Stochastic model for tunneling processes: The question of superluminal behavior

A. Ranfagni and D. Mugnai
Istituto di Ricerca sulle Onde Elettromagnetiche "Nello Carrara," CNR, Via Panciatichi 64, 50127 Firenze, Italy
(Received 14 February 1995)

A stochastic model for tunneling processes that is based on a close correspondence between quantum relativistic and telegrapher's equations is proposed along the lines of a procedure similar to the one already adopted by Kac [Rocky Mountain J. Math **4**, 497 (1974)]. It is hypothesized that reversals of motion imply the inversion of the (imaginary) time rather than of the space, as in the ordinary allowed processes. In this way, superluminal behavior can be predicted in limiting cases. The plausibility of the superluminal motions observed is then discussed on the basis of a signal analysis along the lines of the Sommerfeld-Brillouin criterion which has been suitably modified.

PACS number(s): 03.40.Kf, 02.50.−r, 84.40.Cb

I. INTRODUCTION

It is well known that the upper limit of a signal velocity is represented by the light speed, as demonstrated for wave propagation by Sommerfeld, since the beginning of the century [1]. Similar conclusions can be drawn also for the propagation of a signal on an electric line [2] and for the motion of a relativistic particle. Such arguments, however, are not clearly applicable to classically forbidden situations such as tunneling processes or evanescent waves. Indeed, in the recent literature, a number of results have been reported for tunneling and/or evanescent waves which demonstrate the obtaining of superluminal behavior [3–5]; but it is still an open question as to whether these results can be considered as referring to a genuine signal velocity or not [6]. As we shall see further on, a crucial point is represented by the fact that any practical signal necessarily has a finite spectral extension.

Dealing with the microwave simulation of tunneling, a theoretical interpretation was modeled on the basis of a path-integral solution of the telegrapher's equation [7], analytically continued to imaginary time [8]. In that framework, it was shown that in tunneling processes the effective velocity turns out to be increased by a dissipationlike parameter and can actually exceed the light velocity. What emerges from such an analysis is the inverted role of the "dissipation" that, in tunneling, acts as an accelerator of the motion [9].

Section II is devoted to establishing a close correspondence between quantum relativistic motion (Klein-Gordon equation) and wave propagation in the presence of dissipation (telegrapher's equation). Following the same procedure adopted by Kac [10] in order to individuate a stochastic model related to the telegrapher's equation, we shall try, in Sec. III, to find a stochastic model related to tunneling processes. A possible interpretation of these facts is given in Sec. IV by analyzing the signal velocity in tunneling simulations according to the Sommerfeld-Brillouin method suitably modified. The results are then discussed in Sec. V.

II. QUANTUM RELATIVISTIC MOTION AND WAVE PROPAGATION

Let us summarize the correspondences between quantum relativistic equations and wave propagation established until now. The analogy between particle motion and electromagnetic wave propagation can be first supported on the basis of a similarity in the dispersion relation and of a close correspondence in the wave equations. When dealing with waveguides, Feynman, Leighton, and Sands [11] made the interesting remark that the dispersion relation for a rectangular waveguide is formally identical to that of a relativistic particle provided that the proper substitutions are made. Specifically, while the wave number k for a TE_{01} mode propagation is given by $k = \sqrt{\omega^2/c^2 - \pi^2/b^2}$ (ω is the angular frequency, c the light velocity, and b the width of the waveguide), the one corresponding to a particle of rest mass m is given by $k = \sqrt{\omega^2/c^2 - m^2c^2/\hbar^2}$ where $\hbar\omega$ is the energy of the particle including the rest energy mc^2. These relations are clearly coincident when the substitution $\pi/b \longleftrightarrow mc/\hbar$ is made.

Subsequently, a relation was established between the quantum relativistic motion and the telegrapher's equation which, if analytically continued, results in the Dirac equation [12]. In the same way it is possible to make a connection between the Klein-Gordon equation, for the motion of a particle in the x coordinate,

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = \frac{\partial^2\psi}{\partial x^2} - \frac{m^2c^2}{\hbar^2}\psi \quad \text{(block I, in Fig. 1)}\ , \tag{1}$$

and the telegrapher's equation

$$\frac{1}{v^2}\frac{\partial^2 F}{\partial t^2} = \frac{\partial^2 F}{\partial x^2} - \frac{2a}{v^2}\frac{\partial F}{\partial t} \quad \text{(block III, in Fig. 1)}\ . \tag{2}$$

Here, F represents the voltage or the current in the case of an electric line and v is its propagation velocity in the x direction; a is a positive constant which accounts for dissipation [13]. To connect Eqs. (1) and (2), we perform a phase transformation on ψ letting, as in Ref. [12],

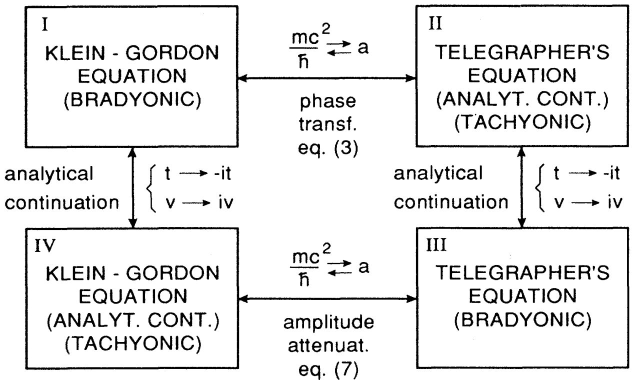

FIG. 1. Block diagram showing the connection between quantum relativistic and telegrapher's equations.

