confined before the crisis. A time scale τ, the mean time between bursts, is defined in order to quantify the observed post crisis behavior. We

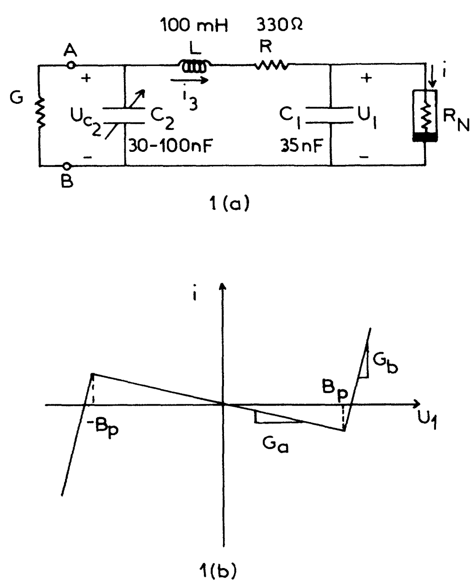

FIG. 1. (a) Third-order electrical (Chua) circuit, (b) i-v_1 characteristic of the nonlinear resistor R_N.

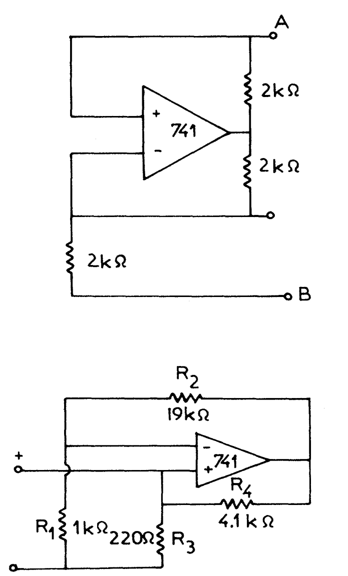

FIG. 2. Laboratory realization of (a) the negative conductance G and (b) the nonlinear resistor R_N.

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examine the dependence of τ on a system control parameter p (the capacitance C_2) as this parameter passes through its crisis value p_c, and we determine the critical exponent γ of this dependence,

$$\tau \cong |p - p_c|^{-\gamma} \tag{1}$$

for p close to p_c [10].

Grebogi *et al.* considered three types of changes that attractors can undergo as a system parameter is varied [10]. The first type leads to the sudden destruction of a chaotic attractor. The second type leads to the sudden widening of a chaotic attractor. In the third type of change two or more chaotic attractors merge to form a single chaotic attractor, which can be extended more in phase space than the union of the attractors before the change. The characteristics temporal behavior, which occurs for the attractor-widening and attractor-merging crisis, is called crisis-induced intermittency [10].

II. EXPERIMENT

The six-element circuit shown in Fig. 1(a) is an electrical realization of the Chua circuit family [11]. Figure 1(b) shows the v-i characteristic of the piecewise linear resistor R_N in Fig. 1(a). Figure 2(a) shows the laboratory realization of the negative conductance with a value of -0.5 mS, while Fig. 2(b) shows the realization of the nonlinear resistor R_N [12].

Figure 3 shows a series of phase portraits of v_2 versus v_1 obtained from the circuit. When we start from $C_2=92.0$ nF, there are no oscillations in the circuit, so the phase portrait is a point, the equilibrium point. At $C_2=91.9$ nF oscillations start with period-1 up to $C_2=81.6$ nF, and we have the limit cycle of Fig. 3(a).

The system follows a bifurcation route as the control parameter C_2 is decreased [Figs. 3(b)-3(d)] until chaotic behavior occurs [Fig. 3(e)]. The spiral attractor of Fig. 3(e) becomes a double-scroll attractor [Fig. 3(f)] at a critical value p_c equal to 60.0 nF. The feature of this transition is the sudden widening of the attractor, which is characteristic of crisis-induced intermittency. As C_2 decreases, the double-scroll attractor expands gradually and eventually becomes a limit cycle at $C_2=40.5$ nF [Fig. 3(h)]. In Fig. 4 time wave forms are shown for different values of the capacitance C_2 very close to the critical value p_c.

