Transport coefficients in a weakly ionized nonequilibrium plasma

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The Boltzmann equation for an electron gas in a weakly ionized plasma is solved for an inhomogeneous plasma in a steady state but outside equilibrium (no Maxwell distribution). The inhomogeneity concerns both the electron density and the imposed dc electric field. With the emphasis on the case of a molecular gas discharge, the perturbed part of the electron distribution is evaluated only to the first order in gradients. The result is incorporated in the electron transport equations, where it leads to a renormalization of transport coefficients. Based on this procedure, a corrected version of the so-called quasihomogeneous equations is given.

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I. INTRODUCTION

In many problems associated with the physics of weakly ionized plasmas the need for an adequate description of plasma inhomogeneity is encountered. This concerns, for instance, the radial dependence of various plasma parameters in the positive column [1], wave phenomena [2], surface phenomena [3], and many other fields. Whereas the case of heavy particles constituting the plasma can usually be treated by standard methods [4], the electron gas merits special attention.

In swarm experiments [5] and a low-current discharge [6] the electron gas is so thin that the mutual electron Coulomb collisions are practically nonexistent and a single electron collides mainly with the neutral atoms or molecules of the background gas. At the same time the electron is accelerated between collisions by the imposed electric field. As a result, an electron gas velocity distribution is established, the shape of which depends largely on the form of collisional cross sections. For a homogeneous one-dimensional plasma a direct solution of the Boltzmann equation is possible, rendering a distribution function that, in principle, is not local.

In the absence of a local electron distribution, the formulation of equations of hydrodynamics is possible if a replacement for the missing local distribution is found. If the distribution of the homogeneous column is substituted for the local distribution, the set of so-called quasihomogeneous equations is derived [7]. For that purpose, the field dependence of the distribution of the homogeneous column is replaced by the dependence on the mean electron energy, which is then regarded as a local quantity governed by the electron energy equation. These somewhat artificially constructed equations of hydrodynamics render a realistic description of plasma inhomogeneity provided the energy correlation length (i.e., the length measured along the field over which a single electron "forgets" its initial energy; see [8]) is small compared with the inhomogeneity scale. In an atomic (noble gas) plasma, in a regime where the elastic energy loss is weak, a direct solution of the Boltzmann equation shows that the nonlocality or memory effects may appear if the correlation length is longer than the acceleration length $U_{\rm in}/E$, $U_{\rm in}$ being the equivalent potential of the first inelastic threshold and E the longitudinal field strength. These phenomena appear since, though the electron gas may be collision dominated with respect to the momentum scattering, it need not be collisional with respect to the energy scattering thanks to the tiny ratio of the electron-to-atom (or molecular) mass. On the other hand, in a molecular gas plasma there is a strong energy scattering mainly on the vibrational degrees of freedom of the molecules (the energy loss due to scattering on the rotational levels is comparable to the elastic scattering). The energy loss incurred by the vibrational excitation acts, in fact, in the same manner as the elastic scattering (for all practical purposes negligible in a plasma of self-sustained glow discharge in a molecular gas) in the case of an atomic gas and thus the correlation length is considerably shortened. The equations of local hydrodynamics should thus become adequate for a description of the molecular gas plasma. It was, however, pointed out in [9,10] that the first-order nonlocality corrections enter the hydrodynamic equations in such a way that they group with the hydrodynamic flow terms, leading to a modification of the transport coefficients. As shown in [9,10], e.g., the diffusion coefficient acquires, owing to the just described corrections, a tensorial character splitting in a lateral and a longitudinal part with respect to the field direction. The purpose of this paper is to find an analogous augmentation of other transport coefficients as they enter the set of quasihomogeneous equations [7].

Over the past ten years the problem of nonlocality corrections to the transport coefficients was discussed several times in the literature [11–18]. Some of these papers include, in addition to the spatial variation of the density and electric field, the temporal dependence [18]. None of them, however, spells out clearly the implica-

tions of the nonlocality for the renormalization of transport coefficients of the quasihomogeneous set of [7].

II. QUASIHOMOGENEOUS EQUATIONS

The Boltzmann equation for the electron gas in a weakly ionized plasma may, quite generally, be written as

$$\partial_t f+\mathbf{v}\cdot\partial_r f-\frac{e}{m}\mathbf{E}\cdot\partial_v f=S(f)\ , \tag{1}$$

where the collisional term on the right-hand side includes in our case only the collisions of the electrons with the neutral particles (and it is thus linear in the electron distribution function). The collision term incorporates, in principle, both the elastic and the inelastic collisions of the electrons with the heavy particles. Some of the inelastic collisions lead just to a rearrangement (shifts) of the electrons in the velocity space (e.g., excitation processes); others, in addition, produce new particles (ionization). These source terms have thresholds lying generally higher than the inelastic collision of the excitation type and are therefore less important. Moreover, the rearrangement of the electrons in the velocity space incurred by the inelastic energy loss (or gain in a superelastic case) is generally more important for the formation of the distribution than the additional electrons created in this kind of collision. In the following the source terms will thus not be considered.

