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Two-dimensional localized solutions for subcritical bifurcations in systems with broken rotational symmetry

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We study spatially localized two-dimensional solutions and their interactions for coupled two-dimensional (2D) equations applicable near a weakly inverted bifurcation for isotropic systems with broken rotational symmetry. Even though the linear operator is substantially different from that for the equations for anisotropic media studied previously, stable localized 2D solutions nevertheless exist. In contrast to the 2D localized solutions for anisotropic media, these solutions have a more complex shape and their interactions show a number of interesting features.

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An important discovery within the last several years has been that of stable localized solutions for the quintic complex Ginzburg-Landau (CGL) equation, an equation with both *dissipation* and *dispersion* which describes systems near a subcritical bifurcation to traveling waves [1–4]. Localized solutions of coupled quintic CGL equations have been found to not only interpenetrate with shape and size unchanged, but also to undergo mutual annihilation for stabilizing cross coupling or to spread and fill in upon interaction for destabilizing cross coupling [2,3]. This is in contrast to solitons, which occur in purely *dispersive* systems such as the nonlinear Schrödinger equation and the Korteweg–de Vries equation (describing shallow water waves), and which only exhibit interpenetration upon interaction [5,6]. Since, in most physical systems, dissipation is not some small perturbation which can be neglected, one would expect the type of solutions found in the quintic CGL equation to be even more common in nature than solitons. Experimental systems in which stable localized solutions have been found are binary fluid convection, a dissipative system exhibiting a subcritical oscillatory instability [7,8]; and the catalytic oxidation of carbon monoxide on Pt(110), a dissipative system in which both the interpenetration and mutual annihilation of counterpropagating localized solutions were observed [9]. For the latter system we have recently pointed out that the behavior of the pulses observed in this experiment shares many features with the pulses occurring in the quintic CGL equation [10].

In addition to the one-dimensional (1D) CGL equations, stable localized solutions have also been found in 2D quintic CGL equations applicable to anisotropic media (where A_{xx} in the 1D equation is replaced by $\nabla^2 A$) [1,11]. Interactions of these 2D solutions are similar to those of the 1D solutions, except for the introduction of a new parameter, the impact parameter [11]. The outcome of a collision—interpenetration, mutual annihilation, or filling-in—depends on the value of the impact parameter. For large values of the impact parameter the solutions simply pass by one another.

Recently, a type of localized solution—which has no analog in integrable systems—was discovered for the quintic CGL equation [12]. For this solution the modulus breathes periodically, quasiperiodically, or chaotically about some fixed shape. These breathing solutions are very different from the "breathers" of the sine-Gordon equation which simply oscillate periodically about zero for the real field. Also the chaotic breathing localized solutions are different from the chaotic localized solutions of the quintic CGL equation that slowly spread with time [13] and that were studied to gain insight into the slowly spreading turbulent slugs of pipe flow [14] and turbulent spots of plane channel flow [15], both of which are subcritical in nature. For pipe flow, localized solutions which maintain a fixed shape on average also exist and are referred to as puffs [16].

In addition, we have studied interactions between these breathing localized solutions of the quintic CGL equation [17]. We found that the interaction behavior is much richer than and qualitatively different from that of the fixed-shape solutions. For a finite range of parameter values, the outcome of a collision—interpenetration, mutual annihilation, or survival of only one solution—depends on the initial conditions, even though there is *no* change in the parameter values. For collisions between *chaotic* localized solutions the outcome depends *sensitively* on initial conditions.

Since the primary ingredient for the existence of localized solutions of the type studied here is a subcritical oscillatory instability, one would expect that the exact form of the quintic CGL equation is not crucial. Showing this is important, since equations for physical systems, which have a subcritical oscillatory bifurcation and for which stable localized solutions exist, may not be of the precise form of the quintic CGL equation. In fact, we have recently shown that for a very different type of nonlinearity—a nonlinearity of the saturation type applicable to a dye laser with saturable absorber, as opposed to that of the polynomial type occurring in the quintic CGL equation—stable localized solutions exist in both one and two dimensions [18]. Also, localized solutions have been found in the equation for optical bistability which not only has a nonlinearity of saturation type but also has a constant term, which is related to an external driving field and which causes the phase to be pinned [19].