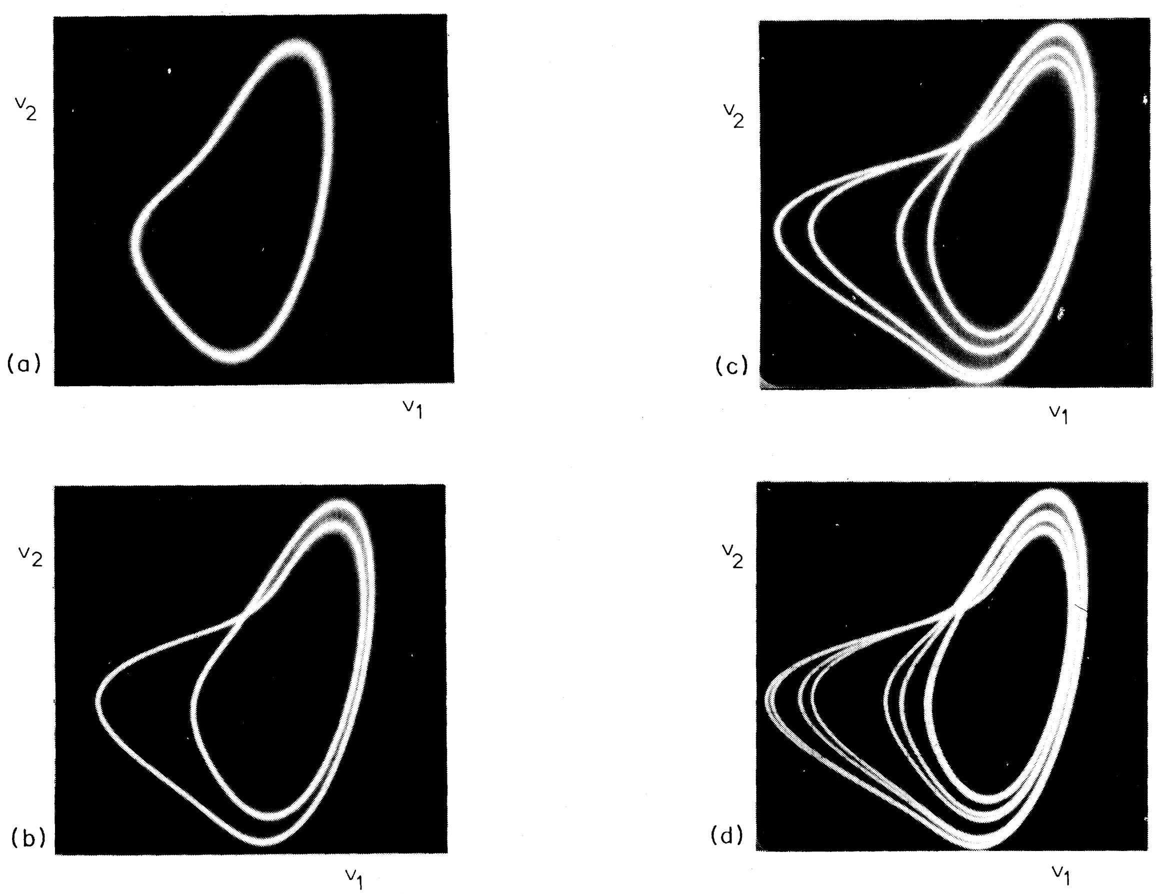

FIG. 3. Experimental phase portraits of $v_2(t)$ vs $v_1(t)$ of the circuits. Horizontal scale (a)–(d), 0.1 V/div; (e)–(h), 0.5 V/div. Vertical scale: (a)–(d), 0.2 V/div; (e)–(g), 1 V/div; (h), 5 V/div. (a) C_2: a period-1 oscillation; (b) C_2: a period-2 oscillation; (c) C_2: a period-4 oscillation; (d) C_2: a period-8 oscillation; (e) C_2: 60.1 nF a spiral attractor just before crisis; (f) C_2: 59.0 nF a double-scroll attractor after crisis occurred; (g) C_2: 44.0 nF the double-scroll attractor is expanded; (h) C_2: 40.5 a period-1 oscillation.