The derivation of the quasihomogeneous equations [7] proceed, however, from the Boltzmann equation expanded in the velocity space:

$$f(\mathbf{r},\mathbf{v},t)=f_0(\mathbf{r},v,t)+\mathbf{f}_1(\mathbf{r},v,t)\cdot\mathbf{v}/v+\cdots\ , \tag{2}$$

$$\begin{aligned}3\left[\frac{mU}{2e}\right]^{1/2}&\partial_t f_0+\mathrm{div}(U\mathbf{f}_1)-\mathbf{E}\cdot\partial_U(U\mathbf{f}_1)\\&-n_g\frac{6m}{M}\partial_U(U^2Q_d f_0)\\&+3n_g\sum_a[UQ_a f_0(U)-(U+U_a)\\&\times Q_a(U+U_a)f_0(U+U_a)]=0\ ,\end{aligned} \tag{3}$$

$$\frac{1}{n_g Q_d}\left[\frac{mU}{2e}\right]^{1/2}\partial_t\mathbf{f}_1+\mathbf{f}_1=\frac{1}{n_g Q_d}(\mathbf{E}\partial_U f_0-\mathrm{grad}f_0)\ , \tag{4}$$

where f_0 and f_1 are the homogeneous and the axial part of the distribution, $\mathbf{E}$ is the vector of the electric field, U is the equivalent potential of the electron kinetic energy $U=mv^2/2e$, m and M are the electron and the neutral masses, e is the elementary charge, Q_d is the transport cross section, Q_a is the inelastic cross section, and n_g is the number density of neutral particles. Equations (3) and (4) differ in their time scales. Whereas the characteristic time of (3) is given by the energy scattering time, (4) is governed by the time scale characteristic of the momentum scattering. Regarding thus the time change in (4) as quasistationary, the time derivative can be omitted (neglect of the electron inertia with respect to the strong collisional momentum scattering):

$$\mathbf{f}_1=\frac{1}{n_g Q_d}(\mathbf{E}\partial_U f_0-\mathrm{grad}f_0)\ . \tag{5}$$

If (5) is substituted into (4), an integration of (4) yields the electron continuity equation

$$\partial_t n_e+\mathrm{div}(n_e\mathbf{v}_e)=0 \tag{6}$$

and the electron energy equation

$$\partial_t(n_e U_e)+\mathrm{div}\mathbf{q}_e+n_e\mathbf{v}_e\cdot\mathbf{E}=-n_e H_e\ , \tag{7}$$

where the electron number density n_e and the electron energy density U_e are given as

$$n_e=\int_0^\infty dU\,U^{1/2}f_0\ ,\quad U_e=\frac{1}{n_e}\int_0^\infty dU\,U^{3/2}f_0\ , \tag{8}$$

the electron energy loss per electron H_e is defined as

$$\begin{aligned}H_e=&\frac{1}{n_e}\left[\frac{2e}{m}\right]^{1/2}\int_0^\infty dU\,n_g\frac{2m}{M}U^2Q_d f_0\\&+\frac{1}{n_e}\left[\frac{2e}{m}\right]^{1/2}\sum_a U_a\int_{U_a}^\infty dU\,n_g UQ_a f_0\ ,\end{aligned} \tag{9}$$

and the energy and particle flows $\mathbf{v}_e$ and $\mathbf{q}_e$ are given by

$$\begin{aligned}\mathbf{v}_e=&\frac{1}{3}\left[\frac{2e}{m}\right]^{1/2}\frac{1}{n_e}\int_0^\infty dU\frac{U}{n_g Q_d}\partial_U f_0\mathbf{E}\\&-\frac{1}{n_e}\mathrm{grad}\left[\frac{1}{3}\left[\frac{2e}{m}\right]^{1/2}\int_0^\infty dU\frac{U}{n_g Q_d}f_0\right]\ ,\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{1}{n_e}\mathbf{q}_e=&\frac{1}{3}\left[\frac{2e}{m}\right]^{1/2}\frac{1}{n_e}\int_0^\infty dU\frac{U^2}{n_g Q_d}\partial_U f_0\mathbf{E}\\&-\frac{1}{n_e}\mathrm{grad}\left[\frac{1}{3}\left[\frac{2e}{m}\right]^{1/2}\int_0^\infty dU\frac{U^2}{n_g Q_d}f_0\right]\ .\end{aligned} \tag{11}$$

The expressions for the flows still contain the unknown isotropic part of the distribution f_0. In the spirit of the quasihomogeneous approximation, the solution of the Boltzmann equation for the homogeneous case $f_0^{(H)}$ is substituted for the unknown isotropic part f_0, in which the field dependence is expressed by the dependence on the mean electron kinetic energy. The relation between the energy and the field is again that derived for the homogeneous case, $U_e=U_e^{(H)}(E)$. In this way it is obtained for the particle and energy flows:

$$\mathbf{v}_e=-b_e\mathbf{E}-D_e\frac{1}{n_e}\mathrm{grad}n_e-\Delta_e\mathrm{grad}U_e\ ; \tag{12}$$

$$\frac{1}{n_e}\mathbf{q}_e=-b_e^*\mathbf{E}-D_e^*\frac{1}{n_e}\mathrm{grad}n_e-\Delta_e^*\mathrm{grad}U_e\ ; \tag{13}$$

$$\begin{aligned}b_e=&-\frac{1}{3}\left[\frac{2e}{m}\right]^{1/2}\int_0^\infty dU\frac{U}{n_g Q_d}\partial_U f_0^{(H)}\ ,\\D_e=&\frac{1}{3}\left[\frac{2e}{m}\right]^{1/2}\int_0^\infty dU\frac{U}{n_g Q_d}f_0^{(H)}\ ;\end{aligned} \tag{14}$$