$$\psi(x,t) = u(x,t)\exp\left(-\frac{imc^2t}{\hbar}\right) . \tag{3}$$

By substituting into Eq. (1) we obtain

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{2im}{\hbar}\frac{\partial u}{\partial t} \quad \text{(block II, in Fig. 1)} , \tag{4}$$

which is equivalent to Eq. (2) with the identifications

$$u \longleftrightarrow F , \quad c \longleftrightarrow v , \quad mc^2 \longleftrightarrow i\hbar a .$$

We can also make the connection through an imaginary time variable considering that, in consequence, also $v\,(\equiv dx/dt)$ takes an imaginary factor. Therefore the telegrapher's equation appears to be a suitable tool for studying the propagation of an electromagnetic pulse which can simulate the motion of a relativistic particle [8].

Referring to the scheme of Fig. 1, we see that the above correspondence connects block I (the Klein-Gordon equation) and block III (the telegrapher's equation) through the equation of block II, which can be considered the analytical continuation of the telegrapher's equation, namely,

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{2ia}{c^2}\frac{\partial u}{\partial t} \quad \text{(block II, in Fig. 1)} \tag{5}$$

with the obvious identification $u \equiv F$. Note that Eq. (5) is derived from Eq. (1), through the phase transformation (3), by identifying mc^2 with $\hbar a$. So, starting from the Klein-Gordon equation, by applying phase transformation (3), we obtain the telegrapher's equation and its analytical continuation for imaginary and real values of the energy mc^2, respectively.

It is interesting to note that Eq. (5) is exactly that we considered as capable of explaining delay-time results in tunneling cases [8] where we considered the analytical continuation of a solution of Eq. (2) in order to describe the motion of the beat envelope of two waves of slightly different frequencies. However, it can be shown that such a solution is also the solution of the analytically continued equation, just Eq. (5). We thus arrive at the result that the equation for tunneling motions can be directly derived from the Klein-Gordon equation without analytical continuation, block I $\longrightarrow$ block II, provided that we are considering imaginary time and velocity. Note that, while the Klein-Gordon equation of block I is of the *bradyonic* type, the equation of block II is of the *tachyonic* type, since the resulting motion is not limited to the light speed c, but can exceed this limit [14].

Now, in order to complete the loop scheme of Fig. 1, let us consider the analytical continuation of the Klein-Gordon equation (1). From the changes $t \longrightarrow -it$ and $c \longrightarrow ic$ we immediately have

$$\frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial x^2} + \frac{m^2c^2}{\hbar^2}\psi \quad \text{(block IV, in Fig. 1)} , \tag{6}$$

which is the Klein-Gordon equation for tachyons [15]: in this case, the dispersion relation is given by $k = \sqrt{\omega^2/c^2 + m^2c^2/\hbar^2}$. In this way we obtained the connection between blocks I and IV in Fig. 1.

Since tachyonic properties have been ascribed to electromagnetic evanescent waves [16], there should be a way to connect Eq. (6) to the telegrapher's equation. This can be done by considering the transformation

$$\psi(x,t) = u(x,t)\exp\left(\mp\frac{mc^2t}{\hbar}\right) \tag{7}$$

which is similar to the transformation of Eq. (3). Here, however, the exponential function represents an attenuation of the wave function, and not merely a phase transformation: as we shall see further on, this fact represents an important practical limitation. [The lower sign in the exponent of Eq. (7), as well as in (8), has to be taken when we consider the inverse transformation, namely, from block III to block IV.] Now, by substituting in Eq. (6), we obtain

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \pm \frac{2m}{\hbar}\frac{\partial u}{\partial t} \quad \text{(block III, in Fig. 1)} , \tag{8}$$

which is equivalent to the telegrapher's equation (2) with the identifications

$$u \longleftrightarrow F , \quad c \longleftrightarrow v , \quad mc^2 \longleftrightarrow \mp\hbar a .$$

These are similar to those relative to Eq. (4), apart from the absence of the imaginary unity. The connection between blocks IV and III of Fig. 1 is thus obtained but in order to arrive at the equation for tunneling processes (block II) we have to repeat an analytical continuation ($t \longrightarrow -it$, $v \longrightarrow iv$).

Once the loop of Fig. 1 is completed, we wonder what kind of motion corresponds to the tachyonic equations (blocks II and IV), since the motion relative to the bradyonic ones (blocks I and III) is well known. The latter is sketched in Fig. 2 by a continuous line, and represents the stochastic model first adopted by Kac [10] in order to reobtain the telegrapher's equation (2), as well as a solution of the Klein-Gordon equation in the bradyonic case [17].

