Electric and thermal resistivities in dense high-Z plasmas

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Analytic expressions for the electric and thermal resistivities in dense high-Z plasmas have been obtained. The expressions incorporate phase-shift calculations of high-Z ion-sphere-model cross sections as well as existing quantum-mechanical transport calculations for hydrogen plasmas, and are applicable to fluid plasmas with $1 \leq Z \leq 26$; strong-coupling effects between electrons and ions are particularly clarified. It has been shown that the heat capacity for a dense plasma may have a considerable effect, modifying the rate of thermal conduction. The results are compared with other theoretical predictions for those plasma parameters appropriate to degenerate stars.

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I. INTRODUCTION

Astrophysical dense matter [1,2] found in the interior of a white dwarf or in the outer crust of a neutron star may be regarded as an electron-ion two-component plasma in which the ionic charge number Z may take on a value greater than unity. Electric and thermal resistivities in dense high-Z plasmas are physical quantities that are essential in the description of the structure and energy transport in such degenerate stars. These issues are also involved in the experimental study of transport properties in dense plasmas produced by intense laser irradiation [3].

In an earlier investigation, Tanaka, Yan, and Ichimaru [4] performed microscopic calculations of the electric and thermal resistivities in dense hydrogen plasmas through solution to quantum-mechanical transport equations for the electrons [5,6]. Strong interparticle correlations were treated rigorously in terms of the local-field corrections [5] obtained through a set of integral equations based on the hypernetted-chain (HNC) modified convolution approximation (MCA) scheme [4]. It has thus been found that in the vicinity of the metal-insulator boundaries, strong Coulomb coupling between electrons and ions brings about an "incipient Rydberg state (IRS)" for the electrons, and acts to enhance the resistivities beyond the Born approximation. Accounting for these strong electron-ion (e-i) coupling effects was the feature in the HNC MCA scheme [4], improving significantly over the existing theories [6–8] that had treated the e-i interaction only through a mean-field theoretic scheme such as the random-phase approximation [9].

Analytic formulas for the resistivities have then been derived on the basis of such an IRS representation of the microscopic plasma states [1]; the results have contributed significantly to our understanding of the physics contents of the formulas over those derived in the earlier expressions [4,6] on the basis of Padé approximants alone. It has also been shown [1] that those formulas accurately reproduce the numerical results [4] for hydrogen plasmas.

In this paper, we extend those results of hydrogen plasmas to the cases of high-Z plasmas, guided by an additional calculation of Z-dependent effects in the Coulomb cross sections for such plasmas. Transport cross sections for quantum-mechanical e-i scattering and their Z dependence are calculated in the strong-coupling regime by the phase-shift analyses with the aid of the ion-sphere model [9,10]. We then rederive analytic formulas for the electric and thermal resistivities in high-Z plasmas, in which the ion-sphere cross sections evaluated in the ion-sphere model are physically incorporated; again no Padé-like parametrizations have been employed. It is shown through comparison with the Coulomb logarithms of hydrogen plasmas computed in the HNC MCA scheme that the enhancement of resistivities arising from the strong e-i coupling is properly taken into account in the formalism. It is also demonstrated that the heat capacity in a dense plasma may introduce a substantial modification in the rate of thermal conduction.

In Sec. II, the phase-shift analyses of the cross section for the ion spheres are described. Parametrized formulas for the electric and thermal resistivities are presented in Sec. III. In Sec. IV, thermal resistivities in dense helium, carbon, and iron plasmas calculated by the present formulas are illustrated and compared with other theoretical predictions. Concluding remarks are given in Sec. V.

II. SCATTERING CROSS SECTIONS FOR THE ION SPHERES

We consider a fully ionized plasma consisting of ions (charge number Z; number density n_i) and electrons (mass m; number density $n_e = Zn_i$) at temperature T. A Coulomb-coupling parameter Γ for the ions and a Fermi-degeneracy parameter Θ for the electrons may be defined as

$$\Gamma = \frac{(Ze)^2}{ak_BT} , \tag{1}$$

$$\Theta = \frac{k_BT}{E_F} = \frac{2mk_BT}{\hbar^2(3\pi^2 n_e)^{2/3}} , \tag{2}$$

where

$$a=\left[\frac{3Z}{4\pi n_e}\right]^{1/3} \tag{3}$$

is the ion-sphere radius [9].