In this paper we study localized solutions in 2D equations applicable to isotropic systems near a subcritical oscillatory instability for which the rotational symmetry in the plane is broken [20]. These equations, which are a generalization of the Newell-Whitehead-Segel equation for stationary rolls to traveling rolls [21], have a very different linear operator from that of the 2D equations studied in Ref. [11], thus further demonstrating that the precise form of the equations is not essential for the existence of stable localized solutions. However, we do find that the solutions have a more complex shape than those studied in Ref. [11]. Also, we find the magnitude and direction of the velocity depend on the rotational orientation of the solutions. Just as in Ref. [11], we find, for collisions between counterpropagating solutions, interpenetration, mutual annihilation, or filling-in, depending on the cross-coupling and the impact parameter. In addition, we find the behavior that, for collisions with nonzero impact parameter, the interaction can cause the individual states to rotate from their initial orientation. This subsequently results in the solutions moving off in directions different from their velocities prior to the collision.

The equations we study are [20]

$$A_t=\epsilon A+\gamma\left(\partial_x-\frac{i}{2k_c}\partial_{yy}\right)^2 A+i\chi A_{xx}-v_g\left(\partial_x-\frac{i}{2k_c}\partial_{yy}\right)A$$
$$-\beta|A|^2A-\delta|A|^4A-\xi|B|^2A, \qquad \text{(1a)}$$

$$B_t=\epsilon B+\gamma\left(\partial_x+\frac{i}{2k_c}\partial_{yy}\right)^2 B+i\chi B_{xx}+v_g\left(\partial_x+\frac{i}{2k_c}\partial_{yy}\right)B$$
$$-\beta|B|^2B-\delta|B|^4B-\xi|A|^2B. \qquad \text{(1b)}$$

Here A and B are the slowly varying complex amplitudes of rolls traveling in the $+x$ and $-x$ directions, respectively (assuming $v_g>0$), and y is in a direction along the rolls. The parameters γ, β, δ, and ξ are in general complex, i.e., of the form $z=z_r+iz_i$, χ is real, ϵ gives the distance above onset of the instability, v_g is the group velocity, and k_c is the critical wave number for the straight rolls. We take $\epsilon<0$ and $\beta_r<0$ so that the system is subcritical, and take $\delta_r>0$ to guarantee saturation. If $\xi_r>0$ the cross coupling will have a stabilizing effect during interactions. If $\xi_r<0$ it will have a destabilizing effect.

Figure 1 shows the formation of a pulse for Eq. (1a) from an initial Gaussian. The parameter values are given in the figure captions. Periodic boundary conditions are used. As can be seen the structure is quite complex during the formation of the pulse and even the final structure (after all transients have settled down) is fairly complex as compared to the 2D solutions of Ref. [11], which are simply axisymmetric and for which there were no unusual transients.

Figure 2 shows a head-on collision for which interpenetration results. The initial states were prepared by initializing with two Gaussians for A and B, respectively, and allowing the resulting solutions to settle down to their asymptotic states. During the interaction the amplitudes of the two states are seen to be reduced as a result of the stabilizing cross coupling. After the interaction the solutions return to the shapes they had before the interaction. This solitonlike behavior has also been observed in the 1D solutions of Refs. [2,3] and the 2D solutions of Ref. [11].

Figure 3 shows a head-on collision for which mutual annihilation results. In this case the stabilizing cross coupling is larger so that the amplitudes are brought to a sufficiently small amplitude to cause them to continue to decay after the interaction (recall that the system is subcritical). Annihilation, which does not occur for solitons, has also been seen in the 1D solutions of Refs. [2,3] and the 2D solutions of Ref. [11].