$$b_e^* = -\frac{1}{3}\left[\frac{2e}{m}\right]^{1/2}\int_0^\infty dU \frac{U^2}{n_g Q_d}\partial_U f_0^{(H)} ,$$
$$D_e^* = \frac{1}{3}\left[\frac{2e}{m}\right]^{1/2}\int_0^\infty dU \frac{U^2}{n_g Q_d} f_0^{(H)} ; \quad (15)$$

$$\Delta_e = \frac{d}{dU_e^{(H)}} D_e , \quad \Delta_e^* = \frac{d}{dU_e^{(H)}} D_e ,$$
$$U_e^{(H)} = \int_0^\infty dU\, U^{3/2} f_0^{(H)} \quad (16)$$

and $f_0^{(H)}$ was assumed to be normalized to one:

$$\int_0^\infty dU \sqrt{U} f_0^{(H)} = 1 . \quad (17)$$

These expressions are usually rewritten in the following manner:

$$\mathbf{v}_e = -b_e\left[\mathbf{E} + \alpha U_e \frac{1}{n_e}\operatorname{grad} n_e + \gamma \operatorname{grad} U_e\right] , \quad (18)$$

$$\frac{1}{n_e}\mathbf{q}_e = \zeta U_e \mathbf{v}_e - b_e U_e\left[\xi U_e \frac{1}{n_e}\operatorname{grad} n_e + \delta \operatorname{grad} U_e\right] \quad (19)$$

to render the continuity and energy equations in the form of the quasihomogeneous set [7]:

$$\partial_t n_e - \operatorname{div}\left[n_e b_e\left[\mathbf{E} + \alpha U_e \frac{1}{n_e}\operatorname{grad} n_e + \gamma \operatorname{grad} U_e\right]\right] = 0 , \quad (20)$$

$$\partial_t (n_e U_e) + \operatorname{div}\left\{n_e\left[\zeta U_e \mathbf{v}_e - b_e U_e\left[\xi U_e \frac{1}{n_e}\operatorname{grad} n_e + \delta \operatorname{grad} U_e\right]\right]\right\} + n_e \mathbf{v}_e \cdot \mathbf{E} = -n_e H_e , \quad (21)$$

with the kinetic coefficients α, γ, ζ, ξ, and δ defined as

$$\alpha = \frac{D_e}{b_e U_e^{(H)}} , \quad \gamma = \frac{\Delta_e}{b_e} , \quad \zeta = \frac{b_e^*}{b_e U_e^{(H)}} ,$$
$$\xi = \frac{1}{b_e (U_e^{(H)})^2}\left[D_e^* - \frac{b_e^* D_e}{b_e}\right] ; \quad (22)$$

$$\delta = \frac{1}{b_e U_e^{(H)}}\left[\Delta_e^* - \frac{b_e^* \Delta_e}{b_e}\right] , \quad (23)$$

where b_e is the electron mobility, α the Einstein coefficient, γ the thermal diffusion coefficient, ζ the coefficient of the thermal convection, ξ the Soret coefficient, and δ the coefficient of the thermal conduction. For a Maxwell distribution function

$$f_0^{(H)} = \frac{2}{\sqrt{\pi}}\frac{1}{U_e^{3/2}}\exp(-U/U_e) \quad (24)$$

the Einstein coefficient $\alpha = \frac{2}{3}$ and the Soret coefficient $\xi = 0$.

The validity of this quasihomogeneous approximation for the flows depends on the ability of the electron gas to establish the "local" homogeneous distribution for given values of n_e and U_e. As mentioned in Sec. I, in a molecular gas plasma where the energy scattering on vibrational degrees of freedom proceeds in many tiny steps (compared to the mean electron energy) the correlation length is short and the homogeneous distribution can be regarded as local to a reasonable degree of accuracy.

Even if the validity of local hydrodynamics is substantiated, the above expressions are inconsistent. The field proportional term in (3) and (4) is of zeroth order when calculating the homogeneous distribution $f_0^{(H)}$, but it joins the gradient terms in the expressions for the flows, which are all first-order terms, disregarded in the evaluation of $f_0^{(H)}$. The way out of this difficulty has been pointed out in [9,10] and it consists in allowing for a small nonlocality of the distribution and in incorporating the resulting correction terms in the gradient terms to obtain contributions proportional to $\mathbf{E}\cdot\operatorname{grad} n_e$. This in effect endows the originally isotropic gradient terms with a tensorial character (with respect to the field direction). In [9,10] the interpretation of the swarm experiments is considered and the field is regarded as homogeneous with the correction originating from the density gradient only; see also [19–24]. However, in the discharge plasma a field inhomogeneity may also turn up, which should be accounted for in much the same way as the density gradient. In the following, the results of [9,10] will be extended to include, besides the density gradient, the field inhomogeneity.

