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α-particle transport-driven current in tokamaks

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It is shown that the radial transport of fusion-born energetic α particles, induced by electrostatic waves traveling in one poloidal direction, is directly connected to a net momentum of α particles in the toroidal direction in tokamaks. Because the momentum change is almost independent of toroidal velocity, the energy required for the momentum generation remains small on an α-particle population sustained by an isotropic time-independent source. By numerical toroidal Monte Carlo calculations it is shown that the current carried by α particles in the presence of intense well penetrated waves can reach several mega-amperes in reactor-sized tokamaks. The current obtained can greatly exceed the neoclassical bootstrap current of the α particles.

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The tokamak plasma current can be driven in various ways, such as by inductive generation, rf current drive, and the bootstrap mechanism. The rf current drive scheme normally involves generation of fast particles, whereas the inductive and bootstrap methods are related to the transport of thermal electrons. In recent years, increasing interest has been devoted to the bootstrap current of nonthermal fusion-born α particles [1,2], the effects of rf power on the α velocity distribution [3,4], and the reverse effect of α particles on the rf power used for current drive [5,6].

The potential of α particles in current drive is based on their high velocity relative to that of thermal ions and on their relatively high concentration in burning plasmas. In a tokamak with a central deuterium and tritium density 0.7×10^{20} m^{-3} and temperature 20 keV, the α-particle production rate 2×10^{18} α particles/m^{3}s may sustain a high energy α-particle density $n_\alpha = 10^{18}$ m^{-3} against their slowing-down in, say, 0.5 s. Assuming an rf-induced shift $\Delta v_\parallel = 10^{-1} v_\alpha$ in the parallel velocity of the α particles having a birth velocity v_α, a current density $n_\alpha \Delta v_\parallel q_\alpha \approx 0.5$ MA/m^2 is found, which in reactor-sized plasma cross sections could produce significant ion currents. In the present work, interaction between the α particles and the rf waves is investigated. Special emphasis is put on mechanisms producing significant shifts in the toroidal component of the α-particle velocity parallel to the plasma current of the tokamak.

Considering an electrostatic wave $\vec{E}\exp(ik_x x + ik_y y + ik_z z - i\omega t) + \text{c.c.}$ with an angular frequency ω much higher than the α cyclotron frequency Ω (e.g., lower hybrid waves), one obtains from the equations of motion

$$\dot{\vec{v}} = (q/m)(\vec{E} + \vec{v}\times\vec{B}), \qquad (1)$$

$$\dot{\vec{x}} = \vec{v}, \qquad (2)$$

the phase-averaged relation

$$W = C + (m\omega\Omega/k_y)x_{\rm GC} \qquad (3)$$

between the total kinetic energy W and the x component $x_{\rm GC}$ of the guiding-center position of the α particle. Here, the magnetic field $\vec{B}$ is taken to be along the z coordinate, C is a constant depending on initial conditions, q and m are the α-particle charge and mass, and $\vec{E} = E_y\hat{y} + E_z\hat{z}$ is assumed to be in the y-z plane. Reference [5] suggested the possibility of amplifying the lower hybrid wave power and enhancing the α-particle radial transport by tapping the free expansion energy of center-born α particles with waves having a properly chosen direction of propagation. If k_y is chosen negative in Eq. (3), the particle energy W would decrease for increasing $x_{\rm GC}$. This mechanism would evidently amplify the wave if the particles would diffuse to larger $x_{\rm GC}$, as demonstrated in Ref. [5].

In a system with a translational symmetry in the z direction, one can additionally find from Eqs. (1) and (2) the phase-averaged relation $\dot{W}_z = \dot{W}_\perp (k_z v_z/\omega)$ between the kinetic energies related to the parallel and perpendicular motion of the particle with respect to the magnetic field. The validity of this relation, too, is based on the assumption $\omega \gg \Omega$, in which limit the particles can have a resonant interaction twice for each cyclotron period with the wave at $\omega = k_y v_y$ [7]. Figure 1 shows the correlation between Δv_z and $\Delta W_\perp$ and between $\Delta x_{\rm GC}$ and ΔW for a group of test particles having initially $x=0$, $z=0$, $-v_\alpha < v_z < v_\alpha$, and y distributed uniformly over the region $0<y<2\pi/k_y$. Various ratios ω/Ω, k_z/k_y, and different wave amplitudes were applied to obtain the correlations. Here, Δ means the deviation from the initial values. As can be seen in Fig. 1, a good correlation between the deviations at any time instant can be observed, in compliance with the phase-averaged relations given above. Any decrease of $W_\perp$ would then indicate either positive or negative change in v_z depending on the sign of k_z. It is of great interest to note that this change does not depend on v_z. This fact implies that for the transport in the positive x direction there is always a current in the z direction for finite k_z in Cartesian coordinates.

