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Intermittent energy bursts and recurrent topological change of a twisting magnetic flux tube

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When continuously twisted, a magnetic flux tube suffers a knot-of-tension instability, like a twisted bundle of rubber bands, and undergoes rapid reconnection with untwisted field lines at the twist-untwist boundary, whereby a giant burstlike energy release takes place. Subsequently, bursts occur intermittently and reconnection advances deeper into the untwisted region. Then the twisted flux tubes, which are reconnected with untwisted field lines, now reconnect with one another to return to the original axisymmetric tube. The process is thus repeatable.

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There are many phenomena in nature that manifest energy release in an intermittent fashion, such as those observed in a geyser and a *shishiodoshi* (a bamboo-made contrivance found in a Japanese garden). The intermittent burstlike energy release can occur in an open system with a steady energy flow if the energy conversion process is nonlinear.

One may recall the propeller of a model airplane which is connected to a bundle of rubber bands. Twisting the rubber bands induces a knot-of-tension instability. Additional knots are created as one continues to rotate the propeller. The creation of a knotted structure lowers the energy of tension. In this sense, the knot-of-tension instability is a good example of an intermittent energy relaxation process.

Magnetic reconnection is believed to be an important, fundamental process that causes a burstlike energy release. Magnetospheric substorms and solar flares are typical examples of conditions considered to be caused by magnetic reconnection. In particular, the driven reconnection process is the most viable process that effectively and swiftly converts magnetic energy into kinetic and thermal energies [1]. Magnetic field lines are easily deformed in accordance with a plasma motion in a collisionless plasma. At a stagnation or a converging point (or line) of the plasma flow, it often happens that field lines with antiparallel components approach one another and an electric current is locally enhanced. Magnetic reconnection, collisional [1] or collisionless [2], is then forced to take place there. This forced reconnection is called "driven" magnetic reconnection.

The photospheric vortex motion is believed to be a primary energy source of a variety of solar phenomena, e.g., the solar dynamo, the coronal heating, and the solar flare. Several authors have attempted to demonstrate the energy conversion process subject to a shear or rotating motion of the footpoint of a magnetic flux tube [3–7].

From the analogy of the knot-of-tension instability of rubber bands, we expect that a repeated, stepwise energy relaxation must take place when a magnetic flux tube is kept twisting. In the case of the magnetic flux, however, it is conceivable that magnetic field lines are reconnected and experience a topological change, which does not happen for the rubber bands except for a sudden break, which never happens for the field lines. In the present paper, we elucidate the energy relaxation and topology change processes of a twisting magnetic flux tube and attempt to reveal a universal self-organizing feature in an open local system in contact with a steady external energy source.

We employ a methodology of a magnetohydrodynamic simulation for a rectangular cylinder model surrounded by a conducting wall in which straight axial field lines are uniformly embedded. Cartesian coordinates (x,y,z) are adopted where the axial boundaries are placed at $z=0$ and $z=L$ and the conducting side boundaries are placed at $x=\pm 1/2L$ and $y=\pm 1/2L$. The simulation box is implemented on a $130\times130\times130$ grid point. At the axial boundaries we impose a circular motion to twist the axial magnetic field.

The axial boundary conditions for the magnetic field **B** and the convection flow **V** are specified as follows.

(1) A prescribed circular motion is applied at $z=0$ and $z=L$ so as to twist continuously and steadily a magnetic flux tube in a circular region of the radius of $0.15L$ centered at the central axis ($x=0$ and $y=0$). The twist profile is described by a cubic equation for a distance r from the central axis which vanishes at the center ($x=0$ and $y=0$) and at the radius of $0.15L$, and is maximized at the radius of $0.1L$. The axial flow component is assumed to be zero at $z=0$ and $z=L$ so that no material transport is allowed through the axial boundaries.