III. GRADIENT CORRECTIONS TO THE HOMOGENEOUS DISTRIBUTION

Throughout this section the electric field will be regarded as inhomogeneous but static. The Boltzmann equation will be expanded to the first order in the density and the field gradients and in such a way the equations governing the gradient corrections to the homogeneous distribution will be derived. It would be possible to start from the Allis-Davydov version of the Boltzmann equation [(3) and (4)], but we shall, for the sake of simplicity, keep the original form of the Boltzmann equation (1) and the expansion in the velocity angular dependence will be carried out only subsequently. Restricting both expansions to the first-order terms, the results are independent of the order in which they are applied. The solution of (1) will be sought as

$$f(\mathbf{r},\mathbf{v},t) = n_e(\mathbf{r},t)\left[f^{(H)}[\mathbf{E}(\mathbf{r}),\mathbf{v}] + f_i^{(N)}(\mathbf{E},\mathbf{v})\frac{1}{n_e}\partial_{r_i} n_e + f_{ij}^{(E)}(\mathbf{E},\mathbf{v})\partial_{r_i} E_j\right] , \quad (25)$$

where $f^{(H)}$ is the solution for the homogeneous case and $f^{(N)}$ and $f^{(E)}$ are the corrections with the following normalization:

$$\int d\mathbf{v}\, f^{(H)}(\mathbf{v}) = 1 ,$$
$$\int d\mathbf{v}\, f_i^{(N)}(\mathbf{v}) = 0 , \quad (26)$$
$$\int d\mathbf{v}\, f_{ij}^{(E)}(\mathbf{v}) = 0 .$$

If (25) is substituted into (1), we obtain

$$\partial_t n_e f^{(H)}+(\partial_{r_i} n_e f^{(H)}+n_e\partial_{r_i}E_j\partial_{E_i}f^{(H)}+\cdots)-\frac{e}{m_e}E_i n_e\left[\partial_{v_i}f^{(H)}+\partial_{v_i}f_j^{(N)}\frac{1}{n_e}\partial_{r_j}n_e+\partial_{v_i}f_{jk}^{(E)}\partial_{r_j}E_k\right]$$
$$=n_e\left[S(f^{(H)})+S(f_i^{(N)})\frac{1}{n_e}\partial_{r_i}n_e+S(f_{ij}^{(E)})\partial_{r_i}E_j\right]. \quad (27)$$

In the first set of parentheses just the lowest-order terms are written explicitly. In the absence of production terms the time derivative of the density can be approximated from the continuity equation to first order in accuracy:

$$\partial_t n_e=-\mathrm{div}(n_e\mathbf{v}_e^{(H)})$$
$$=-\partial_{r_i}n_e v_{ei}^{(H)}-n_e\partial_{r_i}E_j\partial_{E_j}v_{ei}^{(H)}, \quad (28)$$

$$\mathbf{v}_e^{(H)}=\int d\mathbf{v}\,\mathbf{v}f^{(H)}. \quad (29)$$

After a suitable rearrangement of the terms the result is

$$\frac{e}{m_e}E_i\partial_{v_i}f^{(H)}+S(f^{(H)})+\left[\frac{e}{m_e}E_j\partial_{v_j}f_i^{(N)}+S(f_i^{(N)})-(v_i-v_{ei}^{(H)})f^{(H)}\right]\frac{1}{n_e}\partial_{r_i}n_e$$
$$+\left[\frac{e}{m_e}E_k\partial_{v_k}f_{ij}^{(E)}+S(f_{ij}^{(E)})-(v_i\partial_{E_j}f^{(H)}-\partial_{E_j}v_{ei}^{(H)}f^{(H)})\right]\partial_{r_i}E_j=0. \quad (30)$$

The leading terms cancel out since the $f^{(H)}$ was chosen as a solution of the homogeneous case. In a self-consistent model the electron density and the electric field would be tied together through (in the electrostatic approximation) the Poisson equation. The Poisson equation contains on its right-hand side also the ion density, which, together with the electrons, determines the total charge density. Since in the following no self-consistent model will be formulated, the ion density si regarded as arbitrary. Hence both the electron density and the field may be considered independent; the expressions in the two sets of large square brackets must vanish:

$$\frac{e}{m_e}E_j\partial_{v_j}f_i^{(N)}+S(f_i^{(N)})=(v_i-v_{ei}^{(H)})f^{(H)}, \quad (31)$$

$$\frac{e}{m_e}E_k\partial_{v_k}f_{ij}^{(E)}+S(f_{ij}^{(E)})=(v_i\partial_{E_j}f^{(H)}-\partial_{E_j}v_{ei}^{(H)}f^{(H)}). \quad (32)$$

The left-hand sides of (31) and (32) come from the collision as well as the field term; the right-hand sides are given by the space as well as the time derivative terms of the Boltzmann equation. The solutions of Eqs. (31) and (32) are the sought gradient corrections of the homogeneous distribution functions. The corresponding expression for the particle flow following from a substitution of (25) into (10) is thus given as

$$v_{ei}=v_{ei}^{(H)}-D_{ij}\frac{1}{n_e}\partial_{r_j}n_e-A_{ijk}\partial_{r_j}E_k, \quad (33)$$

$$D_{ij}=-\int d\mathbf{v}\,v_i f_j^{(N)}, \quad A_{ijk}=-\int d\mathbf{v}\,v_i f_{jk}^{(E)}, \quad (34)$$

where $\mathbf{v}_e^{(H)}$ is the unperturbed flow velocity for the homogeneous case, D_{ij} is the diffusion tensor, and A_{ijk} is the field gradient correction. The problem involves cylindrical symmetry with respect to the field direction and it is then easy to show (making still use of the relation rot $\mathbf{E}=0$) that Eq. (33) is equivalent to

$$\mathbf{v}_e=\mathbf{v}_e^{(H)}-D_\perp\frac{1}{n_e}\mathrm{grad}_\perp n_e-D_\parallel\frac{1}{n_e}\mathrm{grad}_\parallel n_e$$
$$-2A_{\perp\perp}\mathrm{grad}_\perp E-A_{\parallel\perp}\mathbf{e}\,\mathrm{div}\mathbf{E}$$
$$-(A_{\parallel\parallel}-A_{\parallel\perp})\mathrm{grad}_\parallel E, \quad (35)$$