Figure 4 shows a collision for a nonzero value of the

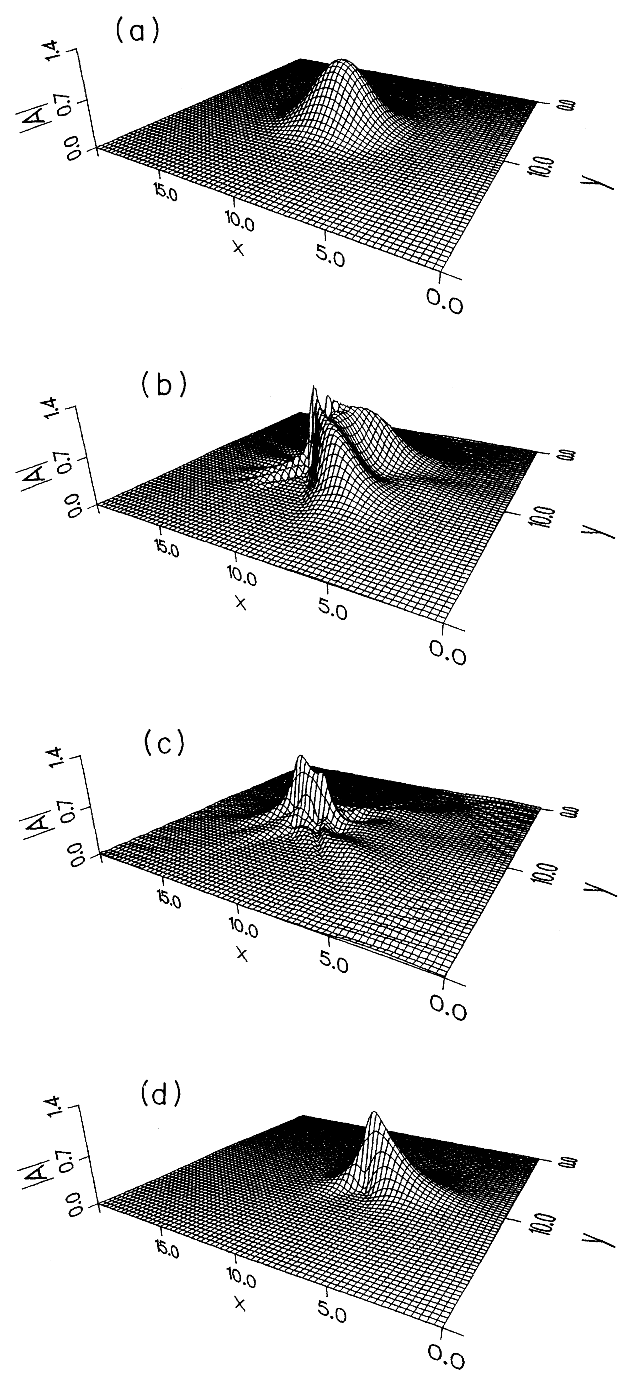

FIG. 1. 3D plots showing the formation of a stable localized solution from an initial Gaussian. The parameter values are $\epsilon=-0.15$, $v_g=0.4$, $\gamma=1+i$, $\chi=0$, $\beta=-3-i$, $\delta=2.75-i$, and $k_c=\pi$. (a) Initial Gaussian ($t=0$). (b) Transient at $t=4$. (c) Transient at $t=12$. (d) Asymptotic state at $t=120$.

