PHYSICAL REVIEW E VOLUME 51, NUMBER 6 JUNE 1995

Observation of coherent Smith-Purcell radiation from short-bunched electrons

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(Received 31 March 1995)

Coherent Smith-Purcell radiation, generated by the passage of short-bunched electrons of relativistic energy of 42 MeV above the surface of a metallic grating, has been observed in the wavelength region from 0.5 to 4.0 mm. The intensity of the radiation is proportional to the square of the beam current. The intensity at $\lambda = 2.5$ mm is enhanced by several orders of magnitude compared with the incoherent radiation. It is found that the intensity decreases proportionally to the square of the modified Bessel function $[K_0(2\pi h/\lambda\beta\gamma)]^2$, as the beam height h from the surface of the grating increases.

PACS number(s): 41.60.−m, 41.75.Ht

It was predicted by Frank [1] in 1942 and by Salisbury [2] in 1949 that electromagnetic waves would be emitted by a charged particle moving near a metallic diffraction grating. In 1953, Smith and Purcell [3] performed an experimental study on radiation from the electron-grating interaction. Since then, several experimental investigations [4–12] have been carried out in which spontaneous and stimulated radiation has been generated over a wide spectral range by a low-energy electron beam. Recently, Doucas *et al.* [13] reported the observation of Smith-Purcell radiation (SPR) in the far infrared using the relativistic electrons of energy 3.6 MeV from a Van de Graaff accelerator.

The dispersion relation of the SPR is

$$\lambda = d\left(\frac{1}{\beta} - \cos\theta\right), \tag{1}$$

where λ is the wavelength of the radiation, d is the grating period, β is the ratio of the velocity of electrons to the light velocity in vacuum, and θ is the angle between the electron trajectory and the radiation.

When the electrons are bunched, the radiation from individual electrons in a bunch adds coherently in the spectral region where the wavelength is comparable to or longer than the longitudinal length of the bunch. Recently, coherent synchrotron radiation [14–18] and coherent transition radiation [17,19–21] generated by relativistic electrons in short bunches have been reported. The investigations of this coherent radiation have clarified that the intensity of coherent radiation is proportional to the square of electron beam current and an enhancement of the intensity by a factor N occurs compared with the incoherent radiation, where N is the number of electrons in a bunch. The advantage of coherent radiation is that we can generate intense radiation by a beam of a low average current.

According to a theoretical study by Toraldo di Francia [22], the intensity of the SPR, U, depends exponentially on the beam height h (see the inset of Fig. 1)

$$U \sim \exp\left(-\frac{4\pi h}{\lambda\beta\gamma}\right), \tag{2}$$

where $\gamma = 1/(1-\beta^2)^{1/2}$ is the Lorentz factor.

In the case of a low-energy electron beam ($\beta \ll 1$ and $\gamma \sim 1$), we have to pass the beam extremely close to a surface of the grating, since the intensity U decreases sharply with increasing beam height h. Hence the observation of the dependence of the emission intensity on the beam height is extremely difficult.

The theoretical relation of Eq. (2) shows that, in the case of relativistic electron ($\beta \sim 1$ and $\gamma \gg 1$), the intensity U decreases slowly with the increase of the height h. This dependence is due to the fact that the electric field is increased by a factor γ compared with the static field in the direction perpendicular to the direction of motion. The advantage of using the relativistic electrons is that we can use a beam of large cross section, because of the extension of the interaction range by the factor γ.

In the present work, we report the experimental observation of the coherent SPR generated by short bunches of relativistic electrons. We compare the experiment with the theoretical relation of Eq. (2). The electron bunches were produced by the L-band electron linear accelerator of the Research Reactor Institute, Kyoto University. The accelerat-

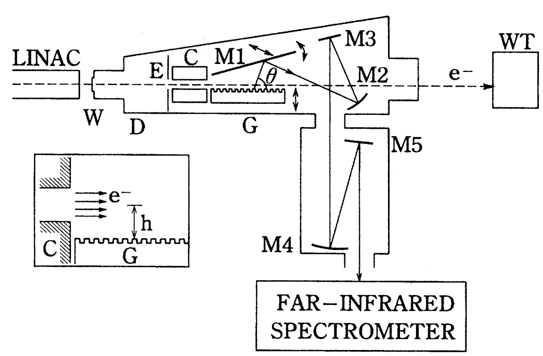

FIG. 1. Schematic diagram of the experimental setup. W: titanium window (thickness of 20 μm); $M1$, $M3$, $M5$: plane mirrors; $M2$, $M4$: spherical mirrors; e^-: 42 MeV electron beam; D: evacuated chamber; E: Ecosorb AN72 (a radio wave absorber); C: beam collimator; G: metallic grating; θ: emission angle; h: beam height; WT: water tank.