The ion-sphere model, originated by Salpeter [10], offers an essentially correct description of the interionic correlation in a strongly coupled ($\Gamma > 1$) plasma. In this model one regards an ion as being surrounded by a sphere of uniform negative charges with the radius a. Physically, the ion sphere represents a Coulomb hole stemming from exclusion of other ions around a given ion, caused by strong Coulomb repulsion at short distances.

To investigate how the resistivities may depend on the ionic charge number Z in the strong-coupling ($\Gamma > 1$) and degenerate ($\Theta \ll 1$) regime, we adopt the ion-sphere model and calculate the quantum-mechanical transport cross sections for the electron-ion scattering, in the ion-sphere potential given by

$$U(r)=\begin{cases} Ze^2\left[-\dfrac{1}{r}+\dfrac{3}{2a}-\dfrac{r^2}{2a^3}\right] & \text{for } r\le a \ , \\ 0, \quad \text{for } r>a \ . \end{cases} \tag{4}$$

The range of the electrostatic potential for an ion with charge Ze is thus confined within the radius a.

Electric and thermal resistivities are proportional to the transport cross section $Q_m(k_F)$ for those electrons on the Fermi surface with the wave number $k_F=(3\pi^2 n_e)^{1/3}$. The cross section may be calculated as

$$Q_m(k_F)=\frac{4\pi}{k_F^2}\sum_{l=0}^{\infty}(l+1)\sin^2(\delta_l-\delta_{l+1}) \ , \tag{5}$$

where δ_l is the phase shift in the radial wave function $R_l(r)$ for a scattered electron with orbital angular momentum l in the potential (4). The wave function may be determined by a solution to the Schrödinger equation,

$$\frac{d^2R_l(r)}{dr^2}+\frac{2}{r}\frac{dR_l(r)}{dr}+\left[k_F^2-\frac{l(l+1)}{r^2}-\frac{2mU(r)}{\hbar^2}\right]R_l(r)=0 \ , \tag{6}$$

under the short-range boundary conditions,

$$\lim_{r\to 0}\frac{R_l'(r)}{R_l(r)}=-\frac{Z}{a_B} \quad \text{for } l=0 \ , \qquad \lim_{r\to 0}R_l(r)=Cr^l \quad \text{for } l\ge 1 \ . \tag{7}$$

Here a_B is the Bohr radius, C is a constant, and the prime denotes a differentiation with respect to r.

Since $U(r)=0$ at $r>a$, the phase shift for a partial wave may be calculated as [11]

$$\tan\delta_l=\frac{k_Fj_l'(k_Fa)-\dfrac{R_l'(a)}{R_l(a)}j_l(k_Fa)}{k_Fn_l'(k_Fa)-\dfrac{R_l'(a)}{R_l(a)}n_l(k_Fa)} \ , \tag{8}$$

where $j_l(x)$ and $n_l(x)$ are the spherical Bessel function and the spherical Neumann function, respectively, of order l. Figure 1 plots the values of $Q_m(k_F)$ in Eq. (5) for $Z=1$, 6, and 26, obtained through a numerical solution to the Schrödinger equation (6) for $0\le r\le a$ with the boundary conditions (7), as functions of the electron density parameter,

$$r_s=a_e/a_B \ , \tag{9}$$

where $a_e=(3/4\pi n_e)^{1/3}$.