We recall that, according to Kac's procedure, the

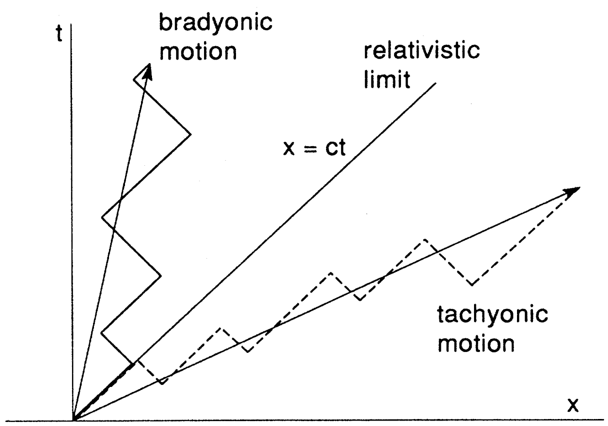

FIG. 2. Trajectory of a stochastic motion, relative to the F^+ function, for a classically allowed case (continuous line) and a forbidden—or tunneling—case (dashed line). In the first case, we have a reversal of the motion in the x space (bradyonic motion), while in the second case the reversal happens in the t (imaginary) time (tachyonic motion).

stochastic motion as depicted in Fig. 2 gives rise to two linear differential equations [Eqs. (14) and (15) in Ref. [10] where a has the dimensions of $(\text{time})^{-1}$]

$$\frac{\partial F}{\partial t} = v\frac{\partial G}{\partial x} , \tag{9a}$$

$$\frac{\partial G}{\partial t} = v\frac{\partial F}{\partial x} - 2aG , \tag{9b}$$

where (see later for the definition of F^+ and F^-)

$$F = \frac{1}{2}(F^+ + F^-) , \quad G = \frac{1}{2}(F^+ - F^-) ,$$

From a further differentiation, we then obtain

$$\frac{1}{v}\frac{\partial^2 F}{\partial t^2} = v\frac{\partial^2 F}{\partial x^2} - \frac{2a}{v}\frac{\partial F}{\partial t}, \tag{10}$$

that is, Eq. (2).

According to the analysis of Ref. [18], we may assume that a description of the tunneling processes can be achieved by inverting x with t, that is, by considering trajectories of the type sketched in Fig. 2 by a dashed line. In this way we will obtain a new pair of differential equations in place of Eqs. (9a) and (9b) and a new equation of motion.

III. DERIVATION OF A NEW EQUATION OF MOTION

Following Kac's procedure, we want to derive the probability that the system has spent a certain time t after a given distance x. As in Ref. [10], we introduce the random variable

$$\epsilon = \begin{cases} 1 & \text{with probability } 1 - \tilde{a}\Delta x \\ -1 & \text{with probability } \tilde{a}\Delta x \end{cases}$$

where $\tilde{a}$ has the dimensions of $(\text{length})^{-1}$. So, with reference to the trajectory represented by the dashed line in Fig. 2, the *effective time*, the time lapsed during the motion, can be expressed as

$$T_n = \frac{\Delta x}{v}(1 + \epsilon_1 + \epsilon_1\epsilon_2 + \cdots) ,$$

[compare with Eq. (16) which, on the contrary, represents the effective displacement in normal processes].

Let us now consider the function

$$\begin{aligned} F_n^+ &= \langle \phi(t + T_n) \rangle \\ &= \left\langle \phi\left[t + \frac{\Delta x}{v} + \frac{\Delta x}{v}\epsilon_1(1 + \epsilon_2 + \epsilon_2\epsilon_3 + \cdots)\right] \right\rangle \end{aligned}$$

which, by introducing the two probabilities associated ϵ_1, becomes

$$\begin{aligned} F_n^+ = {} & \tilde{a}\Delta x \left\langle \phi\left[t + \frac{\Delta x}{v} - \frac{\Delta x}{v}(1 + \epsilon_2 + \epsilon_2\epsilon_3 + \cdots)\right] \right\rangle \\ & + (1 - \tilde{a}\Delta x) \left\langle \phi\left[t + \frac{\Delta x}{v} \right.\right. \\ & \left.\left. + \frac{\Delta x}{v}(1 + \epsilon_2 + \epsilon_2\epsilon_3 + \cdots)\right] \right\rangle . \end{aligned}$$

This can be rewritten as

$$\begin{aligned} F_n^+(t) = {} & \tilde{a}\Delta x F_{n-1}^-\left(t + \frac{\Delta x}{v}\right) \\ & + (1 - \tilde{a}\Delta x)F_{n-1}^+\left(t + \frac{\Delta x}{v}\right) , \end{aligned}$$

and, analogously, we obtain

$$\begin{aligned} F_n^-(t) = {} & \tilde{a}\Delta x F_{n-1}^+\left(t - \frac{\Delta x}{v}\right) \\ & + (1 - \tilde{a}\Delta x)F_{n-1}^-\left(t - \frac{\Delta x}{v}\right) . \end{aligned}$$

