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Cavitation in a flowing liquid

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In this paper, I propose that the cavitation threshold in a flowing liquid could be associated with the maximum tension that the fluid can sustain before undergoing cohesive fracture at a certain point. My criterion is not isotropic; I believe that a liquid will break if the tension in one direction exceeds a threshold, independent of the value of the other principal stresses. I also believe that if a liquid breaks, it is a cohesive fracture in which the liquid molecules disassociate into vapor and recondense as mist.

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Suppose that the vapor pressure of the liquid at a certain temperature θ is $p_s(\theta)$. In general, $p_s(\theta)$ is an increasing function so that the cavitation threshold

$$p<p_s(\theta), \tag{1}$$

where p, the pressure in the liquid, increases with increasing temperature [1,2]. The pressure in a liquid at rest is the mean normal stress. For a liquid in motion, the stress is given by

$$\mathbf{T}=-p\mathbf{1}+\mathbf{S}, \tag{2}$$

where $\mathbf{S}$ is the extra stress due to motion. Here, p is an extra variable needed to satisfy the constraint of incompressibility, and it is not determined by a constitutive equation or equation of state.

The considerations just raised pose the problem of how to impose the cavitation threshold condition in a moving liquid. In fact, the liquid cannot be decomposed into $p\mathbf{1}$ and $\mathbf{S}$. If we cut the liquid, the traction vector on the cut is given by $\mathbf{n}\cdot\mathbf{T}$, where $\mathbf{n}$ is normal to the cut. The fluid feels tractions $\mathbf{n}\cdot\mathbf{T}$ on cuts and the stress $\mathbf{T}$ in the bulk.

There is very substantial literature aimed at determining the maximum tension that a liquid may withstand, and it is found that if nucleation sites are eliminated, large amounts of tension can be maintained [3–5]. In practical applications, $p_s(\theta)$ in (1) can be replaced with an empirical criterion, say, $\tilde{p}_s(\theta)$, which could be a limit associated with degassing or impurities [2–6].

The determination of the maximum tension in a liquid at the fracture point requires that we compare the components on the diagonal of $\mathbf{T}$ in a coordinate system in which it is diagonal:

$$\mathbf{T}_{11}\geqslant\mathbf{T}_{22}\geqslant\mathbf{T}_{33}. \tag{3}$$

Different rupture criteria involving the three principal stresses can be proposed and tested in experiments. An attractive possibility is that the liquid will break (cavitate) at a point if

$$\mathbf{T}_{11}\geqslant\mathbf{T}_m, \tag{4}$$

where $\mathbf{T}_m$ is the breaking threshold. If we think that the breaking stress is determined by the cavitation threshold, then $\mathbf{T}_m\geqslant-\tilde{p}_s(\theta)$ and we should see vapor whenever and wherever

$$\mathbf{T}_{11}\geqslant-\tilde{p}_s(\theta). \tag{5}$$

This criterion implies (1) if the fluid is at rest, but in a Newtonian fluid for which

$$\mathbf{T}_{11}=-p+2\mu\frac{\partial u}{\partial x}, \tag{6}$$

we can expect vapor and mist when and where the rate of stretching

$$\dot{s}=\frac{\partial u}{\partial x} \tag{7}$$

is large enough:

$$\dot{s}>\frac{p-\tilde{p}_s(\theta)}{2\mu}. \tag{8}$$

Equation (8) is our proposal for a cavitation threshold and it can lead to cavity formation under conditions greatly different than (1).

The concept of a breaking threshold in cavitation proposed here is, to a certain extent, analogous to the theoretical strength of solids—the maximum tensile strength between two planes. Apparently, as in the fracture of solids, this idealized criterion could be modified if the fluid contains defects: impurities, bubbles, etc. In this case, a criterion such as

$$\mathbf{T}_{11}\sqrt{a}<K_{cr}(\theta) \tag{9}$$

can be proposed, analogous to similar criteria in the theory of fracture mechanics. Here, a is the size of the defect, $K_{cr}(\theta)$ is a temperature dependent material property.

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