with

$$D_{ij}=(\delta_{ij}-e_ie_j)D_\perp+e_ie_jD_\parallel,$$
$$A_{ijk}=[e_k(\delta_{ij}-e_ie_j)+e_j(\delta_{ik}-e_ie_k)]A_{\perp\perp}$$
$$-e_i[(\delta_{ik}-e_ie_k)A_{\parallel\perp}-e_je_kA_{\parallel\parallel}], \quad (36)$$

where $D_\perp$, $D_\parallel$, $A_{\perp\perp}$, $A_{\parallel\perp}$, and $A_{\parallel\parallel}$ are the only nonvanishing components of the above tensors in the system with $\mathbf{e}\|\hat{z}$ and $\mathbf{e}$ being the unit vector in the field direction $\mathbf{e}=\mathbf{E}/E$.

A quite analogous expression may also be derived for the energy flow:

$$\frac{1}{n_e}\mathbf{q}_e=\frac{1}{n_e}\mathbf{q}_e^{(H)}-D_\perp^*\frac{1}{n_e}\mathrm{grad}_\perp n_e-D_\parallel^*\frac{1}{n_e}\mathrm{grad}_\parallel n_e$$
$$-2A_{\perp\perp}^*\mathrm{grad}_\perp E-A_{\parallel\perp}^*\mathbf{e}\,\mathrm{div}\mathbf{E}$$
$$-(A_{\parallel\parallel}^*-A_{\parallel\perp}^*)\mathrm{grad}_\parallel E. \quad (37)$$

In (35) the split of the diffusion coefficient to the longitudinal and the perpendicular components can be recognized; the last terms in (35) and (37) represent similar corrections to the thermal diffusion and the thermal conductivity coefficients, respectively.

IV. EVALUATION OF THE CORRECTIONS

First, for a practical evaluation of the correction terms in (35) and (37) a coordinate system with the z axis paral-

lel to **E** will be used in which the few nonvanishing components of the above tensors are given as

$$D_{xx}=D_{yy}=D_{\perp}\ ,\quad D_{zz}=D_{\parallel}\ ,$$
$$A_{xxz}=A_{xzx}=A_{yyz}=A_{yzy}=A_{\perp\perp}\ ,$$
$$A_{zxx}=A_{zyy}=A_{\parallel\perp}\ ,\quad A_{zz}=A_{\parallel\parallel}\ . \tag{38}$$

Second, similarly to the case of the unperturbed Boltzmann equation, the distribution function will be expanded in the velocity space [see (3) and (4)] to render from (31) and (32) three separate equations

$$L^{(H)}f_0^{(N)}=2Uf_1^{(H)}+E\frac{d}{dU}\left[\frac{U}{n_gQ_d}\right]f_0^{(H)}+3\left[\frac{mU}{2e}\right]^{1/2}b_eEf_0^{(H)}\ , \tag{39}$$

$$L^{(H)}f_{\perp 0}^{(E)}=\frac{1}{E}Uf_1^{(H)}+3\left[\frac{mU}{2e}\right]^{1/2}b_eEf_0^{(H)}\ , \tag{40}$$

$$L^{(H)}f_{\parallel 0}^{(E)}=2\partial(Uf_1^{(H)})/\partial E-\frac{1}{E}Uf_1^{(H)}+E\frac{d}{dU}\left[\frac{U}{n_gQ_d}\right]\partial f_0^{(H)}/\partial E+3\left[\frac{mU}{2e}\right]^{1/2}b_eEf_0^{(H)}\ , \tag{41}$$

where the linear operator $L^{(H)}$ is defined as the time- and the space-independent left-hand side of (3), out of which $f_0^{(H)}$ is to be calculated:

$$L^{(H)}f_0^{(H)}\equiv\partial_U\left[\frac{U}{n_gQ_d}\partial_Uf_0^{(H)}\right]+n_g\frac{6m}{M}\partial_U(U^2Q_df_0^{(H)})-3n_g\sum_a[Q_af_0^{(H)}(U)-(U+U_a)Q_a(U+U_a)\times f_0^{(H)}(U+U_a)]=0\ , \tag{42}$$

$$f_1^{(H)}=\frac{1}{n_gQ_d}E\frac{d}{dU}f_0^{(H)}\ . \tag{43}$$

It is worth noting that the right-hand sides of Eqs. (39)–(41) cancel out upon integration over energy, i.e., they do not introduce any spurious source terms in the continuity equation and thus the correct behavior of the solutions for $U\to 0+$ is ensured by imposing $(U/n_gQ_d)dfU/dU\to 0$ at infinity [25]. The same property have the right-hand sides of the original equations (31) and (32). Once (39)–(41) are solved, the transport coefficients can be expressed by the following integrals:

$$D_{\perp}=D_e\ ,\quad A_{\perp\perp}=\Delta_e/2\frac{d}{dE}U_e^{(H)}\ , \tag{44}$$

$$D_{\parallel}=-\frac{1}{3}\left[\frac{2e}{m}\right]^{1/2}\int_0^{\infty}dU\frac{U}{n_gQ_d}\partial_Uf_0^{(N)}+D_e\ , \tag{45}$$

