Continued fraction formalism of linear dynamic conductivity by a combined projection technique

Joung Young Sug,[1] Nam Lyong Kang,[1] Jai Yon Ryu,[2] and Sang Don Choi[1]
[1]Department of Physics, Kyungpook National University, Taegu, Korea
[2]Department of Physics, Cheju National University, Cheju, Korea
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Starting with the Liouville equation and with use of the combined projection technique presented earlier [J. Y. Sug, C. H. Choi, and S. D. Choi, Nuovo Cimento B **109**, 177 (1994)], we obtain the ensemble average of current of many-electron systems in the presence of oscillatory electric fields. For quite weak fields the dynamic conductivity is given in a continued fraction form. The present form contains the first- and second-order correction parts, while the Mori type contains only the first-order part.

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I. INTRODUCTION

Studies of dynamic conductivity are of great importance in investigating the electronic transport phenomena in many-electron systems. Among them the study for oscillatory weak electric fields has received special attention in recent years [1]. One of the related topics is optical transition in solids in the presence of a high static magnetic field, which includes intersubband and interband transitions [2]. It is well known that the absorption coefficient for the transition is proportional to the real part of the optical conductivity and the linear part of the conductivity is the most dominant. Thus it suffices to study the linear part only in this case.

On the other hand, the present group introduced a combined projection technique (CPT) by combining the two types of projection techniques; the first one is that of Kenkre and co-workers [3] and the second one is that of Argyres and Sigel [4]. Hereafter the first and second techniques shall be called the "PT-1" and the "PT-2," respectively. The PT-1 contains the equilibrium density operator while the PT-2 contains the electron state index. Kenkre and others, by utilizing the PT-1, succeeded in formulating a response theory, which includes the Kubo theory as the lowest-order approximation [5]. Argyres and Sigel, by applying the PT-2, succeeded in presenting a conductivity formalism and a theory of cyclotron resonance absorption [6]. Choi and Chung also applied the PT-2 to obtain a theory of cyclotron resonance line shapes for electron-phonon systems [7]. Furthermore, our group applied the CPT to the same problem and the derived line-shape function is similar to those obtained by the other techniques [8]. It was shown, however, that the amount of calculation is much smaller in the CPT, compared with that in the other techniques. More recently our group formulated theories of interband transition [9] and of nonlinear conductivity [10] based on the CPT. Thus we may claim that the CPT is quite good.

In this paper we will generalize the CPT to be applicable to the linear conductivity in the continued fraction representation scheme. In Sec. II, by starting with the Liouville equation, the current will be obtained in terms of weak oscillatory electric fields. The form given in a Fourier-Laplace space will be expressed in a linear form in the lowest-order approximation. In Sec. III, we will apply the present formalism to a special case and will see how it will be reduced and compared to others in the same approximations. Section IV shall be devoted to concluding remarks.

II. ELECTRICAL CONDUCTIVITY

We consider a system of many electrons which is subject to an oscillatory electric field $\mathbf{E}=\hat{\mathbf{e}}_l E_l(0)\exp(-i\omega t)$, where $\hat{\mathbf{e}}_l$ is the unit vector in the electric field direction $l\,(l=x,y,z,$ etc.) and ω is the angular frequency. Then the Hamiltonian $H(t)$ and the corresponding Liouville operator $L(t)$, respectively, are given by

$$H(t)=H_s+H'(t)\ , \tag{2.1}$$

$$H'(t)=-P_l E_l \exp(-i\omega t)\ , \tag{2.2}$$

and

$$L(t)=L_s+L'(t)\ , \tag{2.3}$$

$$L'(t)=L_l' \exp(-i\omega t)\ , \tag{2.4}$$

where H_s and L_s are the time-independent part and L_l' corresponds to $-P_l E_l$, $\mathbf{P}$ being the polarization vector, which implies that $L'(t)X=-[P_l,X]E_l(t)$ for an arbitrary operator X. The expectation value of the current operator J_k is defined as