In a toroidal axisymmetric geometry with coordinates r,θ,ϕ, the Hamiltonian describing the particle-wave interaction has the form

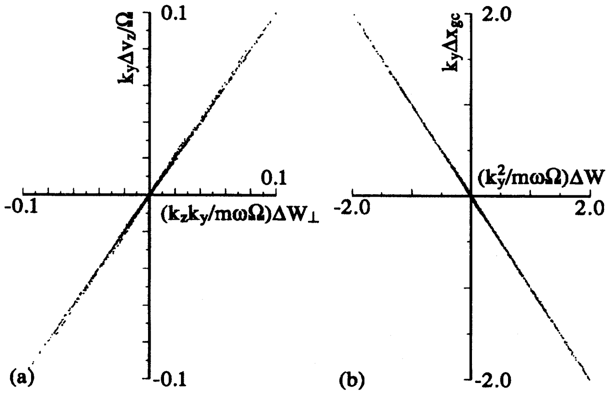

FIG. 1. Stroboscopic sections of trajectories (a) $k_y\Delta x_{\rm GC}$ vs $(k_y^2/m\omega\Omega)\Delta W$ and (b) $k_y\Delta v_z/\Omega$ vs $(k_zk_y/m\omega\Omega)\Delta W_\perp$ of 10 particles in velocity and configuration space having initial parameters $z=x=0$, $v=v_\alpha$, $v_z/v_\alpha=-0.8$, -0.3, 0, 0.3, 0.8, $E_{z0}=9000$ and 15 000 V/m, $\omega/\Omega=40,77,100$, and $k_z/k_y=0.02$, 0.04, 0.06. The dots are recorded at time instants $t=n\Delta t$, $n=1,2,3,\ldots,M$ with $\Omega\Delta t=0.1$ and $M=100$.

$$H=H_0(J_1,J_2,J_3)+qV\exp[i\vec{k}\cdot\vec{r}(Q_i,J_i)-i\omega t], \quad (4)$$

where H_0 is the integrable part, V is the potential of the electrostatic wave, and $\vec{r}$ is the space vector. The canonical coordinates Q_1,Q_2,Q_3 are the angle of the cyclotron rotation, poloidal angle of the guiding center of the particle, and the toroidal angle ϕ of the guiding center, respectively [8]. The canonically conjugated system of momenta is $J_1=W_\perp/\Omega$; $J_2=qB_\phi r_{\rm GC}^2/2$; $J_3=p_\phi$, where $W_\perp=mv_\perp^2/2$ is the energy of the particle perpendicular motion with respect to the magnetic field, $r_{\rm GC}$ is the radius of the guiding center, B_ϕ is the toroidal component of the magnetic field, and p_ϕ is the toroidal momentum. By using the resonance condition $-i\omega+in(\dot{Q}_1+\dot{Q}_2)+ik_\phi R\dot{Q}_3=0$, one can derive from the canonical equations the expression [9]

$$dJ_1/dJ_3=n/k_\phi R \quad (5)$$

for the ratio of perturbations in J_1 and J_3. Here, R is the major radius of the guiding center. Approximating the resonance condition as $\omega=n\Omega$ for $\omega\gg\Omega$, Eq. (5) reads

$$dp_\phi=(k_\phi R/\omega)dW_\perp, \quad (6)$$

where the variation in Ω has been neglected. In the limit $R\rightarrow\infty$, the dependence shown in Fig. 1(a) for Cartesian coordinates is recovered from Eq. (6).