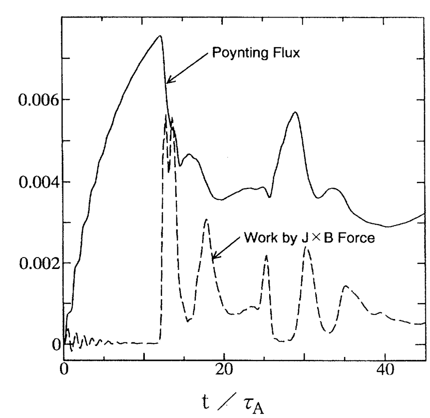

FIG. 1. Time evolutions of the Poynting flux through two axial boundaries and the work done by the **J**×**B** force to accelerate the plasma.

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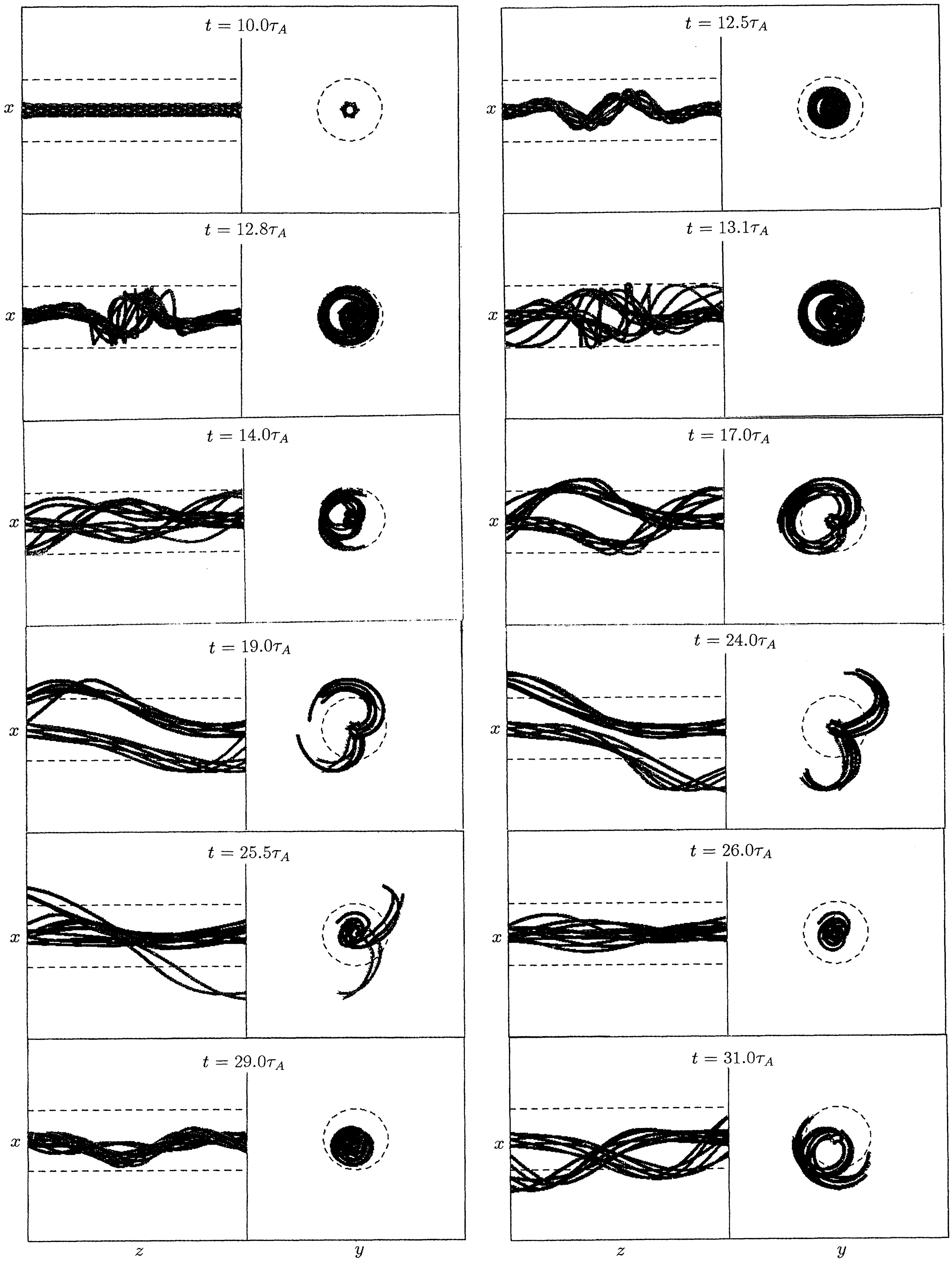
$t = 10.0\tau_A$
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$t = 29.0\tau_A$
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x
z
y

(2) The line-tying condition is imposed on the magnetic field at the axial boundaries, namely, $\partial\mathbf{B}/\partial t=\nabla\times(\mathbf{V}\times\mathbf{B})$ is solved at $z=0$ and L. Therefore, B_z is constant and B_x and B_y are determined by solving the induction equation.