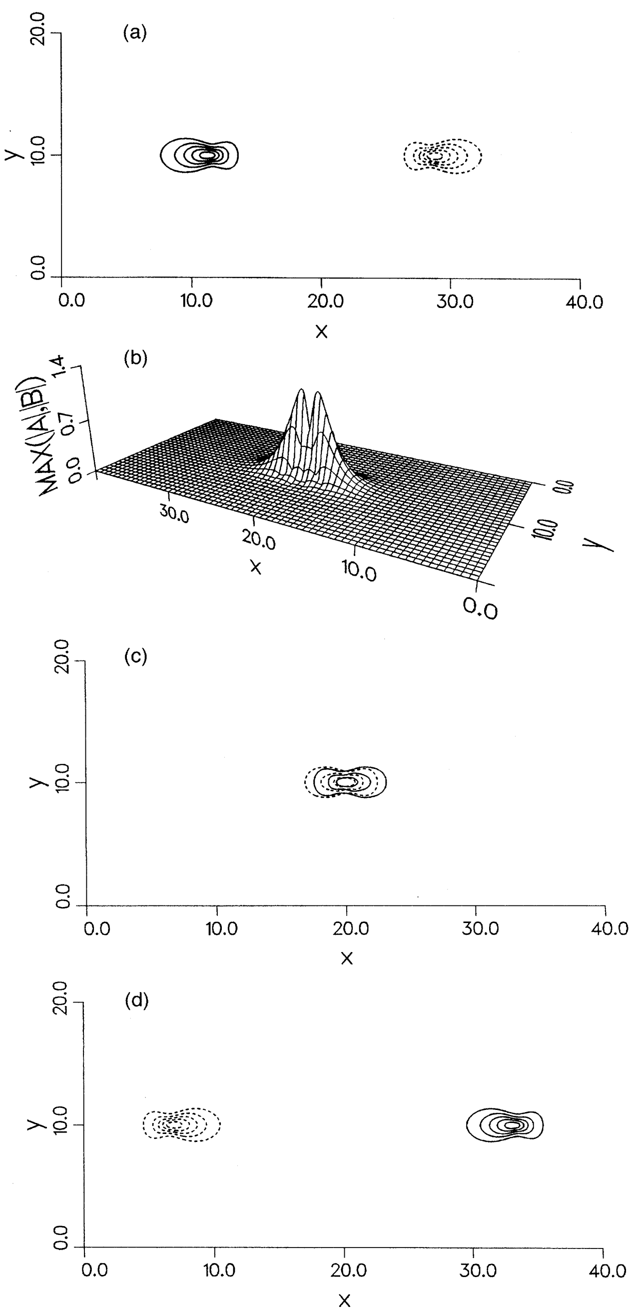

FIG. 2. Head-on collision between two counterpropagating pulses showing interpenetration. The parameter values are the same as in Fig. 1, except for $\xi = 1 + 0i$. (a) Contour plot showing initial state ($t = 0$). (b) 3D plot showing interaction at $t = 12$. (c) Contour plot showing interaction at $t = 16$. (d) Contour plot showing state well after collision at $t = 36$.

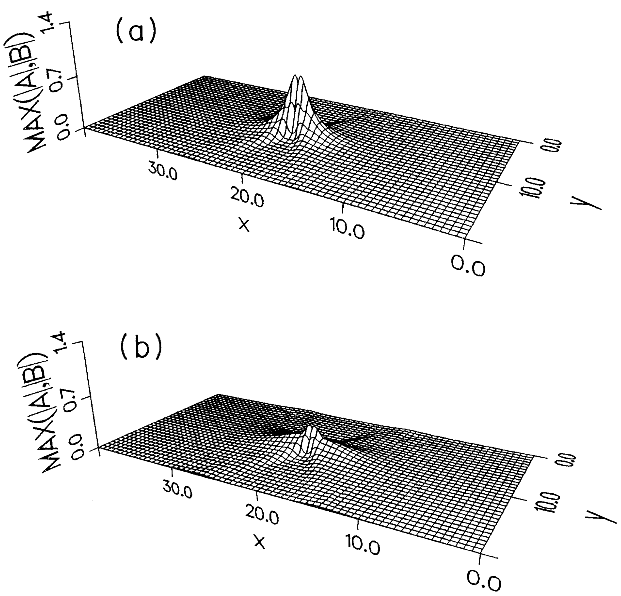

FIG. 3. Head-on collision between two counterpropagating pulses showing mutual annihilation. The parameter values are the same as in Fig. 2, except for $\xi_r = 1.5$. The initial state is the same as in Fig. 2. (a) 3D plot showing interaction at $t = 16$. (b) 3D plot showing decaying remnants at $t = 20$.

impact parameter $b = 0.3125$. As seen in the figure the individual solutions are rotated as a result of the interaction and are no longer symmetric about any line drawn through the solution. The solutions are also seen to drift in the y direction after the interaction. This interaction-induced rotation of the solutions and the subsequent drift in the y direction is very different from that of the solutions of Ref. [11] and is a result of the more complicated structure of the linear operator. For larger impact parameters, the degree of rotation, and therefore the component of velocity in the y direction, is smaller.

For destabilizing cross coupling we have also observed a transition from subcritical to absolutely unstable as a result of the interaction which causes the solutions to spread and eventually fill in the entire box. This behavior has also been observed for the 1D solutions of Refs. [2,3] and the 2D solutions of Ref. [11].