From the definition $p_\phi=mRv_\phi-q\Psi$, where Ψ is the poloidal magnetic flux and v_ϕ is the toroidal component of the particle velocity, one finds for the change in v_ϕ (averaged over the trajectory of the orbit around the magnetic axis of the tokamak plasma torus)

$$dv_\phi=(k_\phi/m\omega)dW_\perp+(qB_p/m)dr. \quad (7)$$

Here, B_p is the poloidal component of the magnetic field and dr is the change in the radius of the particle guiding center. Equation (7) shows that a change in v_ϕ is expected in a toroidal system even for a vanishing toroidal component of the wave number. Assuming $k_\phi=0$, and using Eq. (3) in the form $dW=(m\omega\Omega/k_\theta)dr$, Eq. (7) gives $dv_\phi=(k_\theta/m\omega)(B_p/B)dW$. Consequently, the particle experiences a positive shift in v_ϕ for negative k_θ when W reduces due to the particle transport to larger r. This shift occurs for all particles irrespective of their toroidal velocity. For that reason, a relatively small amount of energy is needed for this current generation. The particle having a positive v_ϕ gains energy from the wave in the shift $v_\phi\rightarrow v_\phi+dv_\phi$, while the particle moving in the opposite direction with the same $|v_\phi|$ loses almost the same energy amount to the wave in the shift $-v_\phi\rightarrow -v_\phi+dv_\phi$, provided dW is the same for these particles.

Similar transport-driven current based on a turbulent spectrum of electrostatic waves has recently been observed in particle-in-cell simulations in axisymmetric tokamak configurations [10]. In these simulations, $\vec{E}\times\vec{B}$ drifts are the dominant transport mechanism and are parallel to the constant electrostatic potential layers. In the present paper, the current is shown to be produced by the asymmetry in the poloidal wave number k_θ of the electrostatic waves, and it can be affected by the choice of the signs of both k_θ and k_ϕ. From Eq. (7) one can obtain an approximation $dv_\phi=(k_\phi v_\perp/\omega)[1+(k_\theta/k_\phi)(B_p/B)]dv_\perp$, if $|dv_\phi|\ll|dv_\perp|$. Consequently, in tokamaks irradiated with sufficiently intense waves that produce a large change in $v_\perp$ comparable to v_α, dv_ϕ may be of the order of $0.1v_\alpha$ or even larger when $B_p/B>0.1$ or $|k_\phi|v_\alpha/\omega>0.1$.

Recently, Fokker-Planck calculations [6] and a full Monte Carlo simulation [11] have been used for analyzing the energy transfer in wave amplification by α particles in tokamak fusion reactors. In the present work, a full Monte Carlo simulation of the wave–α-particle interaction in a toroidal configuration is presented, which covers the self-consistent effects of α-particle distribution in velocity and configuration space, their finite orbits, radial diffusion and current generation in reactor-sized tokamak magnetic configuration. The Monte Carlo code calculates diffusion, power deposition, and energy exchange of the α particles with the wave, and includes the operators for collisional slowing-down, pitch angle scattering, and energy diffusion of particles, in combination with an operator for the wave interaction, assuming a steady α-particle source from the DT fusion reactions. The guiding-center orbits of the α particles are tracked in a tokamak geometry with elongation, D-shape and Grad-Shafranov displacement of the magnetic axis. Based on a binomial statistical distribution for the covariances in the Fokker-Planck model [5] of the α-wave interaction, simple Monte Carlo operators

$$\Delta W_\perp=(1/s)[\partial/\partial W_\perp-(k_\theta/m\omega\Omega)\beta(\partial/\partial\rho)](sD)\Delta t \pm\sqrt{2D\Delta t}, \quad (8)$$

$$\Delta\rho=-\Delta W_\perp(k_\theta/m\omega\Omega)\beta, \quad (9)$$

$$\Delta v_\parallel=(k_\parallel/m\omega)[1+(B_p/B_\phi)(k_\theta/k_\parallel)]\Delta W_\perp, \quad (10)$$

are derived for the changes of α-particle perpendicular energy, position in magnetic surface coordinate ρ, and in the parallel component $v_\parallel$ of the velocity with respect to the magnetic field during the time step Δt. Here, β is given by