The two previous equations can be rewritten as

$$\begin{aligned} & \frac{F_n^+(t) - F_{n-1}^+(t)}{\Delta x} \\ & = \frac{F_{n-1}^+\left(t + \frac{\Delta x}{v}\right) - F_{n-1}^+(t)}{\Delta x} \\ & \quad -\tilde{a}F_{n-1}^+\left(t + \frac{\Delta x}{v}\right) + \tilde{a}F_{n-1}^-\left(t + \frac{\Delta x}{v}\right) \end{aligned}$$

and

$$\begin{aligned} & \frac{F_n^-(t) - F_{n-1}^-(t)}{\Delta x} \\ & = \frac{F_{n-1}^-\left(t - \frac{\Delta x}{v}\right) - F_{n-1}^-(t)}{\Delta x} \\ & \quad +\tilde{a}F_{n-1}^+\left(t - \frac{\Delta x}{v}\right) - \tilde{a}F_{n-1}^-\left(t - \frac{\Delta x}{v}\right) \end{aligned}$$

and, passing to the limit, we get

$$\frac{\partial F^+}{\partial x} = \frac{1}{v}\frac{\partial F^+}{\partial t} - \tilde{a}F^+ + \tilde{a}F^- , \tag{11a}$$

$$\frac{\partial F^-}{\partial x} = -\frac{1}{v}\frac{\partial F^-}{\partial t} + \tilde{a}F^+ - \tilde{a}F^- . \tag{11b}$$

In the present model, as well as in Kac's, the functions F^+ and F^- represent the waves traveling in the positive and negative directions along the x coordinate, respectively, that is, the forward and backward motion. As before, with $F = (F^+ + F^-)/2$ and $G = (F^+ - F^-)/2$, from the sum and the difference of Eqs. (11a) and (11b), we obtain

$$\frac{\partial F}{\partial x} = \frac{1}{v}\frac{\partial G}{\partial t} , \tag{12a}$$

$$\frac{\partial G}{\partial x} = \frac{1}{v}\frac{\partial F}{\partial t} - 2\tilde{a}G . \tag{12b}$$

By a further differentiation, we have

$$\frac{\partial^2 F}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 F}{\partial t^2} - \frac{2\tilde{a}}{v}\frac{\partial G}{\partial t} = \frac{1}{v^2}\frac{\partial^2 F}{\partial t^2} - 2\tilde{a}\frac{\partial F}{\partial x} . \tag{13}$$

Equations (12) and (13) rightfully correspond to Eqs. (9) and (10) once the substitution $t \longleftrightarrow x$ is made. However, Eq. (13) is not equivalent to Eq. (5) [analytically continued from Eq. (2)], as we expected, even if Eq. (13) also represents a tachyonic motion [19].

In order to reobtain Eq. (5), we could operate directly on Eqs. (9a) and (9b) with the transformation $v \longrightarrow iv$ and $t \longrightarrow -it$. In this way, we arrive at the following equation:

$$\frac{1}{v^2}\frac{\partial^2 F}{\partial t^2} = \frac{\partial^2 F}{\partial x^2} + \frac{2ia}{v^2}\frac{\partial F}{\partial t} \qquad \text{(block II, in Fig. 1)} \tag{14}$$

which is the same as Eq. (5), with $u \longleftrightarrow F$ and $c \longleftrightarrow v$. As for the shape of the trajectory, this is not perfectly identified, even if we can argue that, in imaginary time and imaginary velocity, it should be similar to what is schematized in Fig. 2 by a dashed line. In fact, since the product $iv\,(-it) = vt = x$ is real, if we simultaneously change the sign of v and t, their product continues to be real and positive, according to the trajectory represented by the dashed line in Fig. 2.

In order to establish a more direct connection of this kind of trajectory with Eq. (14), we have to consider a generalization of Kac's analysis as follows. Let us consider the following functions [instead of Eqs. (4) and (5) in Ref. [10]]

$$\alpha F_n^+(x) = \langle \phi(x + S_n) \rangle , \tag{15a}$$

$$\beta F_n^-(x) = \langle \phi(x - S_n) \rangle , \tag{15b}$$

where α and β are two coefficients such that $\alpha + \beta = 1$ and S_n is the effective displacement

$$S_n = v\Delta t(1 + \epsilon_1 + \epsilon_1\epsilon_2 + \cdots) . \tag{16}$$

The quantity $\epsilon_i = \pm 1$ is a random variable, and the values ± 1 are associated with the probabilities $1 - a\Delta t$ and $a\Delta t$, respectively [10]. Following the same procedure of Ref. [10] and by passing to the limit, we arrive at the two following relations [instead of Eqs. (11) and (12) in Ref. [10]]

$$\alpha\frac{\partial F^+}{\partial t} = v\alpha\frac{\partial F^+}{\partial x} - a\alpha F^+ + a\beta F^- \tag{17a}$$

$$\beta\frac{\partial F^-}{\partial t} = -v\beta\frac{\partial F^-}{\partial x} + a\alpha F^+ - a\beta F^- . \tag{17b}$$

By adding these equations, we obtain

$$\frac{\partial}{\partial t}(\alpha F^+ + \beta F^-) = v\frac{\partial}{\partial x}(\alpha F^+ - \beta F^-) \tag{18}$$

and by subtracting

$$\frac{\partial}{\partial t}(\alpha F^+ - \beta F^-)$$

$$= v\frac{\partial}{\partial x}(\alpha F^+ + \beta F^-) - 2a(\alpha F^+ - \beta F^-) . \tag{19}$$