$$\langle J_k(t)\rangle=T_R[J_k\rho(t)]=\sum_\alpha J_k^\alpha(t)\ , \tag{2.5}$$

$$J_k^\alpha(t)=[J_k\rho(t)]_{\alpha\alpha}\ , \tag{2.6}$$

where k is the direction index ($k=x,y,z,$ etc.), α is the state index, the density operator for the system $\rho(t)$ can be written as $\rho(t)=\rho_s+\rho'(t)$, ρ_s being the equilibrium part, and T_R denotes the many-body trace. In order to get a useful form of $\langle J_k(t)\rangle$ we should have the explicit form of $\rho(t)$. For that purpose we define the primary projection operator $P_{k\alpha}$ and its Abelian inverse $P_{k\alpha}'$ as [8,10]

$$P_{k\alpha}X=\frac{(J_kX)_{\alpha\alpha}}{(J_kL_l'\rho_s)_{\alpha\alpha}}L_l'\rho_s\ , \tag{2.7}$$

$$P_{k\alpha}'=1-P_{k\alpha}\ , \tag{2.8}$$

for an arbitrary operator X. It is to be noted that the direction of this projection is that of $L_l'\rho_s$ and the projection is time independent. We see from Eqs. (2.7) and (2.8) that this projection is a combination of the PT-1 and PT-2.

We assume that the perturbation of the system may be expanded as [10]

$$L_l'\rho'(t)=L_l'\rho_s\frac{[J_kL_l'\rho'(t)]_{\alpha\alpha}}{(J_kL_l'\rho_s)_{\alpha\alpha}}+\Delta_1\{L_l'\rho'(t)\}\ . \tag{2.9}$$

Then from the Liouville equation $i\partial\rho(t)/\partial t=L(t)\rho(t)$, where we use the unit system in which $\hbar=1$, we have

$$i\frac{\partial\tilde{P}_{k\alpha}\rho'(t)}{\partial t}=\tilde{P}_{k\alpha}L_s\tilde{P}_{k\alpha}\rho'(t)+\tilde{P}_{k\alpha}L_s(1-\tilde{P}_{k\alpha})\rho'(t)+[\tilde{P}_{k\alpha}L_l'\rho'(t)+\tilde{P}_{k\alpha}L_l'\rho_s]E_l(t)\ , \tag{2.10}$$

where $\tilde{P}_{k\alpha}$ is either $P_{k\alpha}$ or $P_{k\alpha}'$. For $\tilde{P}_{k\alpha}=P_{k\alpha}'$, we obtain

$$P_{k\alpha}'\rho'(t)=-i\int_0^t ds G_{k\alpha}(t-s)P_{k\alpha}'L_sP_{k\alpha}\rho'(s)\ , \tag{2.11}$$

where

$$G_{k\alpha}(t)=\exp(-itP_{k\alpha}'L_s)\ . \tag{2.12}$$

On the other hand, if we put $\tilde{P}_{k\alpha}=P_{k\alpha}$ in Eq. (2.10) and consider Eq. (2.11), we obtain

$$\frac{\partial J_k^\alpha(t)}{\partial t}=A_{kl}^\alpha J_k^\alpha(t)-\int_0^t Q_{kl}^\alpha(t-s)J_k^\alpha(s)ds-i\Lambda_{kl}^\alpha E_l(t) \tag{2.13}$$

in the lowest-order approximation, where $j_k^\alpha(0)=0$ for the initial condition and

$$A_{kl}^\alpha=-\frac{i}{\Lambda_{kl}^\alpha}(J_kL_sL_l'\rho_s)_{\alpha\alpha}\ , \tag{2.14}$$

$$Q_{kl}^\alpha(t)=\frac{1}{\Lambda_{kl}^\alpha}[J_kL_sf_1(t)]_{\alpha\alpha}\ , \tag{2.15}$$

