Plasma-beam interaction in a wiggler

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The possibility of obtaining self-bunching of the beam, emission of coherent radiation and strong electrostatic fields in a plasma loaded free electron laser, is studied by means of a set of nonlinear self-consistent equations deduced from the Maxwell equations, the fluid plasma model, and the relativistic equations of motion for the electrons of the beam in the limit of plasma density much larger than the beam density.

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A considerable effort has been made in recent years in the field of high-power and high-frequency generators to invent new configurations and schemes and to improve the efficiency and the operating characteristics of existing devices. Along these lines is the proposal of the plasma free electron laser [1–3], which consists of a free electron laser (FEL) loaded with a cold plasma whose density n_p is much larger than the average density n_b of the electrons of the beam. This system is expected to have a large radiation efficiency due to the presence of two concurrent instabilities, the usual FEL instability, which converts the kinetic energy of the beam into transverse radiative energy, and the beam-plasma instability, which instead converts the kinetic energy of the beam into the electrostatic energy of the Langmuir wave excited in the plasma. The model has been further simplified by assuming that a resonance condition between the longitudinal plasma wave, the magnetic field of the wiggler, and the transverse radiation field is realized, i.e., that $\omega_L=\omega$ and $k_L=k+k_w$, ω (ω_L) and k (k_L) being the frequency and wave number of the transverse (longitudinal) wave and k_w the wave number of the undulator. If we consider the dispersion relation of the transverse mode inside the plasma, $\omega=(\omega_{pe}^2+c^2k^2)^{1/2}$ and that of the Langmuir wave $\omega_L=[\omega_{pe}^2+3v_{\rm th}^2(k+k_w)^2]^{1/2}$, we find that the above resonance condition implies a constraint upon both frequency and wave number of the radiation, namely $\omega=[\omega_p^2+\mu^2k_w^2/(1-\mu)^2]^{1/2}\approx\omega_p$ and $k=\mu k_w/(1-\mu) \ll k_w$, where $\mu=\sqrt{3}v_{\rm th}/c \ll 1$ is the ratio between the thermal velocity $v_{\rm th}$ of the electrons of the plasma and the speed of light c [$\omega_p=(4\pi n_pe^2/m)^{1/2}$ is the plasma frequency of the electrons of the plasma].

It follows from these relations that the frequency of the excited radiation is determined by the plasma density and that the wave number of the radiation in the plasma is much smaller than k_w. Furthermore, by defining $\beta_r=\omega_L/ck_L=\omega(k)/[c(k+k_w)]$ as the ratio between the phase velocity of the Langmuir wave and c, one can argue that the maximum efficiency of the system will occur when the average parallel velocity of the injected electrons $v_\parallel(0)=c\beta_\parallel(0)$ is equal to $c\beta_r$. Obviously, this last condition can only be satisfied when $\beta_r<1$.

The analysis presented in this Brief Report is based upon the following set of nonlinear differential equations, which have been deduced, in the framework of the slowly varying amplitude approximation, from the Maxwell system, the fluid plasma model, and the relativistic equations of motion for the N electrons of the beam:

$$
\begin{aligned}
&\frac{d}{d\tau}\theta_j=\beta_{\parallel j}-\beta_0\ ,\\
&\frac{d}{d\tau}\gamma_j=-iS_3\beta_{\parallel j}\{\langle b\rangle e^{i\theta_j}-\text{c.c.}\}\\
&\qquad\qquad -i(S_4/\gamma_j)\{\tilde{A}e^{i\theta_j}-\text{c.c.}\}\\
&\qquad\qquad -iS_5\beta_{\parallel j}\{\tilde{\alpha}e^{i\theta_j}-\text{c.c.}\}\ ,\\
&\frac{d}{d\tau}\tilde{\alpha}=i\delta\tilde{\alpha}-i\langle b\rangle-i\tilde{A}\ ,\\
&\frac{d}{d\tau}\tilde{A}=i\delta\tilde{A}-iS_1\left[\frac{1}{N}\sum_{j=1}^{N}(e^{-i\theta_j}/\gamma_j)\right]-iS_2\tilde{\alpha}\ .
\end{aligned}
\tag{1}
$$