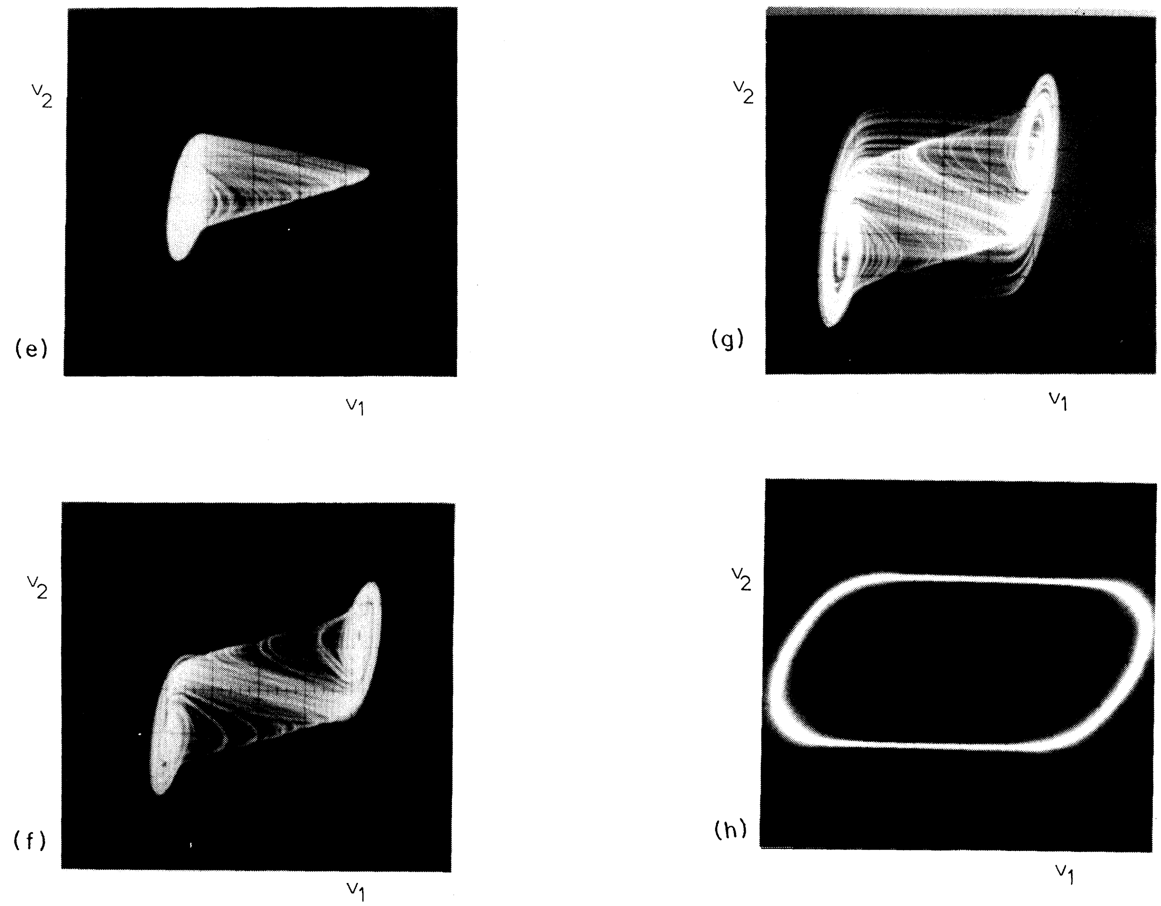

FIG. 3. (*Continued*).