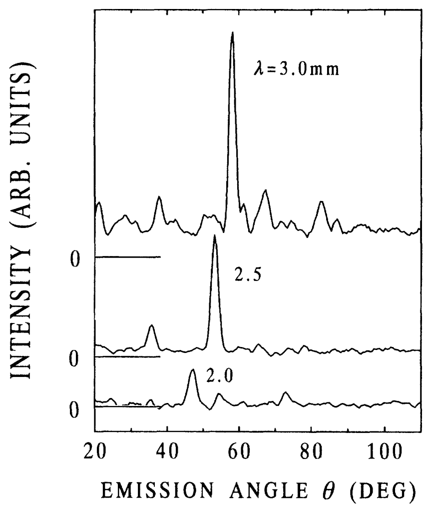

FIG. 2. Dependence of the coherent SPR intensity on the emission angle θ. The intensity was measured with the 42 MeV short-bunched electron beam passing over a 6 mm period grating at the wavelengths 2.0, 2.5, and 3.0 mm.

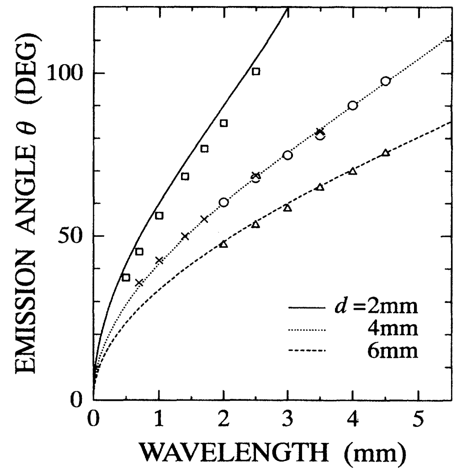

FIG. 3. Dependence of the emission angle of coherent SPR intensity on wavelength for 2, 4, and 6 mm period gratings. Squares, crosses, circles, and triangles show measured values.

ing rf, the energy of electrons, and the energy spread were 1300 MHz, 42 MeV, and 17%, respectively. A duration of a macropulse, or a train of the bunches, was 33 ns and its repetition was 55 pulses/s.

The intensity of the coherent radiation is determined by a bunch shape. The longitudinal bunch shape was observed by a streak camera (Hamamatsu C2909). The bunch length in full width at half maximum (FWHM) was about 56 ps (16.8 mm). The bunch length was estimated also from the observation of coherent transition radiation in the wavelength range from 0.7 to 5.0 mm and the FWHM of about 15 mm was obtained in good agreement with the result of the streak camera. The results indicate that coherent SPR from the bunch should be observable in the longer wavelength region than 0.5 mm [17,20].

A schematic diagram of the experimental setup is shown in Fig. 1. The inset is to show the definition of the beam height h. The electron beam generated by the linac enters an evacuated chamber D through a titanium window W. It passes through the beam collimator C which has a rectangular hole of 10×12 mm^2 in section, 120 mm in length, and passes close to the surface of the metallic grating G. The beam is finally dumped into a water tank, WT.

The beam current was estimated from the charges collected on the water tank. The average current was typically as low as 1.5 μA, and hence the average number N of electrons in a bunch was 4.0×10^9.

SPR emitted in the direction θ was reflected by the mirrors $M1$, $M2$, and $M3$ as shown in Fig. 1. It was collected by a spherical mirror $M4$ which had an acceptance angle of 100 mrad, and was led to the grating-type spectrometer by a plane mirror $M5$. To change the angle θ, the plane mirror $M1$ was slid along a guide and was rotated simultaneously. The optical axis was kept always constant by a mechanism over a range of the angle θ from 20° to 113°. The far-infrared spectrometer with a liquid-He-cooled Si bolometer covered the wavelength from 0.1 to 4.0 mm.

Three gratings with 6, 4, and 2 mm period were used as a radiator. The grating was made of aluminum with a rectangular profile of 1 mm in depth and 30 mm in width. The whole width of the grating along the beam was about 12 cm. The beam height h was variable from 0 to 31 mm.

The angular distribution of the radiation was measured by changing the angle θ. Typical results obtained with the grating of the 6 mm period at wavelengths of 2.0, 2.5, and 3.0 mm are shown in Fig. 2. In the figure, a principal peak and several weak peaks are seen at each wavelength. The angle of the principal peak at each wavelength in Fig. 2 agrees very well with the angle θ calculated from Eq. (1). The experimental results by the three gratings and curves calculated by Eq. (1) are compared in Fig. 3 with satisfactory agreement. There is no doubt that the observed emission is Smith-Purcell radiation. The weak peaks in Fig. 2 are not the higher-order harmonics and they are probably subsidiary diffraction peaks by the grating. When the grating was replaced with a flat aluminum plane, no peak was recorded.

A spectrum at fixed angle (θ) at 70° was measured using the 4 mm period grating of 20 grooves. A sharp peak at a wavelength of 2.68 mm was observed. The FWHM was about 0.21 mm, which was wider than the bandwidth (0.06 mm) of the spectrometer.

An observed dependence of the SPR intensity on the beam current is shown in Fig. 4 for the wavelength of 2.5 mm. The current was varied by changing the grid voltage of the gun pulser of the linac. The solid line shows a quadratic dependence of the SPR intensity on the beam current and it is in good agreement with the observation (circles). The quadratic dependence is clear evidence for coherent radiation from the bunched electrons.