Here $\beta_{\parallel j}=v_{\parallel j}/c$, $\theta_j=(k+k_w)z_j-\omega t-\delta c(k+k_w)t$, is the beam electron phase in terms of the physical time t, $\tau=c(k+k_w)t$, and j ranges between 1 and N. We assume a beam that initially is monokinetic, so that $v_{\parallel j}(t=0)/c=\beta_0$. $S_1=a_{w0}^2/(4\beta_r)$, $S_2=\Omega_p^2a_{w0}^2/(8\beta_r^2)$, $S_3=\Omega_b^2/2$, $S_4=\beta_r\Omega_b^2$, and $S_5=\Omega_p^2\Omega_b^2/(2\beta_r)$ are the coefficients of the equations, with $\Omega_b=\omega_b/[c(k+k_w)]$, $\Omega_p=\omega_p/[c(k+k_w)]$, $\omega_b^2=4\pi n_be^2/m$, the plasma frequency of the electrons of the beam, and with $a_{w0}=eB_w/(k_wmc^2)$, the nondimensional undulator parameter, B_w being the intensity of the magnetic field of the wiggler.

Furthermore, $\delta=\beta_0-\beta_r$ is the detuning factor, $\gamma_j=(1-\beta_{\parallel j}^2-\beta_{\perp j}^2)^{-1/2}$ is the Lorentz factor of the jth electron, and, taking into account that the transverse component of the velocity of the electrons of the beam $\beta_{\perp j}$ is related to the external magnetic field by the condition $\beta_{\perp j}=a_{w0}/\gamma_j$, γ_j can be expressed as $\gamma_j=[(1+a_{w0}^2)/(1-\beta_{\parallel j}^2)]^{1/2}$. Finally, $\langle b\rangle=(1/N)\sum_{j=1}^{N}e^{-i\theta_j}$ is the bunching factor of the electrons of the beam, while $\tilde{A}=[ea_{W_0}/2mc^2\Omega_b^2]e^{i\delta\tau}A$ and $\tilde{\alpha}=[2\beta_r/\Omega_b^2]e^{i\delta\tau}\delta n_p/n_p$ are the nondimensional scaled transverse vector potential A and disturbance of the electron density $\delta n_p/n_p$, respectively.

The previous system is written in the framework of

Cauchy-type problems, and only time variations have been taken into account. The first $2N$ equations are the relativistic equations of motion of the N particles of the beam. The equations for the fields have been written following the line presented in Ref. [3]. In that model, however, γ_j was assumed to remain very close to its initial value $\gamma(0) \approx \gamma_r$ $(\gamma_r = [(1+a_{w0}^2)/(1-\beta_r^2)]^{1/2})$, i.e., $\gamma_j - \gamma(0) \ll \gamma_j$, and was identified with γ_r in the second members of the equations. In our model, on the contrary, no assumptions are made on the magnitude of γ_j, which, therefore, can vary without limitations; that enable us to explore the regime $\gamma(0) \approx 1$ and, in general, all those cases in which γ_j has strong variations.

The linear analysis of system (1) has been made around the equilibrium solution given by

$$\tilde{A}=0, \quad \tilde{\alpha}=0, \quad \langle b \rangle = 0, \quad \gamma_j = \gamma(0) \; .$$

Assuming that all quantities vary exponentially as $\exp(i\lambda\tau)$, the dispersion relation turns out to be a fourth degree equation in λ, and the quantity $G = -\mathrm{Im}\lambda$ represents the growth rate of the unstable mode of the system.

The linear growth rate G versus $\gamma(0)$ is given in Fig. 1 for increasing values of Ω_p^2. The smooth transition from a situation of small plasma effects [curve (a), $\Omega_p^2 = 0.1$], which is similar, therefore, to that of the usual FEL, to a situation in which the plasma effects are dominant [curve (d), $\Omega_p^2 = 1$) is clearly shown. In fact, for a very small value of plasma density, the growth rate has a maximum for a value of the detuning parameter δ larger than zero ($\delta \approx 0.315$) as in the pure FEL case [see Fig. 1(a)]. Increasing the plasma density [Fig. 1(b)], this maximum becomes flat ($\delta \approx 0.27$) and a second peak appears at a negative value of the detuning ($\delta \approx -0.36$). Increasing further Ω_p^2, the original maximum at positive detuning disappears, and only the peak at negative detuning remains. Negative detunings, i.e., $\beta_0 < \beta_r$, correspond to low values of $\gamma(0)$.

