PHYSICAL REVIEW E VOLUME 51, NUMBER 6 JUNE 1995

Early stage scaling in phase ordering kinetics

F. Corberi,[1] A. Coniglio,[1] and M. Zannetti[2]

[1]*Dipartimento di Scienze Fisiche, Università Degli Studi di Napoli, Mostra d'Oltremare, Padiglione 19, 80125 Naples, Italy and Istituto Nazionale di Fisica della Materia, Unità di Napoli, Naples, Italy*

[2]*Dipartimento di Fisica, Università di Salerno, 84081 Baronissi (SA), Italy and Istituto Nazionale di Fisica della Materia, Unità di Salerno, Salerno, Italy*

(Received 22 December 1994)

A global analysis of the scaling behavior of a system with a scalar order parameter quenched to zero temperature is obtained by numerical simulation of the Ginzburg-Landau equation with conserved and nonconserved order parameters. A rich structure emerges, characterized by early and asymptotic scaling regimes, separated by a crossover. The interplay among different dynamical behaviors is investigated by varying the parameters of the quench and can be interpreted as being due to the competition of different dynamical fixed points.

PACS number(s): 64.60.My, 05.70.Fh, 64.60.Cn

I. INTRODUCTION

In recent years a great effort has been made to understand the scaling behavior observed in the late stage of the phase ordering process following the temperature quench of a system from an initial disordered state to a final state inside the coexistence region, below the critical point. A theoretical framework for the description of this phenomenon, well suited for both analytical and numerical approaches, is provided by the time-dependent Ginzburg-Landau (TDGL) model [1].

In the late stage of phase ordering, the order parameter saturates locally to the equilibrium values, giving rise to configurations with domains separated by sharp interfaces. In this regime, the only dynamics left in the system is therefore interface motion and the subsequent time evolution is characterized by the coarsening of domains, while their morphology remains invariant. This leads to the formulation of the scaling hypothesis [2], whereby the residual time dependence in the system is due only to the growth of the size of domains according to the power law $L(t) \sim t^{\frac{1}{z}}$, with $z = 2$ for the nonconserved order parameter (NCOP) and $z = 3$ for the conserved order parameter (COP). According to this hypothesis, the equal time order parameter correlation function obeys an asymptotic form of the type

$$G(|\vec{x} - \vec{x}'|, t) \sim L^{-\alpha}(t)\, F\left[\frac{|\vec{x} - \vec{x}'|}{L(t)}\right] , \tag{1}$$

where $\alpha = 0$ due to the formation of compact domains and F is a scaling function. This in turn implies the form

$$C(\vec{k}, t) \sim L(t)^{d-\alpha} \mathcal{F}[\vec{k} L(t)] \tag{2}$$

for the structure factor, which is the space Fourier transform of $G(|\vec{x} - \vec{x}'|, t)$. Although a full theory of scaling is yet to come, the prediction of the scaling hypothesis for the correlation function is confirmed by experiments [3], numerical simulations [4], and exactly soluble models for the NCOP [5]. Less is known about the behavior of quenched systems in the regime preceding the asymptotic dynamics [6]. At early times, domains are quickly formed, but the order parameter is still away from saturation inside the ordered regions. As a consequence, much more freedom is left to the system as compared to the asymptotic regime, since modulations of the field are also possible.

In this paper we present a global analysis of the scaling behaviors obtained by simulating the TDGL equation for a two-dimensional system and by varying the parameters of the Hamiltonian over the $T = 0$ manifold of the equilibrium phase diagram. The time evolution of the system is followed from the instant of the quench down to equilibration. In so doing, we uncover a structure much more rich than usually realized. In the very early stage, for t smaller than a crossover time t_1, the system relaxes towards equilibrium with a purely diffusive behavior. During this early regime the amplitude of the order parameter shrinks to zero in order to remove spatial inhomogeneities and the correlation function obeys the scaling form (1) with exponents $\alpha = d$ and $z = 2$ or 4, respectively, for the NCOP or the COP. Then, at $t \simeq t_1$, the system enters an intermediate regime characterized by exponential growth of the order parameter towards its local equilibrium value. Eventually, after a characteristic time t_2, the late stage scaling is asymptotically obeyed, with exponents $\alpha = 0$ and $z = 2$ or 3 for the NCOP or the COP. This whole structure is schematically represented in Fig. 1 with symbols to be specified in the following section. We stress that what we call early stage here precedes the usual early time behavior characterized by exponential growth. For common choices of the parameters entering the model the crossover time t_1 is too short to make the early stage observable. However, this can be greatly amplified by a proper choice of the parameters of the TDGL equation. For the COP this amplification can be obtained by performing asymmetric (off-critical) quenches.