$$\beta=(1+k_\parallel v_\parallel/\omega)/[(k_\theta v_\parallel/\omega)(B_p/B_\phi)-1], \quad (11)$$

and we have adopted the convention of using parallel components for both the wave number and velocity instead of the toroidal components. The diffusion coefficient for the Landau-resonant interaction of the lower hybrid wave with α particles satisfying $v_\perp > \omega/k_\theta$ is given by [7] as $D(W_\perp,\rho)=(1/2k_\theta)(\omega/k_\theta)^2(2eE_\theta)^2/\sqrt{v_\perp^2-\omega^2/k_\theta^2}$. $s(\rho)$ is the area of the magnetic surface at radius ρ, and E_θ is the poloidal electric field component of the wave.

The simulations are based on the Gaussian radial dependence of the electric field $|E_\theta|=|E_{\theta 0}|\exp[-2(\rho-\rho_0)^2/\rho_d^2]$ over the plasma cross section. The plasma density and temperature are assumed to have radial dependencies $[n,T]=[n_0,T_0(1-\rho^2/a^2)^{\alpha_{n,T}}]$, where n_0 and T_0 denote the density and temperature values on the plasma axis, and a is the plasma minor radius. The initial coordinates of the α particles are evenly distributed in the poloidal cross section of an axisymmetric plasma in proportion to the local α-particle production rate of the DT plasma. The monoenergetic (3.5 MeV) α particles are evenly distributed in pitch $v_\parallel/v$, and they are followed until they escape the plasma or are thermalized. As suggested in Ref. [5], absorbing boundary conditions at the low energy end $v_\perp=\omega/k_\theta$ are used in order to maximize the wave amplification. By recording the energy exchange with the waves and the time integral of the particle toroidal velocity at each radius and time instant, the steady-state current profile, the power deposition profiles, and the time history of the cumulative energy exchange with the waves can be obtained.

In the calculations, reactor-sized tokamak parameters $R=7.75$ m, $a=2.8$ m, $B=6.157$ T, $I=15$ MA, $T_{i,e}=20$ keV, $n_e=1.5\times 10^{20}$ m^{-3} are chosen for the major radius, minor radius, magnetic field, plasma current, ion (i) and electron (e) temperature, and plasma electron density, respectively. An elongation $\kappa=1.68$ for the cross section with a parabolic current density profile and equal deuterium and tritium concentrations with no impurities are assumed. The lower hybrid wave is modeled with a single poloidal wave number and a frequency of 3.7 GHz. The wave is centered at $\rho_0=1.4$ m with variable ρ_d and $|E_{\theta 0}|$. The value of $|E_{\theta 0}|$ for the lower hybrid wave is calculated from the electrostatic approximation $E_\theta/E_\parallel=(\omega_{pe}^2/\omega^2)/(1+\omega_{pe}^2/\Omega_e^2)$ at $\rho=\rho_0$, where ω_{pe} and Ω_e are the local electron plasma frequency and cyclotron frequency and $E_\parallel$ is the wave electric field component along the magnetic field. The total number of α particles and the time acceleration used in the simulations were varied in the range 2000–8000 and 50–400, respectively, in order to obtain convergence in the results.

Figure 2 shows the cumulative perpendicular energy exchange of the α particles with the wave averaged over the total ensemble of simulated particles as a function of time for different electric fields. Also shown is the radial profile of the α-particle power deposition on the wave. For the wave parameters, we have chosen $k_\parallel=220$ m^{-1} and $\rho_d=0.42$ m, which with $E_{\parallel 0}=3000$ V/m correspond to $k_\theta=-6085$ m^{-1}, $|E_{\theta 0}|=83\,000$ V/m at the wave maximum. As seen in the figure, the average cumulative energy change saturates at a positive value roughly within a collisional slowing-down time ν^{-1} which is about 0.5 s at the wave center. Here, a positive change characterizes a mean energy transfer to the wave. In the field amplitude range shown, the transferred