$$\Lambda_{kl}^\alpha=(J_kL_l'\rho_s)_{\alpha\alpha}\ , \tag{2.16}$$

$$f_1(t)=\exp(-itL_1)f_1\ , \tag{2.17}$$

$$f_1=L_1L_l'\rho_s\ , \tag{2.18}$$

$$L_1=(1-P_{k\alpha})L_s\ . \tag{2.19}$$

Here we have neglected the nonlinear part since we are interested in the case of quite weak electric fields. The Fourier-Laplace transform ($\mathcal{T}_{FL}$) of a time-dependent function $\chi(t)$ can be defined as

$$\tilde{\chi}(z)\equiv\mathcal{T}_{FL}[\chi(t)]=\int_0^\infty \exp(-izt)\chi(t)dt\ . \tag{2.20}$$

Note that the parameter z will disappear in the final stage when the inverse transformation $\mathcal{T}_{FL}^{-1}$ is performed [see Eq. (2.47)]. Then the $\mathcal{T}_{FL}$ of Eq. (2.13) turns out to be

$$\tilde{J}_k^\alpha(z)=\frac{-i\Lambda_{kl}^\alpha\tilde{E}_l(z)}{iz-A_{kl}^\alpha+\tilde{Q}_{kl}^\alpha(z)}\ , \tag{2.21}$$

where

$$\tilde{Q}_{kl}^\alpha(z)=\mathcal{T}_{FL}[Q_{kl}^\alpha(t)]=\frac{1}{\Lambda_{kl}^\alpha}[J_kL_s\tilde{f}_1(z)]_{\alpha\alpha}\ , \tag{2.22}$$

$$\tilde{f}_1(z)=\mathcal{T}_{FL}[f_1(t)]=(iz+iL_1)^{-1}L_1L_l'\rho_s\ . \tag{2.23}$$

In order to calculate the $\tilde{Q}_{kl}^\alpha(z)$ in the denominator of Eq. (2.21) further, we introduce the secondary projection operator $P_{1k\alpha}$ and its abelian inverse $P_{1k\alpha}'$ as

$$P_{1k\alpha}X=\frac{(J_kL_sX)_{\alpha\alpha}}{(J_kL_sL_1f_1)_{\alpha\alpha}}L_1f_1\ , \tag{2.24}$$

$$P_{1k\alpha}'=1-P_{1k\alpha}\ . \tag{2.25}$$

Now we assume that the $f_1(t)$ in Eq. (2.23) can be expanded as

$$f_1(t)=f_1+f_1'(t)\ . \tag{2.26}$$

Note that if the direction of projection is chosen as f_1 instead of L_1f_1, we will have an unsatisfactory result. This does not mean, however, that Eq. (2.24) is the only choice. From Eqs. (2.24) and (2.26) we have

$$\frac{\partial\tilde{P}_{1k\alpha}f_1'(t)}{\partial t}=-i\tilde{P}_{1k\alpha}L_1f_1-i\tilde{P}_{1k\alpha}L_1\tilde{P}_{1k\alpha}f_1'(t)-i\tilde{P}_{1k\alpha}L_1(1-\tilde{P}_{1k\alpha})f_1'(t)\ , \tag{2.27}$$

where $\tilde{P}_{1k\alpha}$ is either $P_{1k\alpha}$ or $P_{1k\alpha}'$. For $\tilde{P}_{1k\alpha}=P_{1k\alpha}$, we obtain

$$P_{1k\alpha}'f_1'(t)=-i\int_0^t K_{1k\alpha}(t-h)P_{1k\alpha}'L_1P_{1k\alpha}f_1'(h)dh\ , \tag{2.28}$$