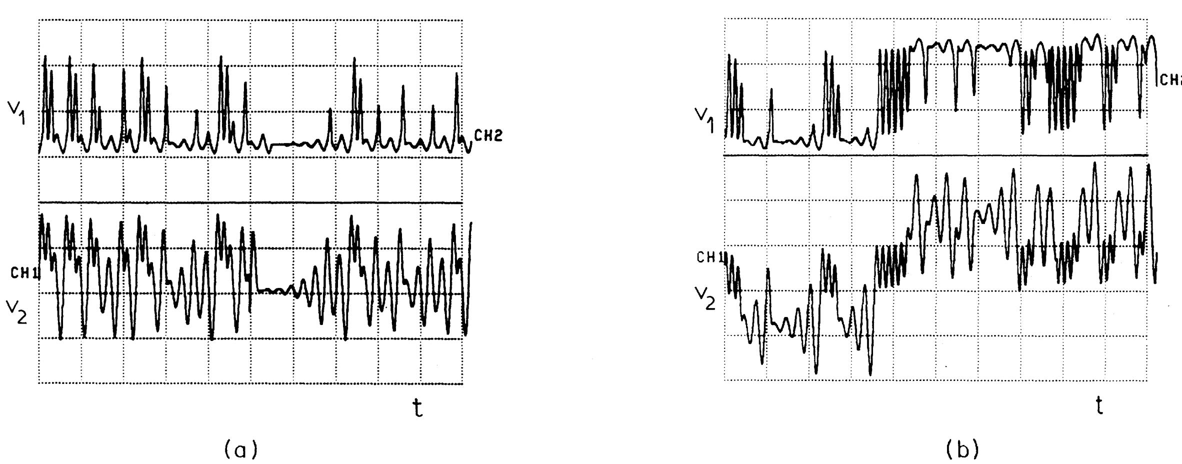

FIG. 4. Wave forms of $v_1(t)$ and $v_2(t)$ of the circuit CH1: $v_2(t)$ (1 V/div); CH2: $v_1(t)$ (1 V/div); time base: 2 ms/div. (a) C_2: 60.1 nF; (b) C_2: 59.2 nF; (c) C_2: 58.0 nF; (d) C_2: 57.5 nF; (e) C_2: 57.0 nF; (f) C_2: 55.0 nF; (g) C_2: 50.0 nF; (h) C_2: 44.0 nF.

III. COMPUTER SIMULATION

The state equations of the circuits in Fig. 1(a) are given by

$$\begin{aligned}
\frac{dv_1}{dt} &= \frac{1}{C_1}[i_3 - g(v_1)] \, , \\
\frac{dv_2}{dt} &= 1 - \frac{1}{C_2}(i_3 + G v_2) \, , \\
\frac{di_3}{dt} &= \frac{1}{L}(v_2 - v_1 - R i_3) \, ,
\end{aligned} \tag{2}$$

where

$$g(v_1) = G_b v_1 + 0.5(G_a - G_b)[|v_1 + B_p| - |v_1 - B_p|]$$

is the v-i characteristic of the nonlinear resistor shown in Fig. 1(b).

We plotted some trajectories for the state equations (2) that come from the ideal circuit in Fig. 1(a) using the normalized parameter values

$$\begin{aligned}
&C_1 = 0.35, \quad G = -0.5, \quad G_a = -0.105, \quad G_b = 4.8 \, , \\
&L_1 = 1, \quad R = 0.3, \quad B_p = 0.68 \, .
\end{aligned} \tag{3}$$