In the present paper this structure is investigated by the numerical solution of the TDGL equation and is interpreted in terms of the interplay between different fixed

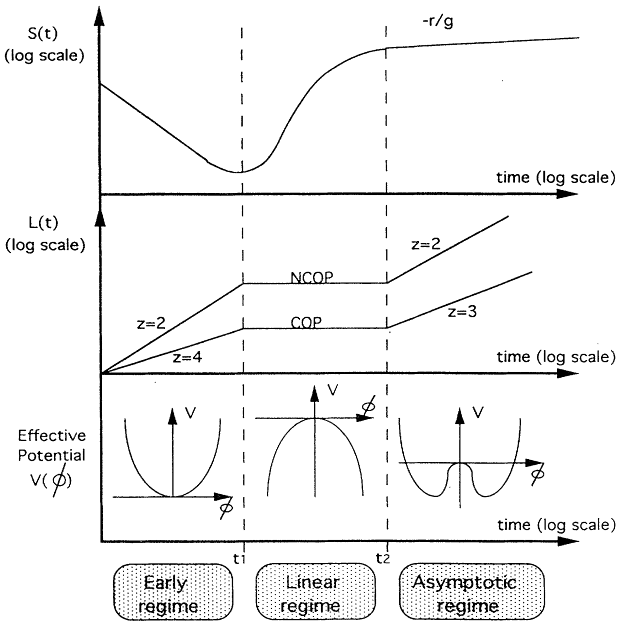

FIG. 1. Schematic representation of the dynamical response of the scalar model for both the NCOP and the COP. Three regimes are shown, separated by the crossover times t_1 and t_2.

points of competing stability. For short times the dynamics is controlled by the trivial fixed point of simple diffusion, whereas asymptotically the attractive ordering fixed point dominates. The intermediate regime corresponds to the crossover in the time interval $t_1 < t < t_2$. This interpretation is supported by the comparison with the exactly soluble large-N model, where similar properties are observed analytically [7].

The outline of the paper is as follows. In Sec. II we introduce the TDGL equation and set up the notation. In Sec. III we present the results of numerical simulations of the model in the scalar case, which clearly show the existence of the early scaling regime. Data are presented for the conserved and the nonconserved order parameter, critical and off-critical quenches. The role of the parameters entering the TDGL equation and their effect on the early stage behavior are elucidated. Section IV is dedicated to the comparison with the exact solution of the large-N model. Finally, in Sec. V a summary of the results is presented and concluding remarks are made.

II. MODEL

We consider a system with a scalar order parameter $\Phi(\vec{x},t)$, initially prepared in a configuration $\Phi(\vec{x},0)$, sampled from a high temperature uncorrelated state with expectations

$$\langle \Phi(\vec{x},0)\rangle = 0 \tag{3}$$

and

$$\langle \Phi(\vec{x},0)\Phi(\vec{x}',0)\rangle = \Delta\delta(\vec{x}-\vec{x}'), \tag{4}$$

where Δ is a constant. For $t \geq 0$ the time evolution is governed by the Langevin equation

$$\frac{\partial \Phi(\vec{x},t)}{\partial t} = -\Gamma(-i\nabla)^p \frac{\partial \mathcal{H}[\Phi,\mu]}{\partial \Phi(\vec{x},t)} + \eta(\vec{x},t), \tag{5}$$

where $\eta(\vec{x},t)$ is the Gaussian white noise produced by the thermal bath at the temperature of the quench T, μ is the set of parameters entering the free energy functional $\mathcal{H}[\Phi,\mu]$, and $p=0$ for the NCOP while $p=2$ for the COP.