where

$$K_{1k\alpha}(t)=\exp(-itP_{1k\alpha}'L_1)\ . \tag{2.29}$$

From Eqs. (2.24), (2.25), and (2.27) we have

$$[J_kL_s\tilde{f}_1(z)]_{\alpha\alpha}=(J_kL_sf_1/iz)_{\alpha\alpha}+\frac{-i(J_kL_sL_1f_1)_{\alpha\alpha}}{iz+i(J_kL_sL_1W_1)_{\alpha\alpha}+[J_kL_sL_1\tilde{f}_2(z)]_{\alpha\alpha}} \tag{2.30}$$

which is the third part in the denominator of Eq. (2.21), where

$$W_1=\frac{L_1f_1}{(J_kL_sL_1f_1)_{\alpha\alpha}}\ , \tag{2.31}$$

$$\tilde{f}_2(z)=\mathcal{T}_{FL}[f_2(5)]\ , \tag{2.32}$$

$$f_2(t)=\exp(-itL_2)f_2\ , \tag{2.33}$$

$$f_2=L_2W_1\ , \tag{2.34}$$

$$L_2=(1-P_{1k\alpha})L_1\ . \tag{2.35}$$

In general, for the nth-order memory effect ($n = 2,3,4,\ldots$), we define the projection operators $P_{nk\alpha}$ and $P'_{nk\alpha}$ as

$$P_{nk\alpha}X = \frac{(J_k L_s L_1 \cdots L_{n-1} X)_{\alpha\alpha}}{(J_k L_s L_1 \cdots L_n f_n)_{\alpha\alpha}} L_n f_n , \tag{2.36}$$

$$P'_{nk\alpha}X = (1 - P_{nk\alpha})X , \tag{2.37}$$

where

$$L_n = P'_{n-1,k\alpha} L_{n-1} , \tag{2.38}$$

$$f_n = L_n W_{n-1} , \tag{2.39}$$

$$W_n = \frac{L_n f_n}{(J_k L_s L_1 \cdots L_n f_n)_{\alpha\alpha}} . \tag{2.40}$$

Then for the nth-order memory effect ($n = 2,3,4,\ldots$), we have

$$\begin{aligned}[J_k L_s L_1 L_2 \cdots L_{n-2} \tilde{f}_{n-1}(z)]_{\alpha\alpha} &= (J_k L_s L_1 L_2 \cdots L_{n-2} f_{n-1}/iz)_{\alpha\alpha} \\ &+ \frac{-i(J_k L_s L_1 L_2 \cdots L_{n-1} f_{n-1})_{\alpha\alpha}}{iz + i[J_k L_s L_1 L_2 \cdots L_{n-1} W_{n-1})_{\alpha\alpha} + (J_k L_s L_1 L_2 \cdots L_{n-1} f_n(z)]_{\alpha\alpha}} ,\end{aligned} \tag{2.41}$$

where $\tilde{f}_n(z) = \mathcal{T}_{FL}[f_n(t)]$. The collision factor $\tilde{Q}^{\alpha}_{kl}(z)$ can be written as

$$\begin{aligned}\tilde{Q}^{\alpha}_{kl}(z) &= [J_k L_s \tilde{f}_1(z)]_{\alpha\alpha}/\Lambda^{\alpha}_{kl} \\ &= \frac{1}{\Lambda^{\alpha}_{kl}} \left[i\gamma_0 + \cfrac{-i\Delta_1}{iz + i\omega_1 + i\gamma_1 + \cfrac{-i\Delta_2}{iz + i\omega_2 + i\gamma_2 + \cfrac{-i\Delta_3}{iz + i\omega_3 + [J_k L_s L_1 L_2 L_3 f_4(z)]_{\alpha\alpha}}}} \right] ,\end{aligned} \tag{2.42}$$

where

$$\omega_n = (J_k L_s L_1 \cdots L_n W_n)_{\alpha\alpha} , \tag{2.43}$$

$$\gamma_n = (-J_k L_s L_1 \cdots L_n f_{n+1}/z)_{\alpha\alpha} , \tag{2.44}$$

$$\Delta_n = (J_k L_s L_1 \cdots L_n f_n)_{\alpha\alpha} . \tag{2.45}$$

As is seen from Eqs. (2.43) and (2.44), the matrix element of ω_n ends with $(L_n W_n)_{m\alpha}$ and γ_n with $(L_{n+1} W_n)_{m\alpha}$, the state index m being determined by the equations. This implies that γ_n is of one more projection than ω_n. By recalling that the eventual projection is performed with respect to $L'_l \rho_s$, we can consider ω_n and γ_n, respectively, as the first-order and second-order correction parts. If γ_n is neglected in approximation, we obtain the Mori-type continued fraction scheme [11].