In conclusion, we have studied stable localized 2D solutions and their interactions for equations applicable to traveling rolls for isotropic systems near a subcritical oscillatory bifurcation with broken rotational symmetry. Even though the linear operator is very different from that for the equations applicable to anisotropic media studied previously [11], stable localized solutions nevertheless exist. This demonstrates that the precise form of the equations is not crucial for the existence of stable localized solutions. We have found that the solutions have a more complex shape than those found in the equations for anisotropic media. Also we have found a number of interesting features for collisions between counterpropagating localized solutions. In addition to the usual behavior of interpenetration, mutual annihilation, and filling-in, we have found that, for nonzero impact parameter, the solutions can rotate during the interaction. This causes them to move off in directions different from their velocities prior to the collision. A likely experimental system in which to find localized solutions of the kind studied in this paper is binary fluid convection, which exhibits a subcritical bifurcation to traveling waves. Although 2D localized solutions have been found to exist at most as long transients for this system [22], this may be due to the fact that there are many defects, with rolls oriented in many directions. It would be

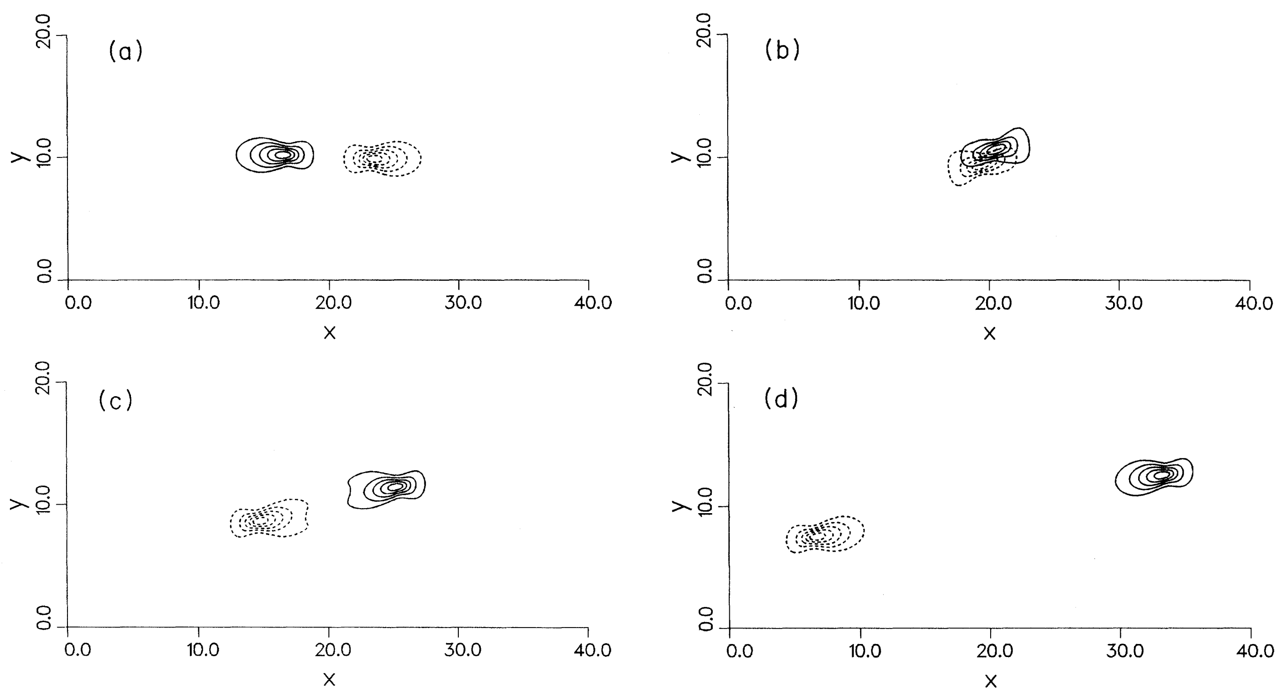

FIG. 4. Contour plots showing a collision for a nonzero impact parameter of $b=0.3125$. The parameter values are the same as in Fig. 3. (a) Approaching solutions at $t=8$. (b) Interaction at $t=16$. (c) Shortly after collision at $t=24$. (d) Well after interaction at $t=36$.

interesting to see if 2D stable localized pulses could be found in a binary fluid system in which the cell size is taken with sufficiently small width so as to stabilize the rolls in one direction, but not so small as to effectively reduce the pulses to 1D pulses, which have already been extensively studied [7,8].

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