The plasma density is assumed to be uniform and constant, ρ_0, and the thermal energy converted from the magnetic and flow energies is expected to be immediately released by radiation. The resistivity and viscosity, η and μ, are assumed to be uniform and constant. All the physical quantities are normalized by the initial axial magnetic field intensity B_0, the constant density ρ_0, and the axial system length L. For example, the energy is normalized by $B_0^2/2\mu_0$ (μ_0 is the permeability), the velocity by $V_A=B_0/\sqrt{2\mu_0\rho_0}$, the time by $\tau_A=L/V_A$. The normalized resistivity, $\eta\tau_A/L^2$, is taken to be 10^{-4} and the normalized viscosity $\nu\tau_A/L^2$ to be 10^{-3}.

Since spatial modes do not develop without any spatial disturbance in the system, we initially impose a random noise with a negligibly small amplitude on the initial uniform configuration. Then we start twisting the magnetic field. The magnitude of the maximum speed V_0 is given as $V_0=0.1V_A$.

The twisted signal propagates axially from both ends (boundaries) as an Alfvén wave and bounces at each end. A half-bounce period of the signal is one Alfvén transit time, $1\tau_A$. An Alfvén wave carries an axial (field-aligned) current, whereby a magnetic field line is helically twisted. Since a constant twisting force (electric field) is continuously supplied at the axial boundaries, the energy supplier acts as a voltage generator. If the energy is not released, the current will increase almost indefinitely and hence the magnetic field would be indefinitely wound up because of a tiny resistivity. This anticipates that an energy releasing process must happen eventually.

The time evolution of the volume integrated rate of conversion of the magnetic energy to the kinetic energy (work done by the ampere force) is shown by the dashed line of Fig. 1. One finds that at around $t=13\tau_A$ the power conversion rate exhibits a burstlike increase. Furthermore, the burst is not a single shot, but repeats quasiperiodically with intervals of $t=5-7\tau_A$. Such a behavior is never observed in Refs. [3–5]. The burst lasts only $1-2\tau_A$. In order to show the energy conversion process, the time evolution of the total Poynting flux through the axial boundaries is simultaneously plotted by the solid line in Fig. 1.

It should be noted that the most drastic energy release takes place in association with the first burst. The energy releases associated with the second ($t\simeq 18\tau_A$) and third ($t\simeq 26\tau_A$) bursts are not so vigorous as the first one. However, the energy release associated with the fourth ($t\simeq 30\tau_A$) burst becomes again vigorous and this is rather similar to the first one. The reason will become clear later on.

We shall now turn our attention to the evolution of the magnetic field configuration. To see the topological change in detail, we present color plots of 12 representative field lines that originate from six equally spaced points located on a central circle with radius $r=0.01L$ at $z=0$ (red) and also at $z=L$ (green). We first illustrate in Fig. 2 the time sequential plots of the 12 field lines (6 red and 6 green) projected on the x-y plane (right-hand panels) and on the x-z plane (left-hand panels). As seen in Fig. 2(a), the field lines are initially almost uniformly twisted and pinched except in the vicinity of the twisting boundary of $z=0$ and L. At about $t=12\tau_A$, they experience a large kink distortion, particularly in the middle part of the tube, just like a knot-of-tension instability of a bundle of twisted rubber bands; see Fig. 2(b) ($t=12.5\tau_A$). Incidentally, the instability starts at about $t=7\tau_A$ when the winding angle of a field line at the axis ($r=0$) exceeds 4.9π which is in good agreement with the linear analysis of Mikic *et al.* [3]. The distortion is drastically accelerated in a very short period ($t=12.8\tau_A$). On looking at the upper panels, one notices the very important fact that while at $t=12.8\tau_A$ the field lines starting from both boundaries of $z=0$ and $z=L$ are terminated on the same circles at the boundaries of $z=L$ and $z=L$, the terminating points start deviating from the original circle as time elapses [see the top panel of Fig. 2(b) at $t=13.1\tau_A$]. On comparing the side views of $t=12.8\tau_A$ and $t=13.1\tau_A$ while keeping this fact in mind, one notices an important difference, namely, at $t=13.1\tau_A$ the red twisted lines in the first half of the tube ($z=0$–$L/2$) remain almost unchanged, but the lines in the second half ($z=L/2$–L) manifest a drastic change, and vice versa for the green lines. This confirms that the reconnection process must have happened around $z=0.5L$ and $r=0.15L$ (border between twisted and untwisted regions).