The output of a typical nonlinear calculation is reported in Fig. 2. Curves (a) and (b) give the intensity $|\tilde{A}|^2$ of the scaled transverse potential and the scaled plasma density disturbance $|\tilde{\alpha}|^2$, respectively, as a function of the scaled time τ. In Fig. 2(c) the bunching factor $\langle b \rangle$ is represented, while Fig. 2(d) gives the average value $\langle \gamma \rangle$ of the energy of the electrons of the beam. These data were obtained with $\gamma(0) = 2.1$, $\Omega_p^2 = 1.1$, $\Omega_b^2 = 10^{-3}$, $\mu = 10^{-3}$, and $a_{w0}^2 = 1$, which may correspond to a plasma of low electron density ($n_p \approx 5 \times 10^{10}$ cm^{-3}) and with an electron temperature of about 0.5 eV, in a 10 cm wavelength wiggler. The initial conditions assumed were $\tilde{A}(0) = 2 \times 10^{-3}$, $\tilde{\alpha}(0) = 2 \times 10^{-3}$, and $\langle b \rangle(0) \approx 0$.

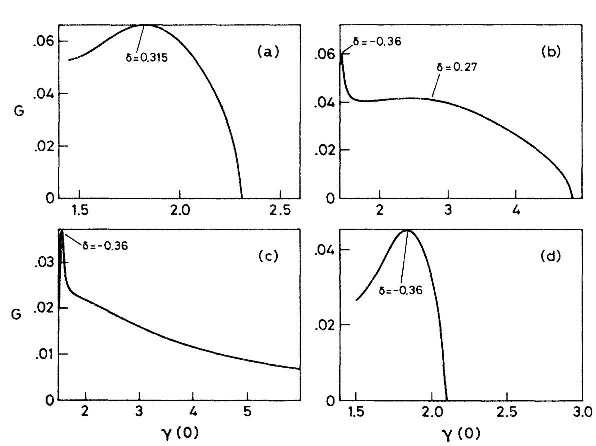

FIG. 1. Linear growth rate G versus $\gamma(0)$ for $\Omega_b^2 = 0.006$, $a_{w0}^2 = 1$, $\mu = 0.01$, and (a) $\Omega_p^2 = 0.1$, (b) $\Omega_p^2 = 0.3$, (c) $\Omega_p^2 = 0.6$, and (d) $\Omega_p^2 = 1$.

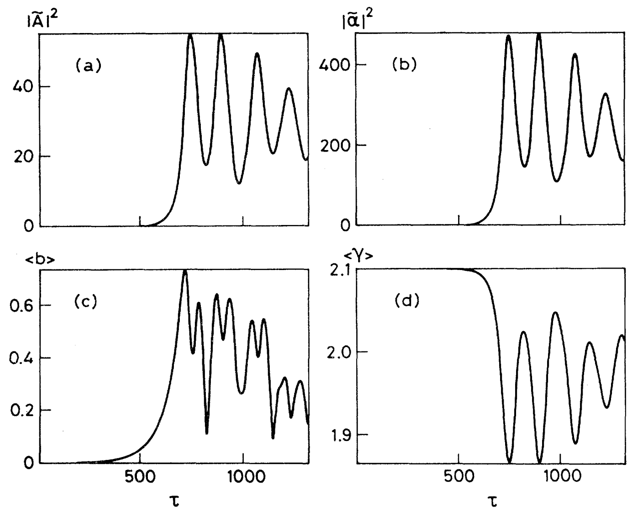

FIG. 2. (a) Intensity of the normalized transverse potential $|\tilde{A}|^2$ vs the normalized time $\tau = c(k+k_w)t$; (b) intensity of the plasma density disturbance $|\tilde{\alpha}|^2$ vs τ; (c) bunching factor $\langle b \rangle$ vs τ; (d) average value $\langle \gamma \rangle$ vs τ for $\Omega_p^2 = 1.1$, $\Omega_b^2 = 10^{-3}$, $a_{w0}^2 = 1$, $\mu = 10^{-3}$, and with the initial values $\gamma(0) = 2.1$, $\tilde{A}(0) = 2 \times 10^{-3}$, $\tilde{\alpha}(0) = 2 \times 10^{-3}$, and $\langle b \rangle(0) \approx 0$.