Now, by putting $\alpha F^+ + \beta F^- = F_{\alpha,\beta}$ and $\alpha F^+ - \beta F^- = G_{\alpha,\beta}$, we obtain [from Eqs. (18) and (19)] the following equations:

$$\frac{\partial}{\partial t}F_{\alpha,\beta} = v\frac{\partial}{\partial x}G_{\alpha,\beta} , \tag{20a}$$

$$\frac{\partial}{\partial t}G_{\alpha,\beta} = v\frac{\partial}{\partial x}F_{\alpha,\beta} - 2aG_{\alpha,\beta} , \tag{20b}$$

which are formally identical to Eqs. (9a) and (9b). By further differentiation and substitution we obtain a generalized telegrapher's equation

$$\frac{1}{v^2}\frac{\partial^2}{\partial t^2}F_{\alpha,\beta} = \frac{\partial^2}{\partial x^2}F_{\alpha,\beta} - \frac{2a}{v^2}\frac{\partial}{\partial t}F_{\alpha,\beta} , \tag{21}$$

which coincides with Eq. (2) for $\alpha = \beta = 1/2$. In this case the functions $F_{\alpha,\beta}$ and $G_{\alpha,\beta}$ describe standing waves [20].

Now let us reconsider the two first members of Eq. (13)—derived from the special stochastic motion described above—which we rewrite here in a generalized way as

$$\frac{1}{v^2}\frac{\partial^2}{\partial t^2}F_{\alpha,\beta} = \frac{\partial^2}{\partial x^2}F_{\alpha,\beta} + \frac{2\tilde{a}}{v}\frac{\partial}{\partial t}G_{\alpha,\beta} . \tag{22}$$

This equation can be compared with Eq. (21)—derived from the usual stochastic motion—which, in the limit of $\alpha \longrightarrow 1$ and $\beta \longrightarrow 0$ (in this limit we have $F_{\alpha,\beta}, G_{\alpha,\beta} \longrightarrow F^+$), is written as

$$\frac{1}{v^2}\frac{\partial^2}{\partial t^2}F^+ = \frac{\partial^2}{\partial x^2}F^+ - \frac{2a}{v^2}\frac{\partial}{\partial t}F^+ , \tag{23}$$

or its analytical continuation ($t \longrightarrow -it$, $v \longrightarrow iv$)—that is, the equation for tunneling processes—as

$$\frac{1}{v^2}\frac{\partial^2}{\partial t^2}F^+ = \frac{\partial^2}{\partial x^2}F^+ + \frac{2ia}{v^2}\frac{\partial}{\partial t}F^+ . \tag{24}$$

In the same limit ($\alpha \longrightarrow 1$, $\beta \longrightarrow 0$) we rewrite Eq. (22)—for the special stochastic motion—as

$$\frac{1}{v^2}\frac{\partial^2}{\partial t^2}F^+ = \frac{\partial^2}{\partial x^2}F^+ + \frac{2\tilde{a}}{v}\frac{\partial}{\partial t}F^+ . \quad (25)$$

Clearly, Eqs. (24) and (25) are equivalent for $\tilde{a} = ia/v$, which is a way to do the analytical continuation.

In this way, we have established a direct connection between the tachyonic-type trajectory (the dashed line in Fig. 2) and the telegrapher's equation analytically continued in the limit of $\alpha \longrightarrow 1$ and $\beta \longrightarrow 0$, that is, for a pure progressive wave like the one that we have considered for interpreting experimental results of tunneling simulations [8] which, in some cases have revealed superluminal behavior [3–5]. In these experiments, the delay time has been identified with the modulation phase shift, interpreted as the transition or traversal time of the barrier. This procedure, however, could cast some doubts on the applicability of the obtained results to a true signal delay. Just this last aspect of the problem deserves a deeper investigation, since the arguments reported until now in the literature regarding propagation in the presence of evanescent waves are not exhaustive.

IV. THE SIGNAL VELOCITY IN TUNNELING

According to Fox, Kuper, and Lipson [21], the front edge of a tachyon wave packet will never exceed the light speed even though the group velocity is greater than light speed. Analogous conclusions have been drawn also in more recent works [22,23]. As anticipated earlier, it is well established that the upper limit of a signal velocity is represented by the light speed, but it is not fully understood whether these conclusions hold true also for tunneling processes and/or propagation in the presence of evanescent waves [24].

Following Brillouin [25], the propagation of a pulse—like a step function equal to $\exp(-i\omega_i t)$ for $t \geq 0$ and zero for $t < 0$—along the x direction is described by a contour integral in the complex plane of ω as

$$\psi(x,t) = \frac{1}{2\pi}\mathrm{Re}\int_\gamma \frac{\exp[-i(\omega t - kx)]}{\omega - \omega_i}\,d\omega , \quad (26)$$

where γ is a closed path including, or not, the pole at ω_i, depending on the sign of the imaginary part of the exponent. The wave number k for a waveguide is given by $k = (1/c)\sqrt{\omega^2 - \omega_0^2}$, ω_0 being the angular cutoff frequency of the waveguide. For $\omega \longrightarrow \infty$ we have $k \longrightarrow \omega/c$, and the exponential in Eq. (26) becomes $\exp[-i\omega(t - x/c)]$. This implies that integral (26) is zero for $t < x/c$ and that the first forerunner of the signal cannot arrive before a time given by $t_0 = x/c$.