So far we have obtained the current for the weak perturbation by the CPT. The current given in Eqs. (2.5) and (2.21) can be rewritten as

$$\begin{aligned}\langle J_k(t) \rangle &= \sum_{\alpha} J^{\alpha}_k(t) \\ &= \sum_{\alpha} \int_0^t \sigma^{\alpha}_{kl}(t-s) E_l(s) ds ,\end{aligned} \tag{2.46}$$

where

$$\sigma^{\alpha}_{kl}(t) = \mathcal{T}^{-1}_{FL}[\tilde{\sigma}^{\alpha}_{kl}(z)] , \tag{2.47}$$

$$\tilde{\sigma}^{\alpha}_{kl}(z) = \frac{-i\Lambda^{\alpha}_{kl}}{iz - A^{\alpha}_{kl} + \tilde{Q}^{\alpha}_{kl}(z)} , \tag{2.48}$$

and $\tilde{Q}^{\alpha}_{kl}(z)$ is given by Eq. (2.42), which is given in the continued fraction form, in terms of ω_n, γ_n, and Δ_n.

In the next section we will apply the present formalism to a system of electrons in impurity background.

III. AN APPLICATION

Now, for the sake of demonstration to see how much this formalism is valid, we apply it to magneto-optical transitions in solids. We consider a system of electrons in isotropic semiconductors which interact weakly with background impurities and assume that the electron-electron interaction is absent. Then we may adopt the single-electron formalism. For a static magnetic field **B** applied in the z axis, the electron energy is quantized. If, in addition, a microwave of angular frequency ω is applied along the z direction, the electromagnetic energy is absorbed at $\omega \approx \omega_c$, ω_c being the cyclotron frequency, i.e., the cyclotron transition arises. Thus we can apply the present formalism to this problem with $z = -\bar{\omega} = -(\omega - ia)(a \to 0+)$ in Eqs. (2.21) and (2.42).

The Liouville operator L_s can be written as

$$L_s = L_e + L_v , \tag{3.1}$$

where L_e and L_v are the Liouville operators corresponding to the electron Hamiltonian H_e and the scattering potential V, respectively.

If the microwave is circularly polarized, the absorption power is proportional to the real part of the conductivity tensor $\sigma_{kl}(\omega)$ with $k = -$ and $l = +$, for which

$$(J_k)_{\beta\alpha} = J^{-}_{\alpha-1,\alpha} \delta_{\beta,\alpha-1} , \quad (J_l)_{\beta\alpha} = J^{+}_{\alpha+1,\alpha} \delta_{\beta,\alpha+1} , \tag{3.2}$$

where $J^{\pm} = J_x \pm iJ_y$ and

$$\Lambda_{kl}^{\alpha}=i\Delta F_{\alpha+1,\alpha}J_{\alpha,\alpha+1}^{-}J_{\alpha+1,\alpha}^{+}\ , \tag{3.3}$$

$$A_{kl}^{\alpha}=-i\varepsilon_{\alpha+1,\alpha}=-i\omega_c\ . \tag{3.4}$$

Here $\varepsilon_{\alpha\beta}\equiv\varepsilon_\alpha-\varepsilon_\beta$ and $\Delta F_{\alpha,\beta}\equiv(F_\alpha-F_\beta)/\varepsilon_{\alpha\beta}$, where ε_α is the energy eigenvalue and F_α the Fermi-distribution function for the state $|\alpha\rangle$ [8]. Thus we have from Eq. (2.21)

$$\tilde{\sigma}_{kl}^{\alpha}(\varpi)=i\frac{F_{\alpha+1}-F_\alpha}{\varepsilon_{\alpha+1}-\varepsilon_\alpha}\frac{J_{\alpha,\alpha+1}^{-}J_{\alpha+1,\alpha}^{+}}{i(\varpi-\omega_c)-\tilde{Q}_{kl}^{\alpha}(\varpi)}\ . \tag{3.5}$$