In the following we take a free energy functional of the Ginzburg-Landau form

$$\mathcal{H}[\Phi,\mu] = \int d^dx \left[\frac{R}{2}(\nabla\Phi)^2 + \frac{r}{2}\Phi^2 + \frac{g}{4}\Phi^4\right], \tag{6}$$

with $\mu = (r,g,R)$. The equation of motion then becomes

$$\frac{\partial \Phi(\vec{x},t)}{\partial t} = -\Gamma(-i\nabla)^p\left[-R\nabla^2\Phi + r\Phi + g\Phi^3\right] + \eta(\vec{x},t). \tag{7}$$

In the study of deep temperature quenches usually one sets $T=0$ [i.e., $\eta(\vec{x},t)=0$ in Eq. (7)] regarding the temperature as an irrelevant parameter and r and g are chosen in the sector $r<0$, $g>0$ corresponding to final equilibrium states inside the ordering region. However, as it will be clear shortly, the edge of this sector is also of considerable interest (i.e., $r=0$, $g \geq 0$), even though no phase ordering occurs there since the equilibrium value of the order parameter vanishes. Thus the states relevant to our discussion are those in the $T=0$ plane of the (T,r,g) space with $r \leq 0$ and $g \geq 0$. We will set $R=1$ and $\Gamma = 1$ for simplicity.

III. NUMERICAL RESULTS

As stated in the Introduction, the quantity of interest is the equal time order parameter correlation function

$$G(|\vec{x}-\vec{x}'|,t) = \langle \Phi(\vec{x},t)\Phi(\vec{x}',t)\rangle - \langle \Phi(\vec{x},t)\rangle^2. \tag{8}$$

We consider first the quench to the trivial state with $\mu_1 \equiv (r=0,\ g=0)$. In this case Eq. (7) can be solved exactly and one finds

$$C(\vec{k},t) = \Delta e^{-2k^{2+p}t}. \tag{9}$$

Therefore, defining $L(t) = (2t)^{1/(2+p)}$, the real space correlation function is in the scaling form (1) with $\alpha = d$ and $z=2$ for the NCOP, $z=4$ for the COP, and $F(x) = \exp(-x^{2+p})$. Notice that in this case Eq. (9) is not just an asymptotic behavior, but it is obeyed exactly along the whole time history, from the instant of the quench onward. In the language of critical phenomena this means that the width of the critical region is maximally amplified, warranting the identification of μ_1 with a (trivial) fixed point on the $T=0$ manifold. The next step is the exploration of the domain of attraction of this fixed point and the search for other fixed points and the crossover induced by their competition.

Since for $g \neq 0$ the theory is not soluble, we proceed by numerical simulation. If scaling holds, from Eq. (1) we have $S(t) = G(0,t) \sim L^{-\alpha}(t) \sim t^{-\alpha/z}$. In the following we will use the behaviors of $L(t)$ and $S(t)$ to get the pair of exponents α and z. The evolution of $G(|\vec{x}-\vec{x}'|,t)$ is obtained by numerical solution of Eq. (7) for a two-dimensional 100×100 lattice. The equal time correlation function is obtained by averaging over different realizations of the time histories (the number of such realizations ranges from 1 to 20, according to the quality of the data). We have found it convenient to extract the characteristic length $L(t)$ from the half-height width of $G(|\vec{x}-\vec{x}'|,t)$.

We consider first the behavior of Eq. (7) for quenches on the $r = 0$ axis. In this case no phase ordering occurs, in the usual sense, since eventually the order parameter vanishes. Nevertheless, one can still observe the formation and the subsequent growth of domains that can be defined by locating their boundary on the contour $\Phi(\vec{x},t) = 0$. $L(t)$ can be thought of as the typical length associated to these structures.

The results of our simulations are presented in Fig. 2 for the NCOP, where the behaviors of $L(t)$ and of $S(t)$ are displayed for different values of g. We do not observe significant differences among the behaviors of $L(t)$ as g is varied and we conclude that $L(t)$ obeys an asymptotic growth law with an exponent consistent with $z = 2$. The quantity $S(t)$, on the other hand, shows a more complex behavior in that the asymptotic scaling $S(t) \sim t^{-\alpha/z}$ sets in after an initial transient, which widens as g becomes larger. For $g = 10$ the value of the exponent $\alpha/z = 1.2$ is slightly different from the one found for $g = 0.1$ and $g = 1$, where $\alpha/z = 1$ implying $\alpha = 2$. This is due the initial transient, which for $g = 10$ is not completed over the time of the simulation.

In conclusion, the data show an asymptotic scaling behavior identical to the one found in the quench to the trivial fixed point with $z = 2$ and $\alpha = d$. The critical regime where scaling holds shrinks, moving away from the trivial fixed point along the $r = 0$ axis.