An evaluation of integral (26) can be made using the saddle point approximation and, to this end, we have to determine the saddle points from the stationary condition of the exponent, namely,

$$\frac{x}{t} = \frac{d\omega}{dk} = c\sqrt{1 - \frac{\omega_0^2}{\omega^2}} .$$

This means that the group velocity $v_g = d\omega/dk$ is less than c for $\omega > \omega_0$, is zero at the cutoff for $\omega = \omega_0$, is imaginary below the cutoff for $\omega < \omega_0$ and, for $\omega < \omega_0/\sqrt{2}$, is greater than c in absolute value. For imaginary ω we always have a real group velocity greater than c. The coordinates of the saddle points of the exponential in Eq. (26) are given by

$$\omega_s = \pm\frac{\omega_0}{\sqrt{1 - \frac{1}{t^2}\left(\frac{x}{c}\right)^2}} , \quad (27)$$

which for $t \geq x/c$ are situated on the real axis of ω and the integration of Eq. (26) can be readily performed [25]. This result can be expressed, for $A = t/t_0 \geq 1$, as

$$\psi(x,t) = F(t_0,\omega_0,\omega_i,A)(A^2-1)^{1/4} \times \cos\left(\omega_0 t_0\sqrt{A^2-1} - \pi/4\right) , \quad (28)$$

where the function F is given by

$$F(t_0,\omega_0,\omega_i,A) = \sqrt{\frac{\omega_0}{2\pi t_0}}\frac{\omega_i}{A^2\omega_0^2 + (1-A^2)\omega_i^2} \approx \frac{1}{\sqrt{2\pi\omega_0 t_0}} , \quad (29)$$

where the last term holds for $\omega_i \simeq \omega_0$.

For $t < x/c$, the saddle points (27) are situated on the imaginary axis of ω, and integral (26) is zero, as can be recognized by deforming the original integration path γ on the real axis of ω (from $-\infty$ to $+\infty$) into a semicircle of radius R in the upper plane of complex ω, with $R \longrightarrow \infty$ [25]. If, however, we consider a finite extension in the ω range, the situation changes. In other words, by dividing the range of integration in the following way:

$$\int_{-\infty}^{\infty} d\omega = \int_{-\infty}^{-\omega_1} d\omega + \int_{-\omega_1}^{\omega_1} d\omega + \int_{\omega_1}^{\infty} d\omega = 0 ,$$

the intermediate integral will, in general, be different from zero. In the case of evanescent waves, we rewrite Eq. (26) as

$$\psi(x,t) \simeq \frac{1}{2\pi}\mathrm{Re}\int_{-\omega_1}^{\omega_1} \frac{\exp[-i\omega t - \kappa x)]}{\omega - \omega_i}\,d\omega , \quad (30)$$

where $\kappa = \sqrt{\omega_0^2 - \omega^2}/c$. For physical reasons, the sign of κ is determined in order to have amplitude attenuation with increasing x. In this way, as given by (27), the saddle point is situated on the positive imaginary axis. When $\omega_1 \longrightarrow \infty$, the integrand tends uniformly to zero over a semicircle whose radius R tends to ∞ in the upper plane of ω and the integral—deforming in this way the contour of integration—goes to zero. However, if we limit the range of integration by selecting $\omega_1 \simeq \omega_0$ (the only range in which the evanescent waves are present), the integral is different from zero and an estimate of Eq. (30) can be made again by the saddle point approximation, obtaining [26]