When the Fermi energy $E_F=(\hbar k_F)^2/2m$ of the electrons is much greater than a characteristic energy for the electron-ion interaction, $mZ^2e^4/2\hbar^2$, the scattering potential may be looked upon as a weak perturbation; the Born approximation applies. One thus finds

$$Q_m^{\text{Born}}(k_F)=\frac{m^2}{2\pi\hbar^4}\int_0^{\pi}d\theta\,\sin\theta(1-\cos\theta)|U(\mathbf{q})|^2 \ , \tag{10}$$

where

$$U(\mathbf{q})=\int d\mathbf{r}\,U(r)\exp(-i\mathbf{q}\cdot\mathbf{r}) =-\frac{4\pi Ze^2}{q^2}\left\{1-\frac{3}{(qa)^3}[\sin(qa)-qa\cos(qa)]\right\} \ , \tag{11}$$

and $\mathbf{q}$ is a scattering vector which satisfies $q=2k_F\sin(\theta/2)$. We then find that numerical results of Eq. (10) can be parametrized in an analytic expression of r_s and Z as

$$\frac{Q_m^{\text{Born}}(k_F)}{4\pi a^2}=9.09\times 10^{-2}r_s^2Z^{8/3}\exp(-1.47Z^{1/3}) \ . \tag{12}$$

Fitting errors of this formula are confined within 5% for $1\le Z\le 20$. Values of $Q_m^{\text{Born}}(k_F)$ so obtained are also exhibited in Fig. 1 for the density range $r_s<(9\pi/4)^{1/3}/Z$, which is identical to

$$E_F > mZ^2e^4/2\hbar^2 \ . \tag{13}$$

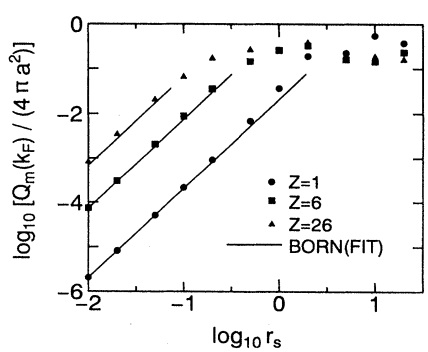

FIG. 1. Transport cross sections of scattering between electrons and ion spheres. Dots, squares, and triangles are the results obtained in the phase-shift analyses for $Z=1$, 6, and 26, respectively. Solid curves denote the corresponding results in the Born approximation, Eq. (12).

In fact, this is a condition for the plasma to be in a fully ionized state, which we assume in the present theory of Coulomb resistivities.

In such a high-density domain, we therefore observe that the results of the phase-shift analyses agree well with those of the Born approximation. Equation (12) thus correctly represents the scattering cross section of an ion sphere in a strongly coupled degenerate plasma for $1 \le Z \le 26$.

III. ELECTRIC AND THERMAL RESISTIVITIES

The electric and thermal resistivities ρ_E and ρ_T due to electron scattering may be expressed as [1]

$$\rho_E = \frac{8}{3}\left[\frac{\pi}{2}\right]^{1/2} \frac{Z^2 e^2 m^{1/2} n_i}{n_e (k_B T)^{3/2}} L_E \ , \tag{14a}$$

$$\rho_T = \frac{c_P^{(0)}}{c_P} \frac{52(2\pi)^{1/2}}{75} \frac{Z^2 e^4 m^{1/2} n_i}{n_e k_B (k_B T)^{5/2}} L_T \ , \tag{14b}$$

where L_E and L_T are the generalized Coulomb logarithms. Here c_P and $c_P^{(0)}$ refer to the specific heat at constant pressure for the plasma and that for the ideal gas of electrons, respectively, per unit volume.

An expression for the thermal resistivity, such as Eqs. (47) and (49) in Ref. [4], has been derived through a solution to a quantum-mechanical transport equation [5,6] for an ideal gas of electrons; in such a calculation, only the heat energy carried by the ideal-gas part of the electronic specific heat, $c_P^{(0)} T$, would be involved. The factor $c_P^{(0)}/c_P$ has been introduced in Eq. (14b) under the *assumption* that the plasma as a whole couples strongly with its partial component of the electrons, which themselves form a system of interacting, nonideal-gas particles. Under these circumstances, we expect that the electrons may transport heat energy by the amount $c_P T$ per unit volume, as in Eq. (14b), rather than by the amount $c_P^{(0)} T$, as in Eqs. (47) and (49) of Ref. [4].