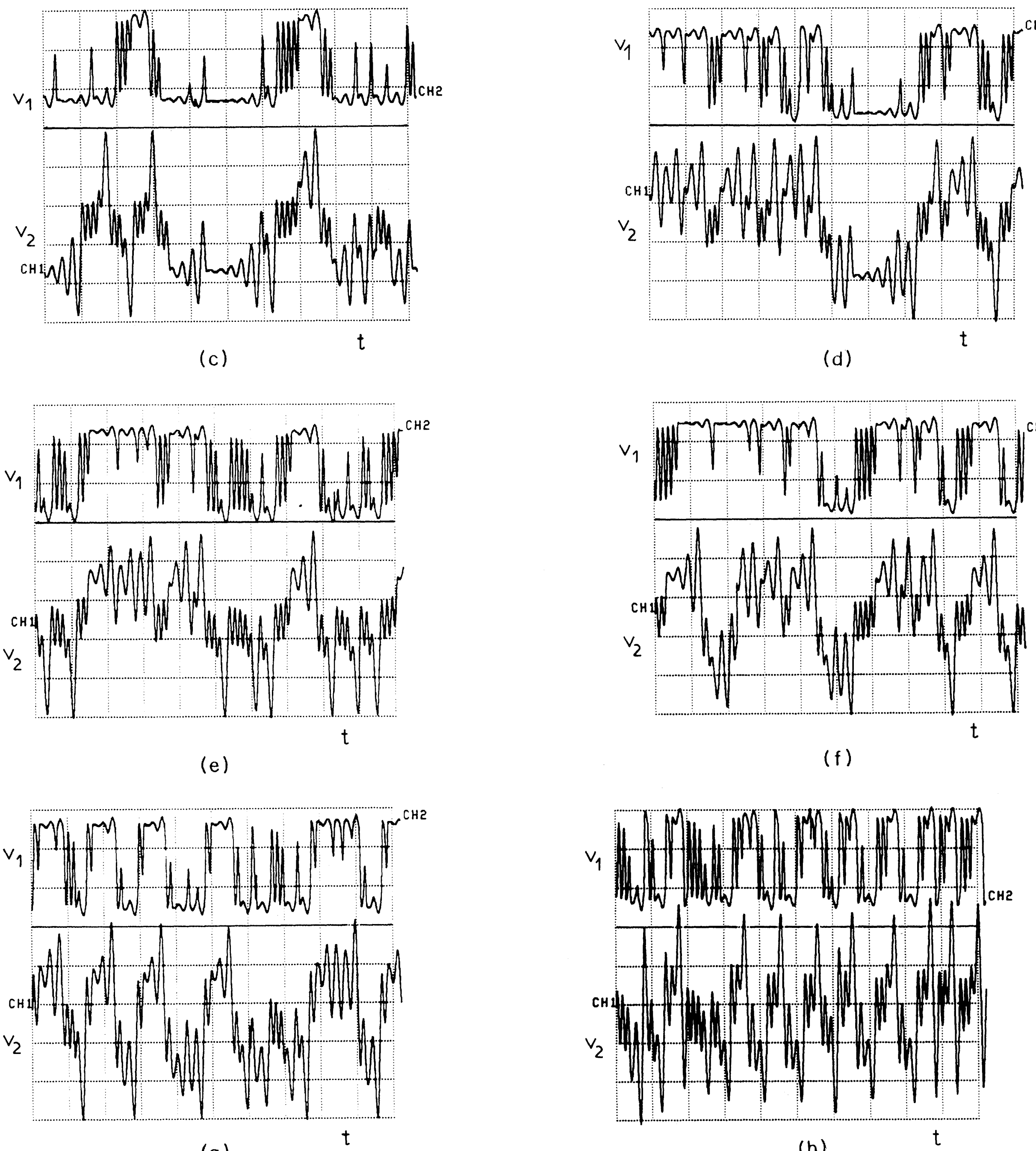

FIG. 4. (*Continued*).

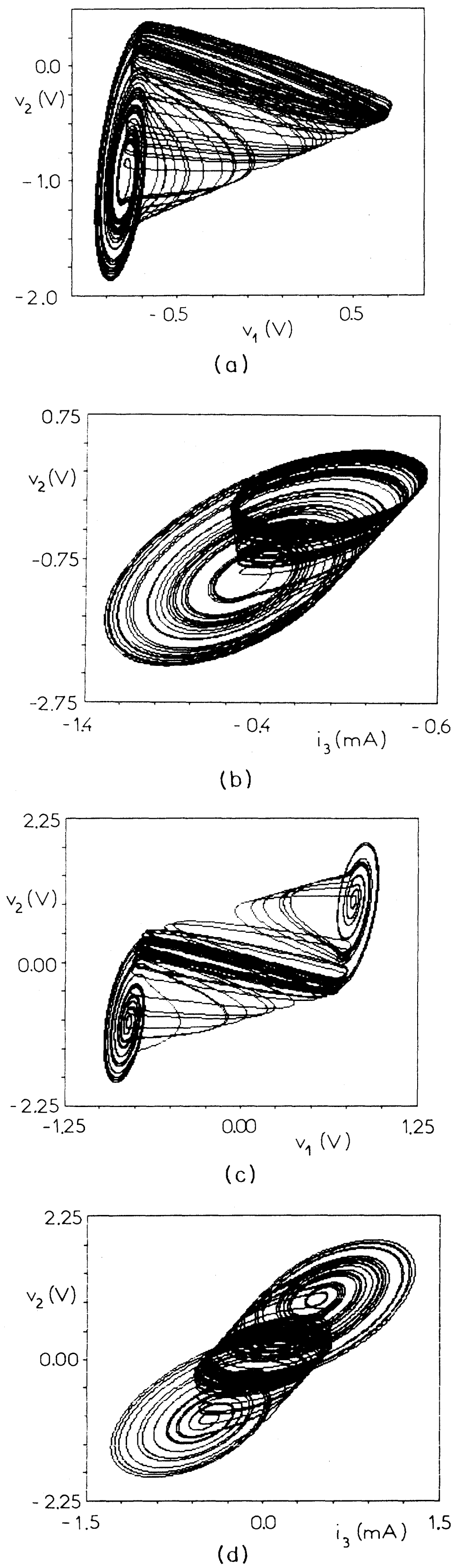

FIG. 5. Computer simulated phase portraits near the critical point. (a) C_2: 61.2 nF: $v_2(t)$ vs $v_1(t)$; (b) C_2: 61.2 nF: $v_2(t)$ vs $i_3(t)$; (c) C_2: 59.9 nF: $v_2(t)$ vs $v_1(t)$; (d) C_2: 59.9 nF: $v_2(t)$ vs $i_3(t)$.