Second, we shall observe the topological change of the magnetic field in the time range of $t=(13–25)\tau_A$. By comparing the top panel of Fig. 2(b) ($t=13.1\tau_A$) and the second one of Fig. 2(b) ($t=14.0\tau_A$), one readily finds that the highly distorted structure at $t=13.1\tau_A$ has relaxed to one pair of rather uniformly separated flux tubes at $t=14.0\tau_A$. An important feature is that toward the end of the first burst [$t=(13–14)\tau_A$] the distorted field lines reconnect quickly near $z=1/2$ with neighboring distorted field lines (see the latter part of the two minor peaks appearing in the first giant peak of Fig. 1). The reconnection proceeds until they reconnect with the untwisted field lines lying at the border between the twisted region and the untwisted region, namely, $r=0.15L$ ($t=14.0\tau_A$). At this time the first giant energy relaxation process ceases. The establishment of a pair of clearly separated flux tubes indicates that this structure must be a local minimum energy state.

As the twisting progresses and an excess free magnetic energy is deposited further in the system, another kink instability arises, as is seen in the last panel of Fig. 2(b) ($t=17.0\tau_A$). As the instability progresses, two distorted flux

FIG. 2. Stereoscopic color plots of the distortion and topological change of the twisted field lines due to a knot-of-tension instability. The dashed line indicates the boundary of the twisted region, which is located at $r=0.15L$. The six red lines are the field lines starting at the $z=0$ boundary and the green ones are those at the $z=L$ boundary. The field lines are seen to be rather uniformly twisted and pinched initially ($t=10.0\tau_A$). Then the knot structure grows rapidly ($t=12.5\tau_A$ and $t=13.1\tau_A$). The distorted twisted field lines suddenly make reconnection with untwisted field lines anchored at the twist-untwist boundary toward the end of the first burst at $t=(13–15)\tau_A$ (cf. Fig. 1). A second rapid reconnection penetrates into a more distant untwisted region in the second burst [$t=(19–21)\tau_A$]. In the third burst [$t=(24–26)\tau_A$] the separated companion pair of reconnected twisted-untwisted field lines (red and green) reconnect with each other ($t=25.5\tau_A$). At the end of the fourth burst, the field line structure returns to a more or less original straight structure ($t=26.0\tau_A$). The straight flux tube of twisted field lines again repeats intermittent bursts similar to that seen in previous panels.

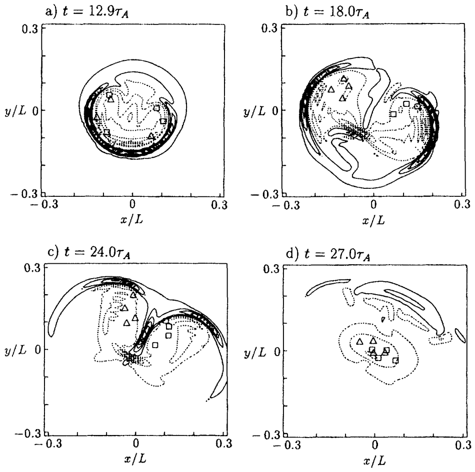

FIG. 3. Contours of the axial current at $z{=}L/2$. Current singularities formed at (a) $t{=}12.9\tau_A$ (first burst), (b) $t{=}18.0\tau_A$ (second burst), and (c) $t{=}24.0\tau_A$ (recurring burst). The last panel (d) shows the contours at $t{=}27.0\tau_A$ when the system returned to an axisymmetric state. The triangles are the intersecting positions of the four field lines starting from $z{=}0$ and $r{=}0.035L$ and the squares are those of the four field lines starting from $z{=}L$ and $r{=}0.035L$.