$$A_{\parallel\perp}=-\frac{1}{3}\left[\frac{2e}{m}\right]^{1/2}\int_0^{\infty}dU\frac{U}{n_gQ_d}\partial_Uf_{\perp 0}^{(E)}\ , \tag{46}$$

$$A_{\parallel\parallel}=-\frac{1}{3}\left[\frac{2e}{m}\right]^{1/2}\int_0^{\infty}dU\frac{U}{n_gQ_d}\partial_Uf_{\parallel 0}^{(E)}+\Delta_e\ , \tag{47}$$

$$D_{\perp}^{*}=D_e^{*}\ ,\quad A_{\perp\perp}^{*}=\Delta_e^{*}/2\frac{d}{dE}U_e^{(H)}\ , \tag{48}$$

$$D_{\parallel}^{*}=-\frac{1}{3}\left[\frac{2e}{m}\right]^{1/2}\int_0^{\infty}dU\frac{U^2}{n_gQ_d}\partial_Uf_0^{(N)}+D_e^{*}\ , \tag{49}$$

$$A_{\parallel\perp}^{*}=-\frac{1}{3}\left[\frac{2e}{m}\right]^{1/2}\int_0^{\infty}dU\frac{U^2}{n_gQ_d}\partial_Uf_{\perp 0}^{(E)}\ , \tag{50}$$

$$A_{\parallel\parallel}^{*}=-\frac{1}{3}\left[\frac{2e}{m}\right]^{1/2}\int_0^{\infty}dU\frac{U^2}{n_gQ_d}\partial_Uf_{\parallel 0}^{(E)}+\Delta_e^{*}\ . \tag{51}$$

For D_e, D_e^{*} and Δ_e, Δ_e^{*} see (14)–(16).

V. SOLUTION OF THE PERTURBED EQUATIONS

Equations (39)–(42) were solved in two different cases: in a model case of constant mean free path, roughly corresponding to the case of neon plasma (with inelastic collisions ignored), and then in a more realistic case of a molecular nitrogen discharge. The constant mean-free-path case may be treated to a large extent analytically, the final results being expressible as integrals over the incomplete γ function, which are then integrated numerically. The more complex case of molecular nitrogen is treated by a direct numerical solution of (39)–(42). The calculations were performed using the data for the molecular nitrogen from [26]. The vibrational temperature was kept zero. The results are presented in Table I. The longitudinal diffusion coefficient for the constant mean-free-path (second column of Table I) case agrees with the result given in [9,10].

VI. IMPROVED QUASIHOMOGENEOUS APPROXIMATION

The quantities $D_{\perp}$, $D_{\parallel}$, $A_{\perp\perp}$, $A_{\perp}$, and $A_{\parallel\parallel}$ cannot be used directly for a construction of the expressions for the currents analogous to [16,11] as the replacement of the field dependence by the mean energy dependence is now less trivial. To preserve the consistency of the first-order gradient expansion, the field dependence in the leading drift term in (10) and (11) must be expressed in terms of the mean energy to first order of accuracy in the gradient expansion

$$U_e=U_e^{(H)}(E)+U^{(N)}\mathbf{e}\cdot\frac{1}{n_e}\mathrm{grad}n_e+U_{\perp}^{(E)}\mathrm{div}\mathbf{E}+(U_{\parallel}^{(E)}-U_{\perp}^{(E)})\mathbf{e}\cdot\mathrm{grad}E\ , \tag{52}$$

where again for the symmetry reasons

$$\int d\mathbf{v}\frac{mv^2}{2e}f_i^{(N)}=e_iU^{(N)}\ , \tag{53}$$

$$\int d\mathbf{v}\frac{mv^2}{2e}f_{ij}^{(E)}=(\delta_{ij}-e_ie_j)U_{\perp}^{(E)}+e_ie_jU_{\parallel}^{(E)}\ , \tag{54}$$

TABLE I. Transport coefficients.

Coefficients (44)–(51)	Constant mfp	N_2 $(E/p_0$ [V/(cm Torr)]) 20	30	40	50
$\alpha_\perp = D_\perp / U_e^{(H)} b_e$	0.763	0.99	0.84	0.79	0.77
$(D_\parallel - D_\perp)/U_e^{(H)} b_e$	−0.388	−0.44	−0.25	−0.22	−0.21
$\gamma_\perp = 2A_{\perp\perp} \Big/ \left[b_e \frac{d}{dE} U_e^{(H)} \right]$	0.381	0.48	0.51	0.50	0.46
$(A_{\parallel\parallel} - 2A_{\perp\perp}) \Big/ \left[b_e \frac{d}{dE} U_e^{(H)} \right]$	−0.396	0.08	−0.01	−0.09	−0.13
$A_{\parallel\perp} E / U_e^{(H)} b_e$	−0.151	−0.11	−0.07	−0.06	−0.06
$(D_\parallel^* - D_\perp^*)/(U_e^{(H)})^2 b_e$	0.509	0.77	0.67	0.48	0.37
$(A_{\parallel\parallel}^* - 2A_{\perp\perp}^*) \Big/ \left[b_e U_e^{(H)} \frac{d}{dE} U_e^{(H)} \right]$	0.763	2.55	1.33	0.74	0.45
$A_{\parallel\perp}^* E / (U_e^{(H)})^2 b_e$	0.206	0.39	0.33	0.23	0.17

or, as in (45)–(47),

$$U^{(N)} = \int_0^\infty dU\, U^{3/2} f_0^{(N)} ,$$
$$U_\perp^{(E)} = \int_0^\infty dU\, U^{3/2} f_{\perp 0}^{(E)} , \tag{55}$$
$$U_\parallel^{(E)} = \int_0^\infty dU\, U^{3/2} f_{\parallel 0}^{(E)} .$$