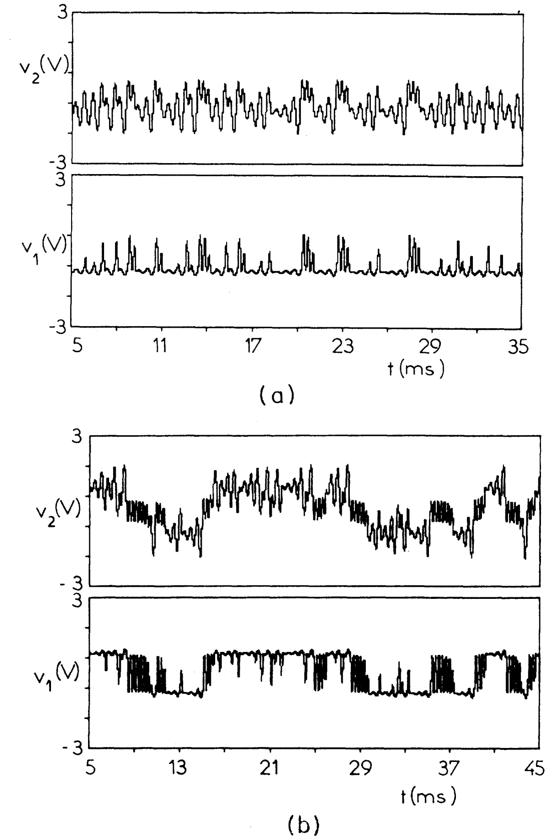

FIG. 6. Computer simulated wave forms of $v_2(t)$ and $v_1(t)$. (a) C_2: 61.2 nF, (b) C_2: 59.9 nF.

C_2 is an adjustable parameter, which varies approximately between 0.35–0.95.

The adopted normalization scale is

$$v_0=1\text{ V},\; i_0=1\text{ mA},\; C_0=0.1\;\mu\text{F},$$
$$L=0.1\text{ H}, R_0=1\text{ k}\Omega\;. \tag{4}$$

Figures 5(a) and 5(c) are the counterparts of Figs. 3(e)–3(f) near the critical value of the control parameter. We observe that each pair of these figures is qualitatively the same. Moreover, the corresponding values of C_2 differ only slightly, due to the tolerance of the circuit elements in the laboratory realization. The corresponding wave forms of v_2 and v_1 are shown in Fig. 6(a) in qualitative agreement with the experimental ones.

IV. ANALYSIS OF EXPERIMENTAL RESULTS

The qualitative behavior of the voltage oscillations, which was described previously, corresponds to a phenomenon referred to as "crisis." The transition from the spiral attractor of Fig. 3(e) to the double-scroll attractor of Fig. 3(f), followed by the sudden widening of the attractor, is characteristic of crisis-induced intermittency.

The qualitative temporal behavior of a system, which undergoes this type of crisis, is characterized by transitions between two chaotic states, a behavior similar to

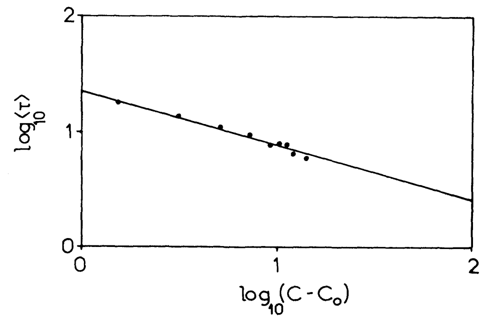

FIG. 7. Plot of the average time $\langle\tau\rangle$ vs $(C-C_0)$.

that of our circuit. The quantitative temporal behavior can be described as follows:

(i) For a smooth distribution of initial conditions, the time τ between successive transitions, for a constant value of the control parameter, is exponentially distributed according to the expression [3]

$$P\langle\tau\rangle=\langle\tau\rangle^{-1}e^{-\tau/\langle\tau\rangle}, \tag{5}$$

where $\langle\tau\rangle$ is the average time between successive transitions.

(ii) For a large class of dynamical systems that exhibit crisis, and in the absence of external noise, the average time $\langle\tau\rangle$ scales with the control parameter p according to the following law:

$$\langle\tau\rangle\sim(p-p_c)^{-\gamma}, \tag{6}$$

where p_c is the crisis value of the control parameter and γ is a critical exponent characteristic of the system $(0.5\leq\gamma\leq1.5)$.

In Fig. 7 a double logarithmic plot of the average time $\langle\tau\rangle$ versus $C-C_0$ is shown in logarithmic scale. C stands for various values of the control parameter C_2 of our system. C_0 is the critical value of C_2, estimated experimentally to have a value of about 60.0 nF. An exponential law obviously holds, resulting in a line whose slope leads to a critical exponent $\gamma=0.5016$, within the theoretically predicted range of values for a strictly dissipative system.