The analogous results for the COP are presented in Fig. 3. Again the scaling behavior is obeyed by $L(t)$ and $S(t)$ with exponents consistent with $z = 4$ and $\alpha = 2$, but, differently from the NCOP, this feature sets in almost immediately after the quench, for every value of g. For both the NCOP and the COP, quenches to the $r = 0$ axis are controlled by the trivial fixed point. Therefore,

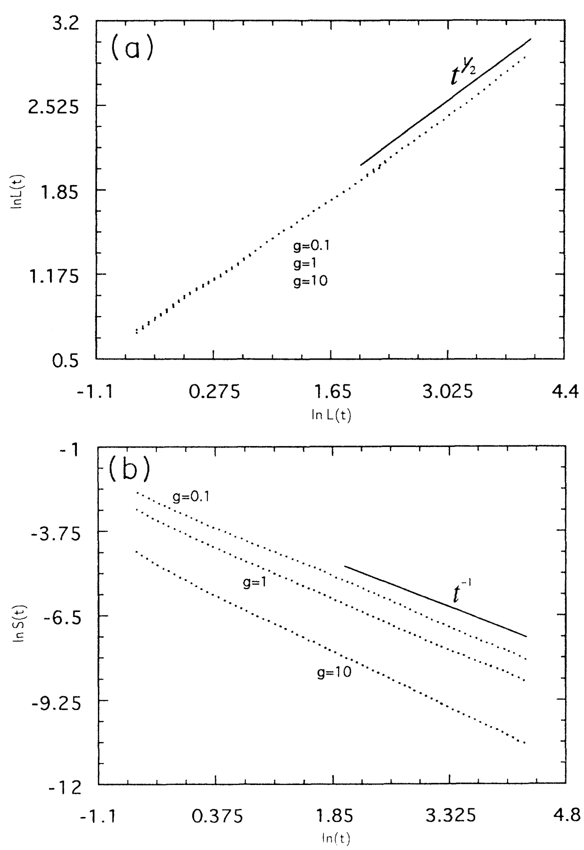

FIG. 2. Behavior of (a) $L(t)$ and (b) $S(t)$ for a nonconserved system quenched on the $r = 0$ axis, with different values of g ($g = 0.1$, 1, and 10). The continuous lines represent, respectively, the power laws $t^{\frac{1}{2}}$ and t^{-1} associated with trivial scaling. The best fit yields $z = 2.01 \pm 0.05$ for every value of g, $\alpha/z = 1.0 \pm 0.1$ for $g = 0.1$ and $g = 1$, and $\alpha/z = 1.2 \pm 0.1$ for $g = 10$.

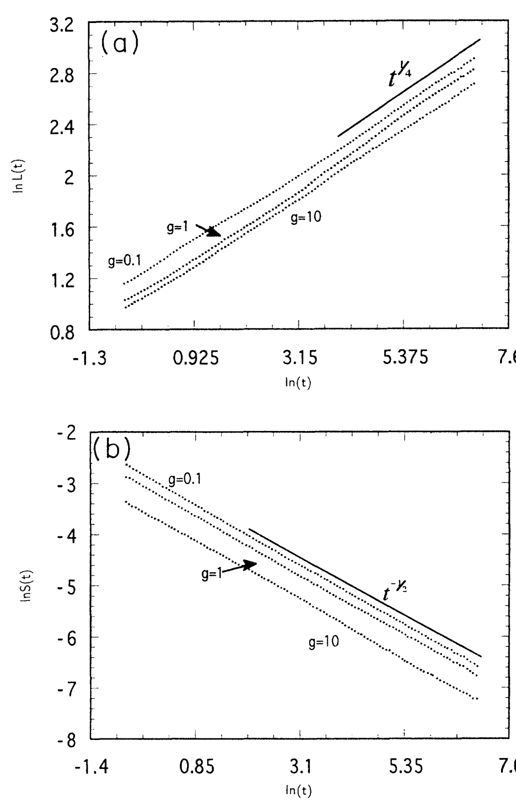

FIG. 3. Behavior of (a) $L(t)$ and (b) $S(t)$ for a conserved system quenched on the $r = 0$ axis, with different values of g ($g = 0.1$, 1, and 10). The continuous lines represent, respectively, the power laws $t^{\frac{1}{4}}$ and $t^{-\frac{1}{2}}$ associated with trivial scaling. The best fit yields $z = 3.98 \pm 0.05$ and $\alpha/z = 0.51 \pm 0.03$ for every value of g.

all quenches with $r = 0$ fall into the same universality class and the cubic term in Eq. (7) is asymptotically irrelevant.