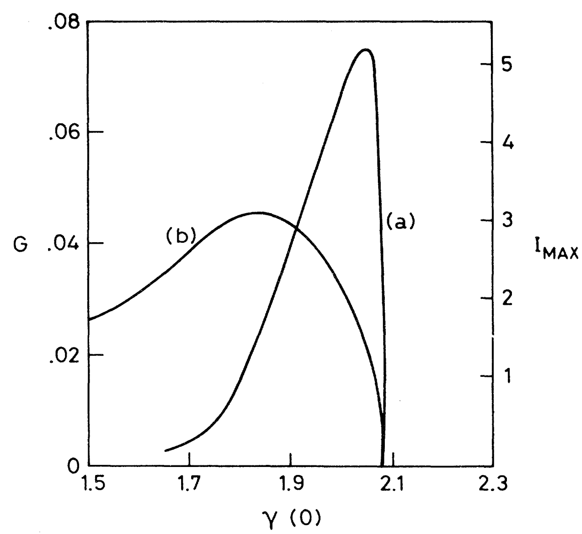

FIG. 3. (a) Maximum value of the intensity of the transverse potential given in FEL units vs $\gamma(0)$ for $\Omega_b^2 = 0.006$, $a_{w0}^2 = 1$, $\mu = 0.01$, and $\Omega_p^2 = 1$. (b) Linear growth rate for the same parameters.

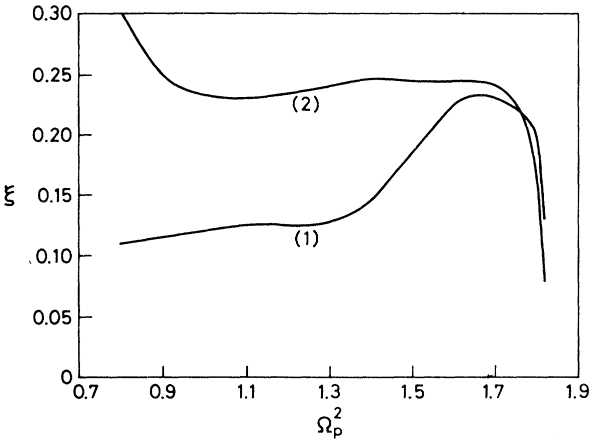

FIG. 4. Radiation efficiency ξ vs Ω_p^2 for $a_{w0}^2=1$, $\mu=2\times10^{-2}$, and (1) $\Omega_b^2=10^{-3}$ and (2) $\Omega_b^2=10^{-2}$.

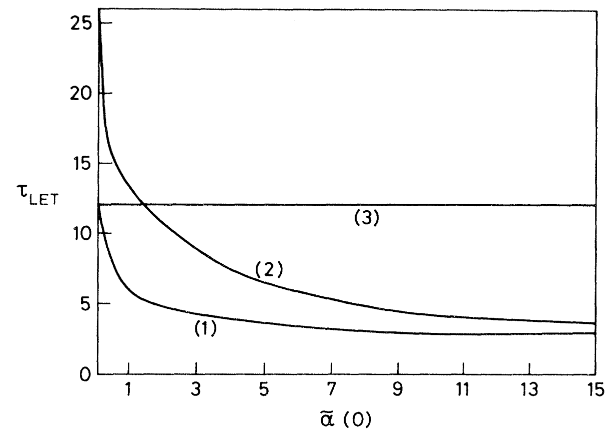

FIG. 5. Lethargy time τ_{LET} of the system vs the initial electrostatic field $\tilde{\alpha}(0)$ for $a_{w0}^2=1$, $\mu=2\times10^{-2}$, $\Omega_b^2=10^{-2}$, and (1) $\Omega_p^2=0.1$, (2) $\Omega_p^2=1.7$, and (3) in the FEL case.