The collision factor $\tilde{Q}_{kl}^{\alpha}(\varpi)$, sometimes called the line-shape function in case of optical transitions, can be obtained in a similar manner. If we assume that the interactions with the electromagnetic field and with the background impurities are weak enough it will suffice to consider only the lowest-order contribution in Eq. (2.42). We consider the fact that terms including an odd number of V's disappear in the average over impurity distribution [14], and consider

$$[(1-P_{-\alpha})L_eL'_+\rho_s]_{\beta\alpha}=0$$

and

$$[J^-L_eX(1-P_{-\alpha})L_vL'_+\rho_s]_{\alpha,\alpha}=0$$

for any operator X. Then the lowest-order part of the memory terms is given as

$$\gamma_0=(A-B-C+D)/\varpi\ , \tag{3.6}$$

$$\Delta_1=F-G-H+I\ , \tag{3.7}$$

$$\omega_1=\frac{K-L-M+N}{F-G-H+I}\ , \tag{3.8}$$

$$\gamma_1=-\alpha+\omega_1^2/(-\varpi)\ , \tag{3.9}$$

where

$$\alpha=\frac{O-P-Q+R}{-\varpi(F-G-H+I)} \tag{3.10}$$

and

$$A=-i\sum_{\beta\neq\alpha+1}\Delta F_{\alpha+1,\alpha}J_{\alpha,\alpha+1}^{-}J_{\alpha+1,\alpha}^{+}V_{\alpha+1,\beta}V_{\beta,\alpha+1}\ , \tag{3.11}$$

$$B=-i\sum_{\beta\neq\alpha+1}\Delta F_{\beta,\beta-1}J_{\alpha,\alpha+1}^{-}J_{\beta,\beta-1}^{+}V_{\alpha+1,\beta}V_{\beta-1,\alpha}\ , \tag{3.12}$$

$$C=-i\sum_{\beta\neq\alpha}\Delta F_{\beta+1,\beta}J_{\alpha,\alpha+1}^{-}J_{\beta+1,\beta}^{+}V_{\alpha+1,\beta+1}V_{\beta,\alpha}\ , \tag{3.13}$$

$$D=-i\sum_{\beta\neq\alpha}\Delta F_{\alpha+1,\alpha}J_{\alpha,\alpha+1}^{-}J_{\alpha+1,\alpha}^{+}V_{\alpha,\beta}V_{\beta,\alpha}\ , \tag{3.14}$$

$$F=-i\sum_{\beta\neq\alpha+1}\Delta F_{\alpha+1,\alpha}J_{\alpha,\alpha+1}^{-}J_{\alpha+1,\alpha}^{+}\varepsilon_{\beta,\alpha}V_{\alpha+1,\beta}V_{\beta,\alpha+1}\ , \tag{3.15}$$

$$G=-i\sum_{\beta\neq\alpha+1}\Delta F_{\beta,\beta-1}J_{\alpha,\alpha+1}^{-}J_{\beta,\beta-1}^{+}\varepsilon_{\beta,\alpha}V_{\alpha+1,\beta}V_{\beta-1,\alpha}\ , \tag{3.16}$$

$$H=-i\sum_{\beta\neq\alpha}\Delta F_{\beta+1,\beta}J_{\alpha,\alpha+1}^{-}J_{\beta+1,\beta}^{+}\varepsilon_{\alpha+1,\beta}V_{\alpha+1,\beta+1}V_{\beta,\alpha}\ , \tag{3.17}$$