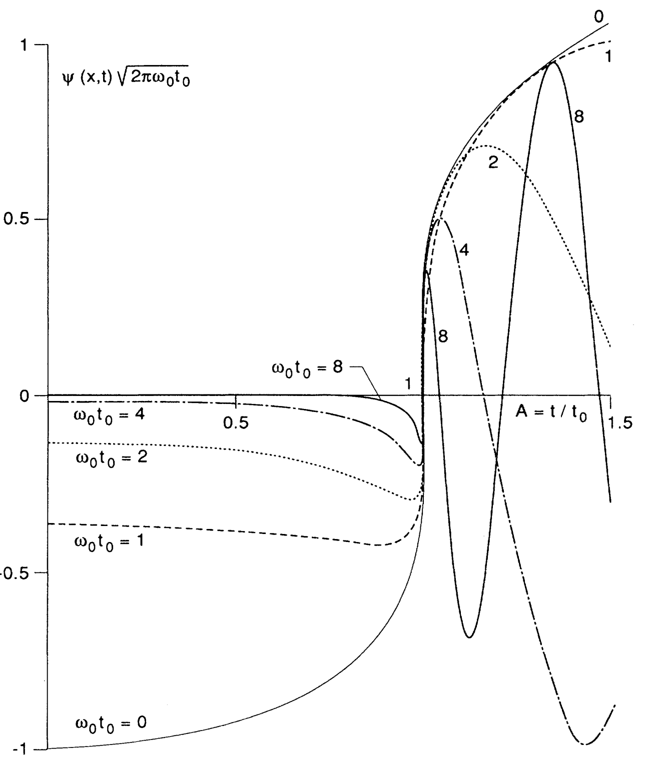

FIG. 3. Behavior of the signal shape computed according to Eqs. (28) and (31) as a function of the normalized time $A = t/t_0$ ($t_0 = x/c$) for ω_i less than but comparable to the cutoff (angular) frequency $\omega_0 = 1$. Note that the contribution for $A \leq 1$ strongly depends on the value $\omega_0 t_0$, that is, on the distance x from the launcher, and tends to disappear when x is of the order of a few wavelengths.

$$\psi(x,t) = -F(t_0,\omega_0,\omega_i,A)(1-A^2)^{1/4} \times \exp\left(-\omega_0 t_0\sqrt{1-A^2}\right) , \quad (31)$$

where now $A = t/t_0 \leq 1$ and F is still given by Eq. (29). This result is shown in Fig. 3, together with the result relative to the case $t \geq x/c$, obtained from Eq. (28). Note that, because of the dependence on $\omega_0 t_0$ $(= 2\pi x/\lambda_0)$ of the exponent in Eq. (31), the contribution for $t/t_0 < 1$ is strongly attenuated by increasing the distance x, so that for sufficiently large distances—say of a few cutoff wavelengths λ_0—the superluminal contribution becomes quite negligible and we again obtain the usual result that nothing arrives before t_0.

V. CONCLUSIONS

We have shown that, by limiting the range of integration in the frequency domain in the case of evanescent waves, we can actually obtain that "something" arrives before the arrival of the usual forerunner, that is, for $0 \leq t \leq x/c$ and for short distances. This contribution rightly tends to zero with an increase in the distance and is zero if the domain of integration is extended from $-\infty$ to $+\infty$, as predicted by Sommerfeld's and Brillouin's analyses. We are aware that a finite spectral extension does not represent a true (front edge) signal which, on the contrary, requires an infinite and continuous spectrum and will obey the usual nonsuperluminal behavior. By limiting the spectral extension, even if the outside spectral components are small, we find that the signal is profoundly modified since the temporal and spatial extension, initially supposed finite, becomes infinite. In this way, the resulting $\psi(x,t)$ does not represent a true signal, even if its profile is traveling with a group velocity $v_g > c$ [21] without contradiction with relativity [27]. In our case (tunneling simulation), the choice of limiting the frequency domain is supported by the fact that we are dealing with evanescent waves, the existence of which is confined by the cutoff frequency, namely, $-\omega_0 \leq \omega \leq \omega_0$. As mentioned before [24], such waves are not properly propagating. However, there is no doubt that, in the experiments to which we are referring [3–5], "something" is propagated even if the spectral width of the signal is well confined in the ω domain of evanescent waves. For an estimate of the order of magnitude of the time scale required for the observation of superluminal effects, the quantity $\omega_0 t_0$ (ranging from 0 to 8 in Fig. 3) can be used. Since the exponent in Eq. (31) has to be of the order of some units so as to have a non-negligible amplitude of ψ, this fact implies that for microwave experiments, with $\omega_0 \approx 10^{10}$ Hz, the resulting time scale is in the range of nanoseconds, precisely in agreement with the experimental results of Refs. [3] and [5]. For a photon tunneling experiment, $\hbar\omega_0$ is of the order of a few eV and the time scale is femtoseconds, in agreement with observations of Ref. [4]. Clearly, this time scale tends to become prohibitive with an increase in the energy. For an experiment with relativistic electrons, for example, we have to consider the quantity $mc^2 t/\hbar$ which, for $mc^2 \approx 0.5$ MeV, gives [according to Eq. (7)] a time scale of the order of 10^{-21} sec. So the theoretical model elaborated previously on the basis of a special stochastic motion (Fig. 2) turns out to be more physically grounded on the signal analysis in the presence of evanescent waves which, for short distances (or short time), demonstrates that superluminal behavior can really be observed and explained.

[1] A. Sommerfeld, in *Wave Propagation and Group Velocity* (Academic Press, New York, 1960), Chap. 2.

[2] G. Doetsch, *Theorie und Anwendung der Laplace Transformation* (Springer, Berlin, 1937).

[3] A. Enders and G. Nimtz, J. Phys. (France) I **2**, 1693 (1992).

[4] A.M. Steinberg, P.G. Kwiat, and R.Y. Chiao, Phys. Rev. Lett. **71**, 708 (1993).

[5] A. Ranfagni, P. Fabeni, G.P. Pazzi, and D. Mugnai, Phys. Rev. E **48**, 1453 (1993).

[6] R. Landauer, Nature **365**, 692 (1993).

[7] C. De Witt-Morette and S.K. Foong, Phys. Rev. Lett. **62**, 2201 (1989).

[8] D. Mugnai, A. Ranfagni, R. Ruggeri, and A. Agresti, Phys. Rev. Lett. **68**, 259 (1992).