Figure 3 gives the maximum value of the transverse field intensity $I_{\max}$ obtained during a temporal run [curve (a)] as a function of $\gamma(0)$, for fixed values of the other parameters. The intensity of the radiation field has been normalized in the following way:

$$I=\frac{|\tilde{A}|\beta_R\Omega_b^2[16(1-\mu)^2]^{2/3}}{2(1-\mu)(a_{W_0}\Omega_b)^{4/3}} .$$

This choice is justified by the fact that, if the transverse field is expressed in this unity, Eqs. (1), written in the limit $\Omega_p^2=0$ and $\gamma_j \gg 1$, turn out to be independent of parameters. Calling $I_{\rm FEL}$ the maximum value attained by the transverse field during a temporal run in this limit, it can be shown that $I_{\rm FEL}$ assumes the universal value 1.4 [4]. The comparison between the performance of the pure FEL and the plasma loaded FEL is therefore easy and immediate. For instance, in the case of Fig. 3, $I_{\max}$ reaches 5, which is more than three times larger than the value without plasma background. Furthermore, the curve (b) on the same figure represents the growth rate G, as provided by the linear theory for the same parameters. As one can see, the region of strong radiation falls inside the instability region predicted by the linear theory, but the peak of $I_{\max}$ does not coincide with the maximum of the growth rate G. It is shifted towards the border of the instability region, which corresponds to smaller negative detunings.

Figure 4 gives the quantity $\xi=[\Omega_b^2a_{w_0}^2/16\gamma(0)(1-\mu)^2]^{1/3}I$ as a function of Ω_p^2, for various values of Ω_b^2. This quantity is defined as the ratio between the output energy due to the transverse radiation field and the initial kinetic energy of the beam. It measures, therefore, the efficiency of the radiation generated by the system. Each point on the graph corresponds to the maximum value of ξ, obtained by varying $\gamma(0)$ and for fixed values of both Ω_p^2 and Ω_b^2. As one can see, the efficiency ξ reaches large values (even more than 25% in some cases), as compared with that of the usual FEL, which is only a few percent. Furthermore, the presence of the critical plasma density pointed out in Ref. [1] is confirmed. In our case the value of the critical plasma density corresponds to a value of Ω_p^2 of about 1.85 and we attribute the reduction in the efficiency occurring above this value to the fact that the phase velocity of the Langmuir wave $c\beta_r\cong\Omega_p$ exceeds c and, therefore, the electrons of the beam cannot be trapped anymore by the electrostatic potential of the Langmuir waves.

Another possible application of the plasma FEL is to use it as a buncher of electron beams. In fact, if a small electrostatic field is applied at $\tau=0$, the lethargy time of the system decreases considerably, while the value attained by the bunching factor $\langle b\rangle$ remains large ($\langle b\rangle\approx0.7$). Figure 5 shows this effect: the lethargy time $\tau_{\rm LET}=c(k+k_w)t_{\rm LET}$ (measured as the time corresponding to the first maximum of $\langle b\rangle$) is presented versus the applied Langmuir field $\tilde{\alpha}(0)$. The lethargy time of the usual FEL [curve (3)] is also reported for comparison. A third possible use of this system is for generating large electrostatic Langmuir waves in the plasma with phase velocity larger than c, which is one of the basic ingredients for the acceleration of electrons [see, for instance, Fig. 2(b)].

As a conclusion, we can say that, in a plasma loaded FEL, the concurrence of two instabilities (the usual FEL instability and the plasma-beam instability) permits us to obtain values of the efficiency of radiation production very much larger than in a usual FEL. The study of the radiation intensity as a function of the basic parameters of the system shows that the maximum efficiency is reached when the initial energy of the beam is small and when the phase velocity of the excited Langmuir wave is larger than c. The presence of a Langmuir wave can also contribute to diminish the time lethargy. Finally, preliminary results seem to indicate the possibility of obtaining, with this configuration, high-gradient acceleration of the electrons of the beam either when a strong electrostatic field is applied or when an external electromagnetic wave is injected into the system.

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