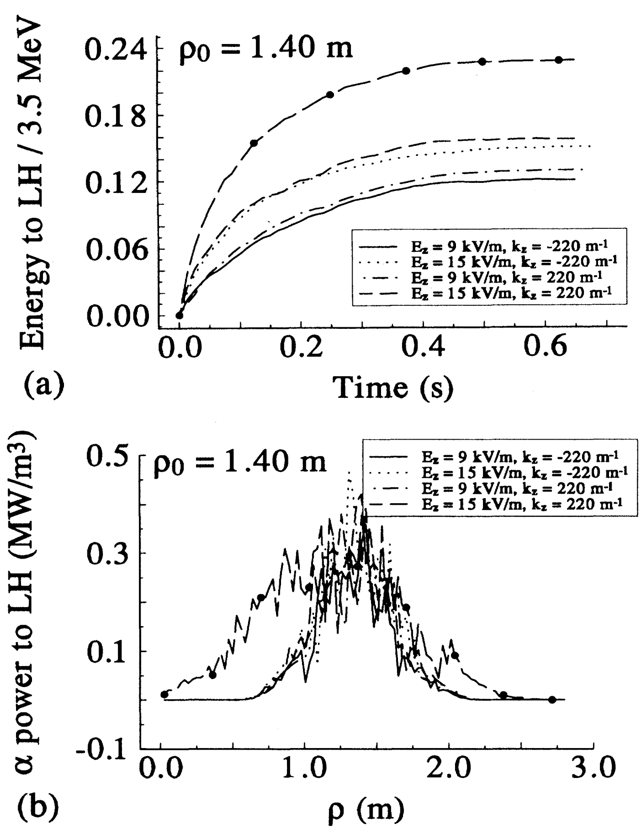

FIG. 2. (a) The average fraction of the α-particle birth energy transferred to the wave as a function of time for two different wave electric fields $|E_z|\equiv|E_{\parallel 0}|$ and parallel wave numbers $k_z\equiv k_\parallel$. The result for $|E_z|=15\,000$ V/m and $k_z=-220$ m^{-1} is shown also with a broader wave field distribution having $\rho_d=0.8$ m (curve with bullets). (b) Wave power deposition profiles of the α particles for the cases in (a).

energy fraction varies between 12% and 16%. Consequently, from the total fusion α-particle power 811 MW produced with the present reference parameters some 100 to 130 MW is transferred from the α particles to the lower hybrid waves. Because the wave interaction is strong only within the region $\rho\in[\rho_0-\rho_d,\rho_0+\rho_d]$, the relative conversion rate would be slightly larger if only that fusion α-particle power is taken into account which is born or interacts in the wave region (about 600 MW in the present case). For increasing field amplitude, the power transfer appears to saturate at the level of about 130 MW for $|E_{\parallel 0}|>9000$ V/m. Figure 2 also shows the result with a broader lower hybrid wave electric field distribution having $\rho_d=0.8$ m. In this case the interaction region between the α particles and the wave is larger, and about 187 MW is transferred from α particles to the wave. The present results are in fairly good agreement with those obtained earlier with a simplified model of the α-particle–wave interaction [6,11], where $\beta=-1$ and no variation in $v_\parallel$ was considered.

The α-particle current density profile is shown in Fig. 3 for $|E_{\parallel 0}|=9000$ V/m and 15 000 V/m. The results are shown separately for $k_\parallel=+220$ m^{-1} and -220 m^{-1}. In accordance with Eq. (10), positive α-particle current in the direction of the plasma current appears when k_θ is negative and opposite to the poloidal magnetic field, and when $k_\parallel$ is negative and

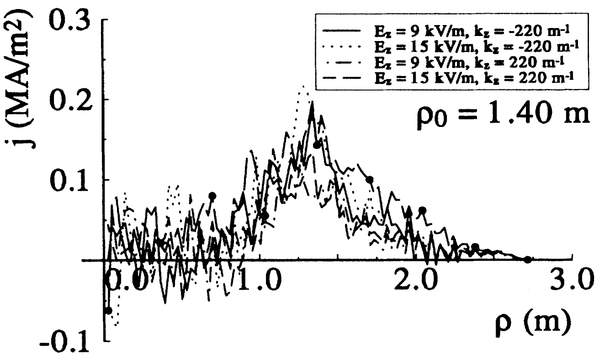

FIG. 3. The current density of the α particles for two different wave electric fields $|E_z| \equiv |E_{\|0}|$ and parallel wave numbers $k_z \equiv k_\|$ of the lower hybrid wave. The result for $|E_z| = 15\,000$ V/m and $k_z = -220$ m^{-1} is shown also with a broader wave field distribution having $\rho_d = 0.8$ m (curve with bullets).