tubes start reconnecting with untwisted field lines farther away, as is clearly observed in the top view of Fig. 2(c) ($t{=}19.0\tau_A$), and the system relaxes to another local minimum energy state. This corresponds to the second burst seen in Fig. 1 [$t{=}(16{-}19)\tau_A$].

We shall go on to the topological evolution during the third burst [$t{=}(24{-}26)\tau_A$]. The largely separated two fluxes (red and green ones) approach each other at around $z{=}L/2$ ($t{=}24.0\tau_A$) and reconnect with each other ($t{=}25.5\tau_A$) to return to the original one flux tube, as seen in the top panel of Fig. 2(d) ($t{=}26.0\tau_A$).

From these sequential events, we can conclude that when a flux tube is continuously twisted, a knot-of-tension instability, or a localized kink instability, arises and reconnection progresses among the distorted field lines until the twisted field lines execute reconnection with field lines in an untwisted region. During this burst event, magnetic energy stored in the form of tension (twisted) energy is swiftly released, mostly, as the thermal energy. As the twisting continues further, another instability arises and reconnection advances to a more distant outer region. Then, the third instability stimulates reconnection among the companion field lines whereby the magnetic field configuration returns to the original axisymmetric one. The energy releases associated with the second and third instabilities are not so vigorous as the first one.

The above fact that a twisting flux tube returns to the original state motivates us to conclude that as a bundle of magnetic field lines is kept twisting, an energy relaxation and a magnetic topology change occur intermittently and exhibit recurrence.

The energy buildup and abrupt drop of the Poynting flux around $t{=}30\tau_A$ of Fig. 1 manifest a recurrent energy release. Figure 2(d) illustrates the topological evolution of the magnetic field lines during the fourth burst [$t{=}(29.0{-}32.0)\tau_A$] that starts from the more or less original topology ($t{=}29.0\tau_A$). As is seen in this figure, a distortion, which is very similar to that seen at $t{=}12.5\tau_A$ of Fig. 2(a), appears and reconnection advances up to the boundary of the untwisted region ($t{=}31.0\tau_A$). This temporal behavior suggests that the stable state (a more or less straight structure) between $t{=}26{-}29\tau_A$ is the global minimum energy state.

Finally, we look into the physical process of reconnection as a primary cause of the energy burst and topological change. Figure 3 shows the contours of the axial current on the x-y plane at $z{=}L/2$ at several featured events. As seen in Fig. 3(a), a thin-arc-like-current sheet (current singularity) is formed on the compressed front side of the kinking motion which is coincident with the twist-untwist boundary. This corresponds to the first burstlike relaxation. Meanwhile, a second kinking starts and expands radially in a splitting fashion. Two arclike current singularities are then formed whereby the second burst occurs, as shown in Fig. 3(b). In the meantime, the two kinked flux tubes approach each other and a straight current singularity is formed at their interface, as shown in Fig. 3(c). Figure 3(d) indicates that after the reconnection at the central current singularity, the field line structure returns to a gentle and stable axisymmetric one. These features strongly support the argument that the kink driven plasma motion and resulting current singularity formation rapidly accelerate reconnection. In other words, driven reconnection plays the vital role in the intermittent and recurrent energy bursts and topological changes of a twisted flux tube.

This work was carried out to advance our formulation of the quantitative scientific basis of the physics of the complexity of plasma. A demonstration is given that in an open magnetohydrodynamic system where a free magnetic energy is continuously supplied and a produced thermal energy (entropy) is immediately expelled, self-organization proceeds in two cycles, a minor cycle and a grand cycle. Energy release and topology change exhibit intermittently a burstlike transition between a local minimum state and an excited state in a minor cycle [$(2{-}3)\tau_A$]. In the grand cycle [$(14{-}15)\tau_A$], which consists of a few minor cycles, the system exhibits a return to the original state in both energy and topology, which is the global minimum energy state.