In a classical ($\Theta \gg 1$) and weak-coupling ($\Gamma \ll 1$) limit, both L_E and L_T approach their Debye-Hückel (DH) limiting value [6],

$$L_0 = -\tfrac{1}{2}\ln\zeta - \frac{1}{2}\left[\gamma + \frac{1}{Z}\ln(Z+1)\right] + O(\zeta) \ , \tag{15}$$

where $\zeta = \hbar^2 (Z+1) k_{\mathrm{De}}^2 / 8 m k_B T$, $k_{\mathrm{De}} = (4\pi n_e e^2 / k_B T)^{1/2}$, and $\gamma = 0.577\,21\ldots$ is Euler's constant. Equation (15) was first derived by Kivelson and DuBois [12] with the aid of the quantum-mechanical version of the Balescu-Guernsey-Lenard equation [13].

For a degenerate ($\Theta \ll 1$) plasma with strong coupling ($\Gamma > 1$), we expect that both ρ_E and ρ_T may be proportional to the ion-sphere cross section, Eq. (12). In fact, in such a quantum limit, the Wiedemann-Franz relation [14],

$$\frac{\rho_E}{\rho_T} = \frac{\pi^2 k_B^2 T}{3 e^2} \ , \tag{16}$$

holds true.

Tanaka *et al.* [4] performed a set of microscopic calculations for L_E and L_T in hydrogen ($Z=1$) plasmas by solving the quantum-mechanical transport equations for electrons [5,6]. The effects of electron screening, ion-ion correlation, and local-field corrections, which are the functions accounting for the strong Coulomb coupling between charged particles beyond the random-phase approximation [1], have been all taken into consideration through numerical solutions to the HNC MCA integral equations in the density-response formalism; values of L_E and L_T are available for parametric combinations of Γ and Θ in the ranges $0.01 \le \Theta \le 10$ and $0.05 \le \Gamma \le 43.441$ [4].

Taking account of the limiting behaviors exhibited in Eqs. (12) and (15), the Wiedemann-Franz relation (16), and the raw data obtained in the HNC MCA theory [4], we now derive the analytic expressions for the Coulomb logarithms L_E and L_T, applicable over $1 \le Z \le 26$. The results are expressed in the parametrized formulas as

$$L_E = \tfrac{1}{2}\ln\left[1 + \alpha_E\left[\frac{1}{\zeta_{\mathrm{DH}}} + \tanh\frac{1}{\zeta_{\mathrm{Born}}}\right]\right] \times \{1 + A_E x_b^2 \exp(-C r_s^D) + B_E x_b^{10}[\exp(-C r_s^D)]^5\} \ , \tag{17a}$$

$$L_T = \tfrac{1}{2}\ln\left[1 + \alpha_T\left[\frac{1}{\zeta_{\mathrm{DH}}} + \tanh\frac{1}{\zeta_{\mathrm{Born}}}\right]\right] \times \{1 + A_T x_b^2 \exp(-C r_s^D) + B_T x_b^{10}[\exp(-C r_s^D)]^5\} \ . \tag{17b}$$

Here $\alpha_E = 1$, $\alpha_T = 75/13\pi^2$, and we have introduced dimensionless parameters,

$$\zeta_{DH} = \frac{(Z+1)^{1+1/Z}\exp\gamma}{(12\pi^2)^{1/3}} \frac{\Gamma_e}{\Theta} \ , \tag{18}$$

$$\zeta_{\mathrm{Born}} = \frac{1}{K\Theta^{3/2} Z^{4/3} \exp(-1.47 Z^{1/3})} \ , \tag{19}$$

with $\Gamma_e = e^2 / a_e k_B T$;

$$x_b = \left\{ r_s \tanh\left[\hbar\left[\frac{2\pi}{m k_B T}\right]^{1/2} n_e^{1/3}\right]\right\}^{1/2} \tag{20}$$

represents the IRS fractional parameter [1], which we shall discuss shortly. In these formulas, the parameters take on the values $A_E = 0.42$, $B_E = 0.063$, $A_T = 0.38$, $B_T = 0.049$, $C = 6\times 10^{-4}$, $D = 2$, and $K = 2.5$, which have been determined by fits to the values of Coulomb logarithms obtained in the HNC MCA calculations for hydrogen plasmas [4].