opposite to the magnetic field. In Fig. 3, the current is not reversed but is reduced when $k_\|$ becomes positive, which is in agreement with the prediction of Eq. (10), because the term proportional to B_p in Eq. (10) dominates with the present parameters. With the values $k_\| = 200$ m^{-1}, $k_\theta = -6085$ m^{-1}, $B_p/B \approx 0.2$ at $\rho = 1.4$ m, and $\Delta W_\perp \approx 0.15 W_\alpha$, where $W_\alpha = m v_\alpha^2/2$, one finds the averaged value $\Delta v_\| \approx 4 \times 10^5$ m/s from Eq. (10). Consequently, with an average α-particle density $n_\alpha = 10^{18}$ m^{-3}, the current density $q n_\alpha \Delta v_\|$ would be 2.5×10^5 A/m^2, which roughly agrees with the average current density obtained in Fig. 3 in the wave regime.

The total α-particle current obtained in the cases shown in Fig. 3 varies between 0.94 and 1.41 MA with the chosen tokamak cross section. A larger current of 1.87 MA is obtained with a broader lower hybrid wave electric field distribution having $\rho_d = 0.8$ m. The current values obtained clearly exceed and are much more localized than the bootstrap current of the α particles. Simulations with a much larger number of α particles ($\sim$40 000–80 000) have indicated a fairly broad bootstrap current profile being finite also at the axis but with a maximum current density less than 30 000 A/m^2. In the result shown in Fig. 3, this current is overwhelmed by the statistical noise, and cannot be resolved. Omitting $\Delta v_\|$ and putting $\beta = -1$ in Eqs. (8)–(10) gives a somewhat larger energy transfer to the wave, because no current is generated and no power is consumed for this. The reduction of the power transfer to the wave in the presence of current generation is found to increase with the field amplitude. This agrees with the observation of the larger current with a larger field amplitude in Fig. 3.

As noted previously, the α-particle current generated by the present mechanism can be affected by the choice of the signs of $k_\|$ and k_θ. A particular choice in the sign of k_θ is required to make ΔW negative for the particles diffusing outwards from the plasma center. Therefore, if the latter term in Eq. (10) dominates, the current appears to flow in the same direction as the plasma current. Because this part of the current is proportional to B_p, it vanishes on the plasma axis ($\rho = 0$). However, the first term, which is proportional to $k_\|$, does not vanish on the axis. The sign of this term can be chosen by directing the waves properly in the toroidal direction to generate current in the direction of the plasma current. As the rf waves may also generate electron current in the direction of $\omega/k_\|$ by Landau damping, this part of the α-particle current would always run in the same direction as the rf-driven electron current. On the other hand, the total current by the α particles includes also the electron screening part $-(Z_\alpha/Z_{\text{eff}}) j_\alpha$ in addition to the pure α-particle ion current j_α calculated in the present work. Here, Z_α is the α-particle charge number and Z_{eff} is the effective charge of the plasma. If Z_{eff} is less than 2, the total α-particle induced current would flow in the direction opposite to j_α. This would be unfavorable, because the total α-particle induced current would then oppose both the plasma current and the rf-driven electron current.

As lower hybrid waves have penetration problems in reactor-sized tokamaks [9], much interest has been recently devoted to ion Bernstein waves generated by mode conversion in fast wave heating in the ion cyclotron range of frequencies [12]. For this wave, enhanced transport of α particles and the concomitant current generation by the poloidally asymmetric wave spectrum may well be possible. As the wave angular frequency is now of the order of the fundamental ion cyclotron frequency, the formulas presented in this work for the variations of the toroidal momentum, radius, and parallel velocity of the α particles have to be somewhat modified by the different resonance condition. Investigation of this case is beyond the scope of the present Rapid Communication, but the method described can be used for its analysis.

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