The intensity of the principal peaks in Fig. 2 is of comparable order of magnitude to those of coherent synchrotron radiation [17] and of coherent transition radiation [20] from the electron beam of the same average current. The intensity of the peak at $\lambda=2.5$ mm is 2.7×10^{13} photons/[s(1% bandwidth)]. It is difficult to calculate the intensity of incoherent SPR in the actual situations. Assuming an ideal radiation process of the highest efficiency, the intensity of incoherent SPR is estimated by the theory of di Francia using Eq. (49)

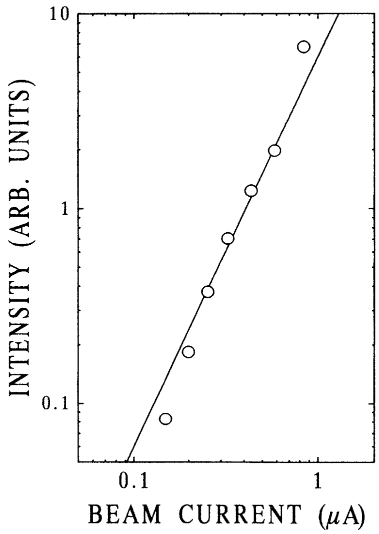

FIG. 4. Dependence of the SPR intensity on the beam current. The solid line represents a square dependence.

of Ref. [22] of 9×10^5 photons/[s(1% bandwidth)]. Compared with this value, the one observed in Fig. 2 is enhanced by a factor 3×10^7. This value of the factor of enhancement is smaller than the number of electrons in the bunch ($N=4.0\times10^9$) and the enhancement is not complete at $\lambda=2.5$ mm because of the longer length of the bunch. Similar results of the enhancement have been observed for synchrotron radiation [17] and transition radiation [20].

The dependence of the intensity of SPR on the beam height was observed at the wavelength of 2.5 mm with the 6 mm period grating. The result is shown by the open circles in Fig. 5, where the beam height stands for the distance between the center of the beam and the grating surface. In the figure, the theoretical relation of the exponential dependence of Eq. (2) is shown by the dashed curve. The observed intensity increases more rapidly than the exponential curve as the beam height decreases. For the beam height h larger than about 20 mm, the theoretical curve is in agreement with the present experiment. The dependence at the smaller value of h, however, is not in agreement with the theory of di Francia.

According to the two dimensional theory of SPR [23], where the source is considered to be a charged wire, the intensity is predicted to decrease exponentially with the beam height as given by Eq. (2). However, applying a three dimensional theory to SPR of the configuration of the present experiment, we can deduce that the relation between the intensity and the beam height is expressed as follows:

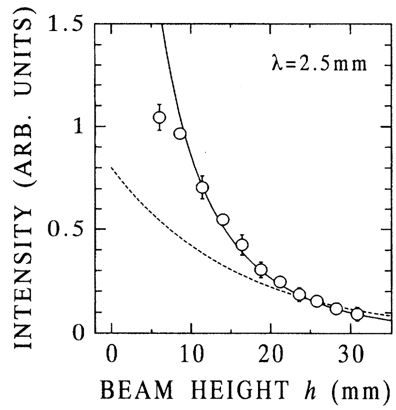

FIG. 5. Dependence of the intensity of coherent SPR on the beam height. Open circles show the dependence observed at the wavelength of 2.5 mm. The solid and dashed curves are the theoretical function of the modified Bessel function of Eq. (3) and of the exponential function of Eq. (2), respectively.

$$U \sim [K_0(2\pi h/\lambda\beta\gamma)]^2, \tag{3}$$

where K_0 is the modified Bessel function of the second kind of the zeroth order. The relation of Eq. (3) is related to the fact that the Fourier component of the electric field parallel to the trajectory of the electron is proportional to $K_0(2\pi h/\lambda\beta\gamma)$ at the surface of the grating [24,25]. The theoretical relation of Eq. (3) is shown by the solid curve in Fig. 5, and is in good agreement with the present experiment. When the argument of K_0 is large, the modified Bessel function is approximated by an exponential function: $K_0(x)\sim\exp(-x)$. Therefore, when the beam height is higher than a certain value, we can hardly distinguish the relation of Eq. (3) from that of Eq. (2).

In conclusion, coherent Smith-Purcell radiation from the bunched electrons of the relativistic energy is intense and monochromatic light in the far infrared. The intensity is enhanced by the coherent effect and also by the relativistic effect. The radiation from the electron beam of the linac is a useful source for spectroscopy. When a Smith-Purcell radiator is inserted into a straight section of a storage ring of a short-bunched relativistic electron beam, we will be able to obtain a useful source in the far-infrared region.

We thank Mr. T. Tsutaya and the staff in the Research Reactor Institute, Kyoto University, for their technical support and assistance. We would also like to express our thanks to Hamamatsu Photonics Co. for the loan of a streak camera (C2909). This work was partially supported by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan.

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