The above dependence of U_e on E has to be inverted and the result substituted into the longitudinal drift term in (10) and (11). These additional gradient terms give contributions of the same order as the earlier calculated contributions listed in Table I. They thus combine in the following manner, leading to the modified values presented in Table II:

$$\bar{D}_\parallel = D_\parallel - E U^{(N)} db_e / dU_e^{(H)} ,$$
$$\bar{A}_{\parallel\perp} = A_{\parallel\perp} - E U_\perp^{(E)} db_e / dU_e^{(H)} ; \tag{56}$$
$$\bar{A}_{\parallel\parallel} = A_{\parallel\parallel} - E U_\parallel^{(E)} db_e / dU_e^{(H)} ,$$
$$\bar{D}_\parallel^* = D_\parallel^* - E U^{(N)} db_e^* / dU_e^{(H)} ; \tag{57}$$
$$\bar{A}_{\parallel\perp}^* = A_{\parallel\perp}^* - E U_\perp^{(E)} db_e^* / dU_e^{(H)} ,$$
$$\bar{A}_{\parallel\parallel}^* = A_{\parallel\parallel}^* - E U_\parallel^{(E)} db_e^* / dU_e^{(H)} . \tag{58}$$

It is interesting to note that the modified value of the longitudinal diffusion coefficient lies closer to the perpendicular coefficient than the original longitudinal coefficient corresponding to the field dependence of the mobility in the longitudinal drift term retained. The modified values of the tensorial components make possible a direct calculation of the modified values of the longitudinal transport coefficients in the expressions for the flows (18) and (19). In the gradient terms the gradient of the field magnitude may be expressed through the mean energy gradient using the simple relation derived for the homogeneous case, just as described in Sec. II. The perpendicular components of the currents remain the same as given by rela-

TABLE II. Modified transport coefficients.

Coefficients (56)–(58)	Constant mfp	N_2 $(E/p_0$ [V/(cm Torr)]) 20	30	40	50
$(\bar{D}_\parallel - D_\perp)/U_e^{(H)} b_e$	−0.073	−0.38	−0.20	−0.13	−0.09
$(\bar{A}_{\parallel\parallel} - 2A_{\perp\perp}) \Big/ \left[b_e \frac{d}{dE} U_e^{(H)} \right]$	0.184	0.25	0.10	0.05	0.05
$\bar{A}_{\parallel\perp} E / U_e^{(H)} b_e$	−0.021	−0.08	−0.04	−0.03	−0.02
$(\bar{D}_\parallel^* - D_\perp^*)/(U_e^{(H)})^2 b_e$	0.028	−0.28	−0.15	−0.08	−0.03
$(\bar{A}_{\parallel\parallel}^* - 2A_{\perp\perp}^*) \Big/ \left[b_e U_e^{(H)} \frac{d}{dE} U_e^{(H)} \right]$	−0.080	−0.17	−0.20	−0.18	−0.16
$\bar{A}_{\parallel\perp}^* E / (U_e^{(H)})^2 b_e$	0.01	−0.008	−0.04	−0.02	−0.00

TABLE III. Coefficients of quasihomogeneous equations.

Coefficients of (59) and (60)	Constant mfp	N_2 (E/p_0 [V/(cm Torr)]) 20	30	40	50
$\alpha_\parallel$	0.690	0.61	0.64	0.66	0.68
$\gamma_\parallel$	0.586	0.79	0.65	0.58	0.52
$\xi_\perp$	−0.250	−0.16	−0.11	−0.13	−0.15
$\xi_\parallel$	−0.111	0.16	0.06	−0.01	−0.04
$\delta_\perp$	0.789	1.70	0.98	0.77	0.71
$\delta_\parallel$	0.388	1.10	0.60	0.48	0.45
ζ	1.526	1.58	1.58	1.55	1.52
$(\overline{A}^*_{\parallel\perp}-b_e^*\overline{A}^*_{\parallel\perp}/b_e)E/(U_e^{(H)})^2 b_e$	0.041	0.05	0.02	0.02	0.02

tions (18) and (19). The longitudinal components, however, will now read

$$\mathbf{v}_{e_\parallel}=-b_e\left[\mathbf{E}+\alpha_\parallel U_e\frac{1}{n_e}\mathrm{grad}_\parallel n_e+\gamma_\parallel\mathrm{grad}_\parallel U_e+\frac{\overline{A}_{\parallel\perp}}{b_e}\mathbf{e}\,\mathrm{div}\mathbf{E}\right], \tag{59}$$

$$\frac{1}{n_e}\mathbf{q}_{e_\parallel}=\zeta U_e\mathbf{v}_{e_\parallel}-b_e U_e\left[\xi_\parallel U_e\frac{1}{n_e}\mathrm{grad}_\parallel n_e+\delta_\parallel\mathrm{grad}_\parallel U_e+\frac{1}{b_e}\left[\overline{A}^*_{\parallel\perp}-\frac{b_e^*}{b_e}\overline{A}_{\parallel\perp}\right]\mathbf{e}\,\mathrm{div}\mathbf{E}\right], \tag{60}$$

where the longitudinal transport coefficients are expressed in terms of the tensor components as

$$\alpha=\frac{\overline{D}_\parallel}{b_e U_e^{(H)}},\quad \gamma_\parallel=\left[\frac{d}{dE}U_e^{(H)}\right]^{-1}(\overline{A}_{\parallel\parallel}-\overline{A}_{\parallel\perp})/b_e, \tag{61}$$

$$\xi_\parallel=\frac{1}{b_e(U_e^{(H)})^2}\left[\overline{D}^*_\parallel-\frac{b_e^*\overline{D}_\parallel}{b_e}\right]. \tag{62}$$