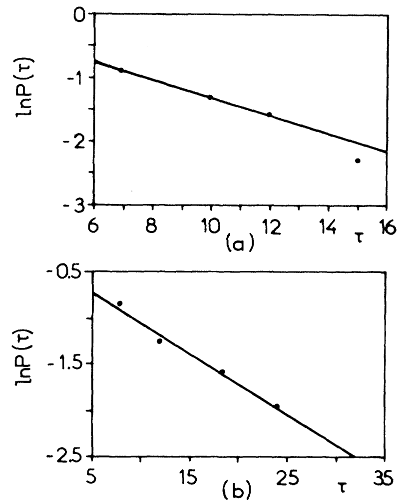

FIG. 8. Distribution $P(T)$ vs τ for two representative values of the control parameter C_2. (a) C_2: 57.0 nF; (b) C_2: 50.0 nF.

In Fig. 8 the distribution $P(\tau)$ versus τ is shown for two representative values of the control parameter C_2. An exponential law can describe our results. The average $\langle\tau\rangle$ values can be calculated from the slope of these curves. We have $\langle\tau\rangle=7.24$ for $C_2=50.0$ nF and $\langle\tau\rangle=15.12$ for $C_2=57.0$ nF. Averaging the directly measured τ values from a number of time series for these values of C_2 we find $\langle\tau\rangle=7.74$ and13.4, respectively, in good agreement with the results calculated from Eq. (5). In conclusion, taking into account the qualitative and quantitative analysis of our observed voltage time series, we deduce that our system undergoes the transition from the spiral to the double-scroll chaotic attractor following the route of a crisis-induced intermittency.

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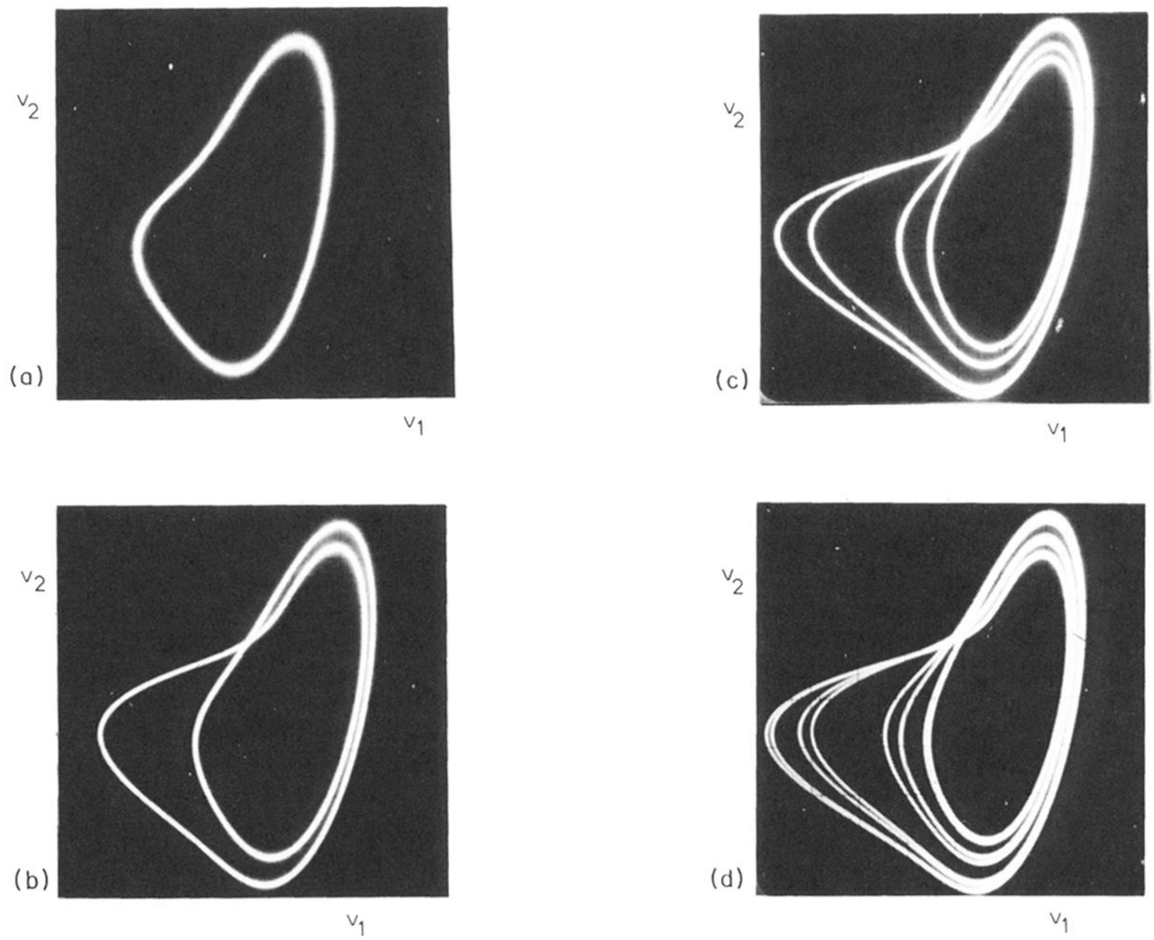