The parameter α_T has been determined on account of Eq. (16). The parameters, Eqs. (18) and (19), have stemmed from Eqs. (15) and (12), respectively. The principal logarithmic factors in the Coulomb logarithms, Eqs. (17a) and (17b), do not therefore involve any adjustable parameters; only the curly-bracketed factors describing an IRS effect contain adjustable parameters, which have been fitted to the raw data with the HNC MCA analyses. We stress the significance of these physical determinations, in contrast to arithmetic Padé fits in the Coulomb logarithms of Refs. [4] and [6].

The parametrized formulas thus retain the following features:

(i) In the classical ($\Theta \gg 1$) and weak-coupling ($\Gamma \ll 1$) regime, where $\zeta_{\rm DH} \ll 1$ and $\zeta_{\rm Born} \ll 1$, Eqs. (17) reproduce the Debye-Hückel values, Eq. (15).

(ii) In the degenerate ($\Theta \ll 1$) and strong-coupling ($\Gamma \gg 1$) regime, where $1 \ll \zeta_{\rm Born} < \zeta_{\rm DH}$, we have $L_E \approx \alpha_E / 2\zeta_{\rm Born}$ and $L_T \approx \alpha_T / 2\zeta_{\rm Born}$. We thus find in this regime that a transport cross section $Q_m(k_F)$ derived from ρ_E or ρ_T via a relation of the Drude type,

$$Q_m(k_F) = \frac{n_e e^2 \rho_E}{\hbar n_i k_F} , \tag{21}$$

becomes proportional to $Q_m^{\rm Born}(k_F)$ of Eq. (12). Consequently, Eqs. (17) correctly account for the Born scattering of ion spheres via the parameter $\zeta_{\rm Born}$.

(iii) The Wiedemann-Franz relation, Eq. (16), is appropriately satisfied.

When the density and/or temperature are lowered toward the vicinity of metal-insulator boundaries, given by

$$Z^2 e^4 m / 2\hbar^2 = E_F \ , \quad Z^2 e^4 m / 2\hbar^2 = k_B T \ ,$$

Coulomb coupling between electrons and ions becomes significant; probabilities of electrons assuming IRS need to be taken into consideration [1,4]. These states represent those of the electrons being scattered repeatedly in the short-range fields of the ions and thereby act to enhance the resistivities over those in the Born approximation.

Strengths of such an e-i coupling may be measured by the IRS fractional parameter x_b of Eq. (20). It is a parameter representing a ratio of Rydberg energy to an average kinetic energy of the electrons; in the classical and quantum limits, respectively, we find

$$x_b^4 = \begin{cases} (36\pi)^{1/3} \dfrac{e^2/2a_B}{k_B T} & \text{for } \Theta \gg 1 \ , \\ \left[\dfrac{9\pi}{4} \right]^{2/3} \dfrac{e^2/2a_B}{E_F} & \text{for } \Theta \ll 1 \ . \end{cases} \tag{22}$$

As the system approaches the metal-insulator boundaries, values of x_b may exceed unity and consequently Eqs. (17) may take on values enhanced over the Born resistivities.

It should be remarked, however, that the probabilities of electrons assuming an IRS should vanish in the low-density ($r_s \gg 1$) limit, since the electrons may rarely be found within a Bohr radius of an ion or an atom in these circumstances. The factor $\exp(-C r_s^D)$ accounts for such an effect.

In Table I, the values of L_E and L_T evaluated with Eqs. (17) for $Z=1$ are compared with those in the HNC MCA calculations; the accuracy of the analytic expressions (17) is clearly manifested. The formulas (14) and (17) are applicable for those high-Z plasmas with $1 \le Z \le 26$ over the entire parameter regime in the fluid phase.