$$I=-i\sum_{\beta\neq\alpha}\Delta F_{\alpha+1,\alpha}J_{\alpha,\alpha+1}^{-}J_{\alpha+1,\alpha}^{+}\varepsilon_{\alpha+1,\beta}V_{\alpha,\beta}V_{\beta,\alpha}\ , \tag{3.18}$$

$$K=-i\sum_{\beta\neq\alpha+1}\Delta F_{\alpha+1,\alpha}J_{\alpha,\alpha+1}^{-}J_{\alpha+1,\alpha}^{+}\varepsilon_{\beta,\alpha}^2V_{\alpha+1,\beta}V_{\beta,\alpha+1}\ , \tag{3.19}$$

$$L=-i\sum_{\beta\neq\alpha+1}\Delta F_{\beta,\beta-1}J_{\alpha,\alpha+1}^{-}J_{\beta,\beta-1}^{+}\varepsilon_{\beta,\alpha}^2V_{\alpha+1,\beta}V_{\beta-1,\alpha}\ , \tag{3.20}$$

$$M=-i\sum_{\beta\neq\alpha}\Delta F_{\beta+1,\beta}J_{\alpha,\alpha+1}^{-}J_{\beta+1,\beta}^{+}\varepsilon_{\alpha+1,\beta}^2V_{\alpha+1,\beta+1}V_{\beta,\alpha}\ , \tag{3.21}$$

$$N=-i\sum_{\beta\neq\alpha}\Delta F_{\alpha+1,\alpha}J_{\alpha,\alpha+1}^{-}J_{\alpha+1,\alpha}^{+}\varepsilon_{\alpha+1,\beta}^2V_{\alpha,\beta}V_{\beta,\alpha}\ , \tag{3.22}$$

$$O=-i\sum_{\beta\neq\alpha+1}\Delta F_{\alpha+1,\alpha}J_{\alpha,\alpha+1}^{-}J_{\alpha+1,\alpha}^{+}\varepsilon_{\beta,\alpha}^3V_{\alpha+1,\beta}V_{\beta,\alpha+1}\ , \tag{3.23}$$

$$P=-i\sum_{\beta\neq\alpha+1}\Delta F_{\beta,\beta-1}J_{\alpha,\alpha+1}^{-}J_{\beta,\beta-1}^{+}\varepsilon_{\beta,\alpha}^3V_{\alpha+1,\beta}V_{\beta-1,\alpha}\ , \tag{3.24}$$

$$Q=-i\sum_{\beta\neq\alpha}\Delta F_{\beta+1,\beta}J_{\alpha,\alpha+1}^{-}J_{\beta+1,\beta}^{+}\varepsilon_{\alpha+1,\beta}^3V_{\alpha+1,\beta+1}V_{\beta,\alpha}\ , \tag{3.25}$$

$$R=-i\sum_{\beta\neq\alpha}\Delta F_{\alpha+1,\alpha}J_{\alpha,\alpha+1}^{-}J_{\alpha+1,\alpha}^{+}\varepsilon_{\alpha+1,\beta}^3V_{\alpha,\beta}V_{\beta,\alpha}\ . \tag{3.26}$$