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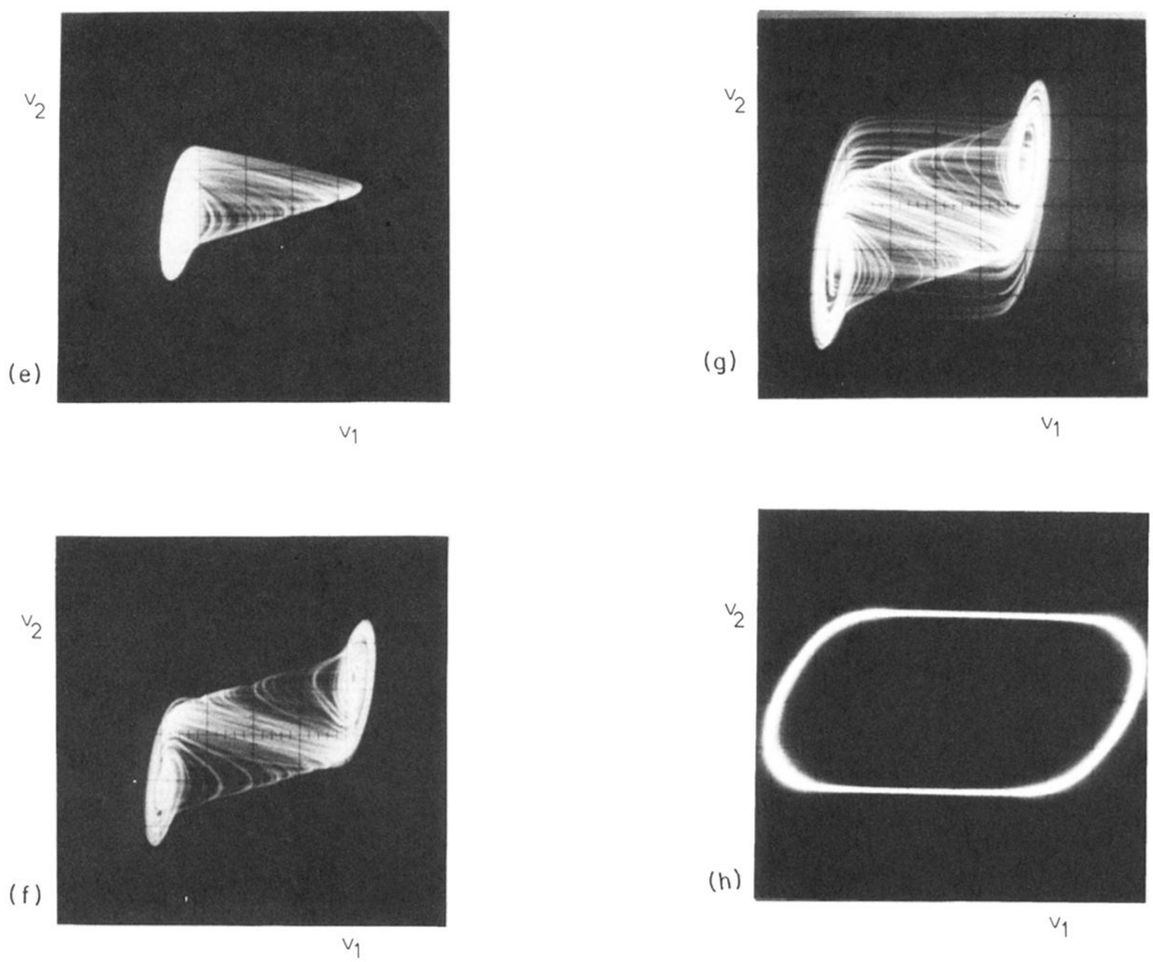

FIG. 3. (*Continued*).