It should also be remarked in these connections that, in the classical limit of $\Theta \gg 1$, Eqs. (14a) and (14b) take on values 1.97 and 1.66 times as large as the Spitzer values [15,16], respectively, for $Z=1$. The origin of the discrepancies may be traced to the microscopic calculations [4], in which it has been assumed that the deforma-

TABLE I. Generalized Coulomb logarithms for hydrogen plasmas in the HNC MCA theory ($L_E^{\rm HNC\ MCA}$ and $L_T^{\rm HNC\ MCA}$) compared with those of the fitting formulas (17) (L_E and L_T) at selective combinations of Θ and Γ.

Θ	Γ	x_b	$L_E^{\rm HNC\ MCA}$	L_E	$L_T^{\rm HNC\ MCA}$	L_T
10	0.05	0.565	2.732	2.803	2.363	2.473
10	0.1	0.800	2.710	2.716	2.256	2.330
10	0.2	1.131	3.295	3.129	2.557	2.530
10	0.35	1.496	8.041	7.758	5.573	5.480
5	0.1	0.659	2.097	2.128	1.725	1.795
5	0.3	1.142	2.506	2.311	1.853	1.770
5	0.5	1.474	5.002	5.202	3.384	3.485
1	0.1	0.388	0.9560	1.132	0.7507	0.8847
1	0.5	0.867	0.7073	0.7078	0.4836	0.4840
1	1.1	1.286	0.9144	0.9400	0.5828	0.5704
0.27151	0.2	0.312	0.2314	0.3652	0.1657	0.2419
0.27151	1.0	0.698	0.1448	0.1434	0.09325	0.08644
0.27151	2.5	1.104	0.1466	0.1219	0.09061	0.06958
0.1	0.5	0.303	0.04618	0.07530	0.02879	0.04517
0.1	2.0	0.606	0.02971	0.02962	0.01795	0.01728
0.1	5.4301	0.999	0.02663	0.02252	0.01580	0.01277
0.01	5.4301	0.316	7.062×10^{-4}	9.600×10^{-4}	4.130×10^{-4}	5.592×10^{-4}
0.01	10.0	0.429	5.776×10^{-4}	6.805×10^{-4}	3.377×10^{-4}	3.952×10^{-4}
0.01	30.0	0.743	4.450×10^{-4}	4.967×10^{-4}	2.601×10^{-4}	2.850×10^{-4}
0.01	43.441	0.894	4.483×10^{-4}	4.973×10^{-4}	2.619×10^{-4}	2.829×10^{-4}

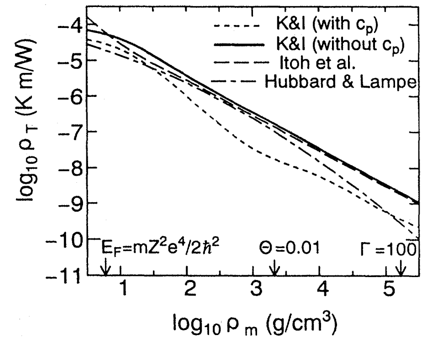

FIG. 2. Thermal resistivity of ^{4}He plasmas at $T=10^{5.5}$ K in various theories. "K&I (with c_P)" represents the present result with Eqs. (14b) and (17b); "K&I (without c_P)" is the result in which $c_P=c_P^{(0)}$ is assumed.

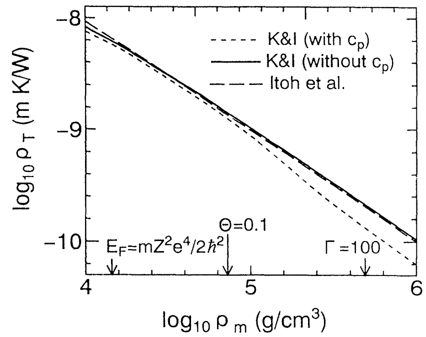

FIG. 4. Thermal resistivity of ^{56}Fe plasmas at $T=10^{7.5}$ K in various theories.

tion of the electron distribution is of a dipole shape corresponding to the adopted single Sonine polynomial approximation, while the Spitzer values have effectively taken into account all the terms in the Sonine polynomial expansion. In the application of Eqs. (14) for classical plasmas, the corrections arising from these considerations should be taken into account.