[9] This apparent paradox can be explained by considering that when we say "dissipation in tunneling" we are not dealing with a true dissipation—as in the allowed processes—but rather with an imaginary quantity introduced in order to obtain the analytical continuation of the wave equation. A true dissipation, introduced *ad hoc* in a tunneling experiment, seems to produce just the opposite effect, that is, a slowing down of the motion. G. Nimtz, H. Spieker, and H.M. Brodowsky, J. Phys. (France) I **4**, 1379 (1994).

[10] M. Kac, Rocky Mountain J. Math. **4**, 497 (1974), reprinted from Magnolia Petroleum Company and Socony Mobil Oil Company, Colloquium Lectures in the Pure and Applied Sciences, No. 2, October, 1956.

[11] R.P. Feynman, R.B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1977), Vol. 2, pp. 24–27.

[12] B. Gaveau, T. Jacobson, M. Kac, and L.S. Schulman, Phys. Rev. Lett. **53**, 419 (1984).

[13] The function $F(x,t) = \lim_{n\to\infty}(1/2)\langle\phi(x+S_n)\rangle + (1/2)\phi(x-S_n)\rangle$ is the solution of Eq. (2): $F(x,0)=\phi(x)$ is an arbitrary function such that $(\partial F/\partial t)_{t=0}=0$, $\phi(x,t)$ is a solution of the wave equation (2) without dissipation ($a=0$), and S_n [see Eq. (16)] the effective displacement.

[14] We recall that this result refers to the beat envelope of two waves, so that the involved velocity v has to be considered as the beat or group velocity. Whether or not this result is also applicable to a true signal is not immediate, as explained in the text.

[15] D. Shay and K.L. Miller, in *Tachyons, Monopoles, and Related Topics*, edited by E. Recami (North-Holland, Amsterdam, 1978), p. 189.

[16] J. Strnad and A. Kodre, Phys. Lett **51A**, 139 (1975).

[17] K. Takano and K. Yonemitu (unpublished).

[18] D. Mugnai, A. Ranfagni, R. Ruggeri, and A. Agresti, Phys. Rev. E **49**, 1771 (1994); **50**, 790 (1994).

[19] We can verify that, by increasing $\tilde{a}$ from zero to $\tilde{a}^2<1$, the wave propagation velocity (intended as beat velocity) constantly increases.

[20] We wish to note, however, that the procedure adopted here [which allows us to obtain Eq. (21)] is rather formal since $F_{\alpha,\beta}$ and $G_{\alpha,\beta}$ are exactly equal to $\langle\phi(x+S_n)\rangle \pm \langle\phi(x-S_n)\rangle$, respectively, just as in Kac's analysis. In spite of this, when we consider the limiting case of $\alpha\to 1$ and $\beta\to 0$, we can obtain (we assume F^+ and F^- to be in any case finite) Eqs. (23) and (24) for a pure progressive wave. A more convincing approach could be based on the analysis by See Kit Foong [in *Developments in General Relativity. A Jubilee Volume in Honour of Nathan Rosen*, edited by F. Cooperstock *et al.* (Institute of Physics, Bristol, 1990), p. 367] where it is shown that the linear combination

$$F(x,t)=\lim_{n\to\infty}\alpha\langle\phi(x+S_n)\rangle+\beta\langle\phi(x-S_n)\rangle$$

with $\alpha+\beta=1$ and $(\partial F/\partial t)_{t=0}=(\alpha-\beta)(\partial\phi/\partial t)_{t=0}$ is still the solution of Eq. (2). Therefore, in the limit of $\alpha\to 1$ and $\beta\to 0$, we obtain Eq. (23).

[21] R. Fox, C.G. Kuper, and S.G. Lipson, Proc. R. Soc. London Ser. A **316**, 515 (1970).

[22] K. Hass and P. Busch, Phys. Lett. A **185**, 9 (1994). The analysis reported here is not fully convincing even if the conclusions are presumably correct [K. Hass (private communication)].

[23] P. Moretti and A. Agresti (unpublished).

[24] Just speaking of propagation with evanescent waves is improper, but this should be the case in the quoted experiments [3–5].

[25] Ref. [1], Chap. 3.

[26] A similar situation was indeed considered by Brillouin (Ref. [1], p. 83) but the fact that "a result different from zero, i.e., forerunners would exist which propagate with a velocity greater than that of light" was reputed impossible. In our opinion, the case considered in the present work (tunneling simulation) deserves a different consideration.

[27] We wish to note, however, that any practical signal necessarily has a finite spectral extension. The relative delay time, sometime referred to as *technical signal delay*, is not necessarily coincident with the front delay which requires us to consider the limit of $\omega\to\infty$. This is a delicate point since, as is known, infinite frequencies do not exist in physical phenomena. See A. Papoulis, *The Fourier Integral and its Applications* (McGraw-Hill Book Company, New York, 1962), Chap. 7; G. Nimtz, A. Enders, and H. Spieker, J. Phys. (France) I **4**, 565 (1994); W. Heitmann and G. Nimtz, Phys. Lett. A **196**, 154 (1994), where, for frequency band limited microwave experiments, the term *technical information* (rather than signal) is used.