Thus after some systematic calculation we have

$$\tilde{Q}^{\alpha}_{kl}(\varpi)=i\left\{\sum_{\beta\neq\alpha+1}\frac{1}{\varpi}\left[V_{\alpha+1,\beta}V_{\beta,\alpha+1}-\frac{\Delta F_{\beta,\beta-1}}{\Delta F_{\alpha+1,\alpha}}\frac{J^{+}_{\beta,\beta-1}}{J^{+}_{\alpha+1,\alpha}}V_{\alpha+1,\beta}V_{\beta-1,\alpha}\right]\right.$$
$$+\sum_{\beta\neq\alpha}\frac{1}{\varpi}\left[V_{\alpha,\beta}V_{\beta,\alpha}-\frac{\Delta F_{\beta+1,\beta}}{\Delta F_{\alpha+1,\alpha}}\frac{J^{+}_{\beta+1,\beta}}{J^{+}_{\alpha+1,\alpha}}V_{\alpha+1,\beta+1}V_{\beta,\alpha}\right]$$
$$+\sum_{\beta\neq\alpha+1}(\varepsilon_{\beta,\alpha})\frac{V_{\alpha+1,\beta}V_{\beta,\alpha+1}\dfrac{\Delta F_{\beta,\beta-1}}{\Delta F_{\alpha+1,\alpha}}\dfrac{J^{+}_{\beta,\beta-1}}{J^{+}_{\alpha+1,\alpha}}V_{\alpha+1,\beta}V_{\beta-1,\alpha}}{i\varpi-i\varepsilon_{\beta,\alpha}+i\left[\theta_1-\theta_3-\dfrac{(2\varepsilon_{\beta,\alpha}\theta_1-\theta_1^2)}{\varpi}\right]-\Gamma_2}$$
$$\left.+\sum_{\beta\neq\alpha}(\varepsilon_{\beta,\alpha+1})\frac{V_{\alpha,\beta}V_{\beta,\alpha}-\dfrac{F_{\beta+1,\beta}}{F_{\alpha+1,\alpha}}\dfrac{J^{+}_{\beta+1,\beta}}{J_{\alpha+1,\alpha}}V_{\alpha+1,\beta+1}V_{\beta,\alpha}}{i\varpi-i\varepsilon_{\alpha+1,\beta}+i\left[\theta_2-\theta_4-\dfrac{(2\varepsilon_{\beta,\alpha+1}\theta_2-\theta_2^2)}{\varpi}\right]-\Gamma_2}\right\}, \tag{3.27}$$

where

$$\theta_1=\frac{E_1(F-G-H+I)-(K-L-M+N)}{(F-G-H+I)}, \tag{3.28}$$

$$\theta_2=\frac{E_2(F-G-H+I)-(K-L-M+N)}{(F-G-H+I)}, \tag{3.29}$$

$$\theta_3=\frac{(O-P-Q+R)-E_1^2(F-G-H+I)}{\varpi(F-G-H+I)}, \tag{3.30}$$

$$\theta_4=\frac{(O-P-Q+R)-E_2^2(F-G-H+I)}{\varpi(F-G-H+I)}, \tag{3.31}$$

$$\Gamma_2=(-J^{-}L_sL_1\tilde{f}_2(\varpi))_{\alpha\alpha}\,. \tag{3.32}$$

This result is similar to Sawaki's expression based on the Stark ladder representation [12] and Ryu and Choi's result based on the method of Argyres and Mori [13]. In this theory, γ_0 and γ_1 are the correction part; γ_0 gives the first two terms in Eq. (3.27) and γ_1 yields θ_3 and θ_4. If the interaction with the background is quite weak and θ_1, θ_2, θ_3, and θ_4 are neglected, the third and fourth terms are similar to Kawabata's result [14] based on Mori's method.

IV. CONCLUDING REMARKS

In Sec. II, we derived the linear dynamic conductivity in a system of many electrons by the CPT introduced earlier. The conductivity tensor is given in a continued fraction form, in terms of the scattering factors ω_n, γ_n, and Δ_n ($n=1,2,3,\ldots$). The ω_n and Δ_n, respectively, correspond to the characteristic frequencies and the reciprocal decay times in the Mori-type formalism [11]. But the present formalism includes the additional factor γ_n, which plays a role of second-order correction. This factor comes from the fact that the CPT deals with the time-dependent and time-independent perturbations in the unified category.

In Sec. III we demonstrated that the theory could be applied to a simple problem and made comparison with some other works. In real cases, however, further calculation including computer work is needed. This part is left for a future study.

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