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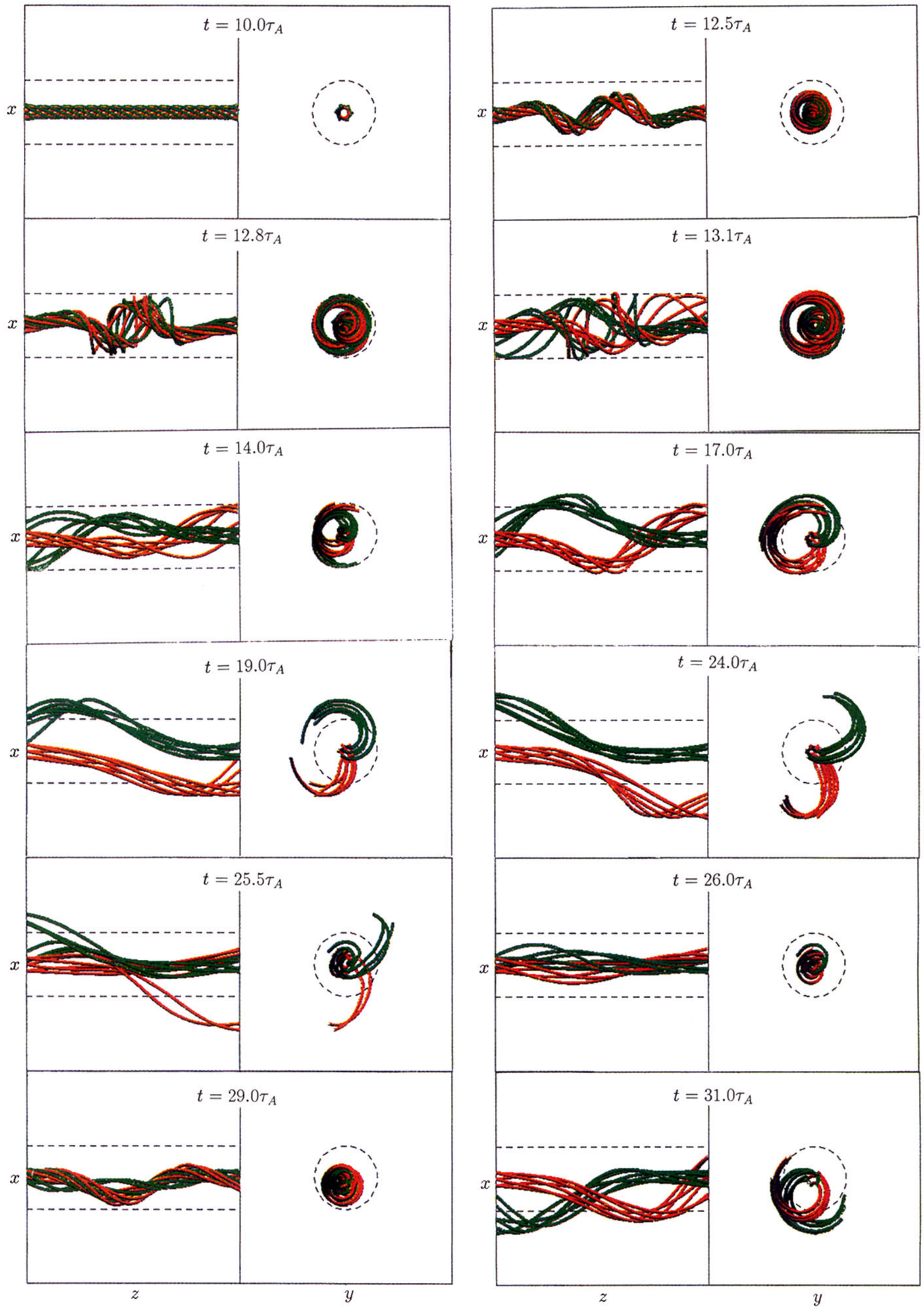

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