IV. COMPARISON WITH OTHER THEORIES

Hubbard and Lampe [17] computed the thermal resistivities for stellar matter over a wide range of density, temperature, and charge-number parameters. In the weak-coupling regime, the Chapman-Enskog method was used for the calculations. The resistivities in the solid phase were calculated by assuming plane waves for the Bloch states and Einstein oscillators for the ionic lattice vibrations. In the strong-coupling fluid regime of $10 \lesssim \Gamma \lesssim 100$, these authors then interpolated between the weak-coupling fluid and strong-coupling solid calculations.

Itoh *et al.* [18] calculated the resistivities in dense stellar matter on the basis of a relativistic version of the Ziman formula [19], in which they took into account the dielectric function for relativistically degenerate electrons in the random-phase approximation and the ionic structure factor for classical one-component plasmas. Since

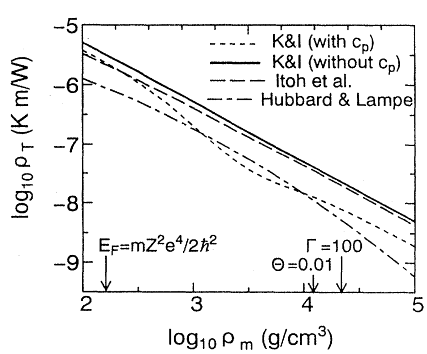

FIG. 3. Thermal resistivity of ^{12}C plasmas at $T=10^6$ K in various theories.

the present results are based on a nonrelativistic theory, the relativistic effects in the final expressions of Itoh *et al.* have been ignored in the present comparison.

In Figs. 2 and 3, various theoretical evaluations for the thermal resistivities are compared for ^{4}He plasmas ($Z=2$) at $T=10^{5.5}$ K and 3 g/cm^3 $\leq \rho_m \leq 3\times10^5$ g/cm^3, and for ^{12}C plasmas ($Z=6$) at $T=10^6$ K and 10^2 g/cm^3 $\leq \rho_m \leq 10^5$ g/cm^3, where $\rho_m = m_i n_i$, with m_i denoting the mass of an ion. These are typical conditions near the surfaces of white dwarfs [2]. We find that the present results "without c_P" agree fairly well with those of Itoh *et al.* in the quantum limit $\Theta \ll 1$. It seems, however, that the calculations of Hubbard and Lampe underestimate the resistivities, owing probably to their use of the fluid-solid interpolation scheme.

We remark that the possible effects of the total heat capacity, i.e., the differences between the present results "with c_P" and "without c_P," are substantial. The ratio $c_P^{(0)}/c_P$ takes on a value significantly smaller than unity due to strong Coulomb coupling between ions in a dense matter; hence, the thermal resistivity may assume a reduced value in such a condition. In the present evaluation of c_P, we have used the equation of state based on the IRS concept [1], which accurately reproduces the thermodynamic functions of hydrogen plasmas computed in the HNC MCA theory [4].

In Fig. 4, a similar comparison of the thermal resistivities for ^{56}Fe plasmas ($Z=26$) is exhibited for $T=10^{7.5}$ K and 10^4 g/cm^3 $\leq \rho_m \leq 10^6$ g/cm^3. Here again, a good agreement is observed between the present results with $c_P^{(0)}/c_P=1$ and those of Itoh *et al.*; the electrons are strongly degenerate under these conditions. The importance of accurate evaluation for the thermal resistivity in such a parametric regime was pointed out [20] in conjunction with the analysis of thermal structures in neutron star envelopes.

V. CONCLUDING REMARKS

We have presented the analytic expressions for the electric and thermal resistivities for high-Z plasmas in the fluid phase, where quantum-mechanical cross sections of scattering between the electrons and the ion spheres have been accurately incorporated in the strong-coupling regime. Enhancement of the resistivities due to strong electron-ion coupling has been taken into account in terms of IRS descriptions; the parametrized formulas accurately reproduce the resistivities of hydrogen plasmas obtained through the HNC MCA microscopic calculations and are applicable for $1 \le Z \le 26$. It has been shown that the heat capacity in a dense plasma may have a considerable effect modifying the rate of thermal conduction.

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