The numerical values of the longitudinal coefficients are given in Table III and their dependence on the reduced electric field in the case of a N_2 plasma is shown in Fig. 1. It is seen in (59) and (60) that terms proportional to divE appear whose coefficients, however, remain fairly small. As implied above, the difference between the perpendicular and the longitudinal diffusion coefficients is somewhat moderated due to the introduction of the energy representation, the other transport coefficients undergoing similar modifications; cf. Table I.

VII. DISCUSSION AND CONCLUSIONS

The nonlocality corrections modify the kinetic coefficients of the quasihomogeneous equations [7] describing the particle and energy balance of the electron gas in a weakly ionized plasma. In particular, the longitudinal component of both the particle and the energy flows is changed, thus making a distinction not only between the longitudinal and the perpendicular (with respect to the direction of the imposed electric field [9,10]) diffusion coefficients, but changing in this way also the other kinetic coefficients In addition, a term appears in the expressions for the particle and the energy flows, proportional to the field divergence.

The replacement of the electric field magnitude E by the mean electron energy U_e using the relation between E and $U_e^{(H)}$ based on (16) [or the extended expression (52)], which is the method originally used in [7], requires the

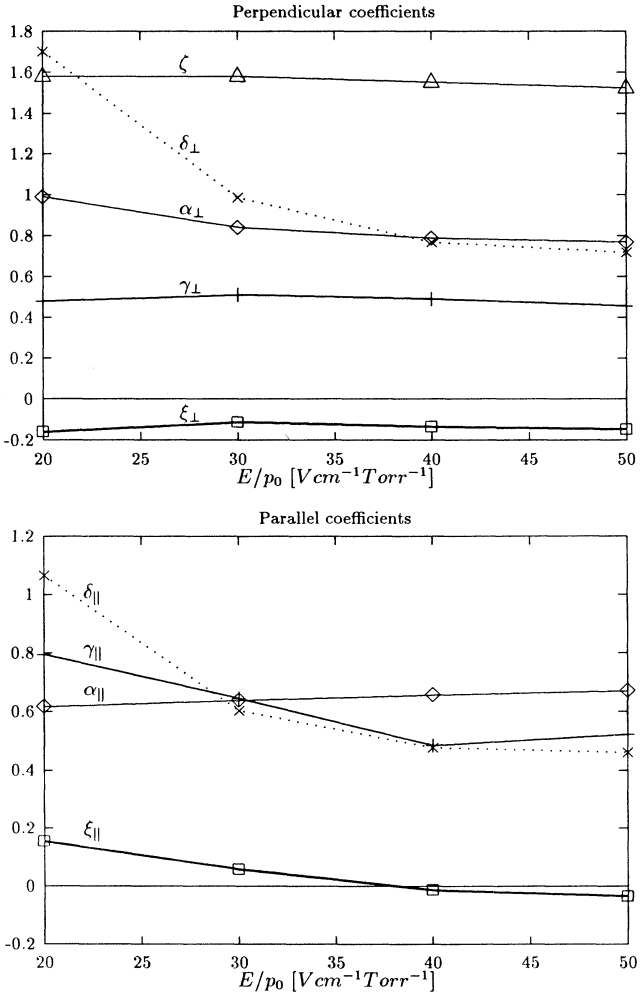

FIG. 1. Transport coefficients (Tables I and III) in the directions perpendicular and parallel to the imposed electric field in a N_2 plasma dependent on the reduced electric field E/p_0.

neglect of contributions from the higher-order derivatives of n_e and U_e to the flows. The same approximation should be applied to the energy loss term H_e (9) on the right-hand side of the energy equation (7). To maintain consistency, the correction terms in derivatives to second order would have to be included, which would eventually combine with analogous expressions on the left-hand side of (7). A more rigorous derivation of the transport equations could at best be made by applying the projection operator technique [27]. In this analysis, where a time-independent electric field was assumed, a more intuitive approach was used, which corresponds closely to the original derivation in [7].

A question may arise whether the higher-order derivatives cannot in some cases represent a non-negligible contribution and thus invalidate the solution of the quasihomogeneous set. This is indeed the case as it follows from a comparison between the direct solution of the Boltzmann equation and the solution of the quasihomogeneous set [8]. As mentioned earlier, the energy scattering in the case of plasma electrons is not dominated by the elastic collision frequency, but by its m/M multiple. With a high inelastic threshold U_{in} lying above the mean electron energy, the memory effects combine with the electron motion in the potential field between the inelastic collisions. For instance, for a small sinusoidal deviation from the homogeneous equilibrium, the vulnerable wavelengths are equal to U_{in}/E and its integral fractions. However, in the case of molecular gases the (vibrational) inelastic threshold is fairly low, it is smaller than the mean electron energy, and the inelastic collisions on vibrational levels act in the same way as the elastic collision, thus shortening the energy memory length. For that reason, the hydrodynamic approximation for the description of an electron gas in a weakly ionized molecular gas plasma makes sense.

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