We turn now to the phase ordering region by setting $r < 0$ and $g > 0$. When considering quenches inside this region, as has been well documented in the literature [4], the asymptotic scaling behavior is different from the one just found on the $r = 0$ axis. In the phase ordering region there exists a nontrivial fixed point characterized by $z = 2$ and $\alpha = 0$ for the NCOP and $z = 3$ and $\alpha = 0$ for the COP, whose domain of attraction is the entire sector $(r < 0, g > 0)$. For quenches sufficiently close to $r = 0$ axis, but inside the ordering region, we expect to observe the crossover from trivial to nontrivial scaling behaviors.

We have performed simulations with $g = 1$ and three different values of r, for both the NCOP (Fig. 4) and the COP (Fig. 5). For the NCOP, $L(t)$ scales with the same exponent $z = 2$ when the dynamics is dominated both by the fixed point of the ordered region or by the trivial one. More interesting is the behavior of $S(t)$, which is characterized by the sequence of three regimes. In the early regime, whose duration is shorter the farther away from the $r = 0$ axis the quench occurs, we find that $S(t)$ decreases with a behavior similar to the one found in the case of a quench on the $r = 0$ axis. During this early regime the order parameter evolves as if the potential did not have a double well and relaxes locally toward zero in order to eliminate spatial inhomogeneities as in the quenches to the trivial fixed point. This occurs because, as a consequence of the initial disordered state, the gradient term dominates in Eq. (6). Then, at some time t_1 (see the schematic representation of Fig. 1), the double well structure starts to play a role. At this point $S(t)$ stops decreasing and enters the intermediate time regime characterized by exponential growth of the order parameter toward local equilibrium at the bottom of the wells. In this regime $L(t)$ is approximately constant, as in the linear theories [8]. The intermediate regime terminates with the formation of domains within which the order parameter is close to saturation. From this point onward the late stage is entered with dynamics dominated by interface motion. Since the system now is close to saturation $S(t) \sim S(\infty) = -\frac{r}{g}$, implying scaling controlled by the nontrivial fixed point with $\alpha = 0$.

The case of the COP, shown in Fig. 5, is particularly interesting because this whole crossover structure manifests also in the growth law of the size of domains $L(t)$. In the first regime, whose duration is again controlled by the distance from the $r = 0$ axis, the dynamics is dominated by the trivial fixed point, with scaling exponents $z = 4$ and $\alpha = d$. As stressed before for quenches on the $r = 0$ axis, no initial transient is observed for the COP. This allows a precise determination of the exponents α and z in the early regime obtaining $\alpha = 2$ and $z = 4$. Then, when the system "feels" the presence of the local potential, the exponential growth of $S(t)$ is accompanied

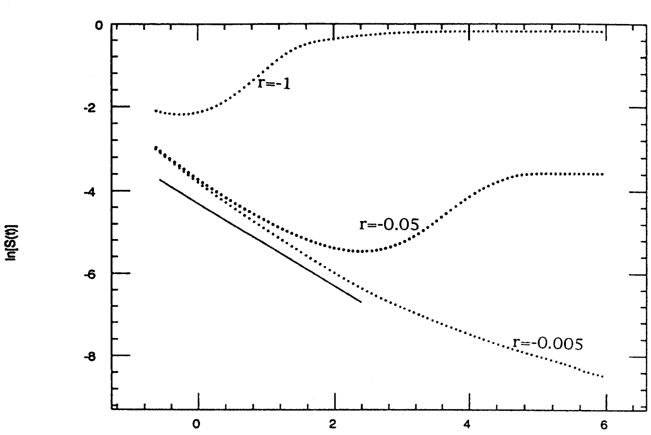

FIG. 4. Behavior of $S(t)$ for a nonconserved system quenched inside the phase consistence region, with $g = 1$ and different values of r ($r = -0.005$, $r = -0.05$, and $r = -1$). The duration of the early scaling regime increases as $|r|$ decreases.

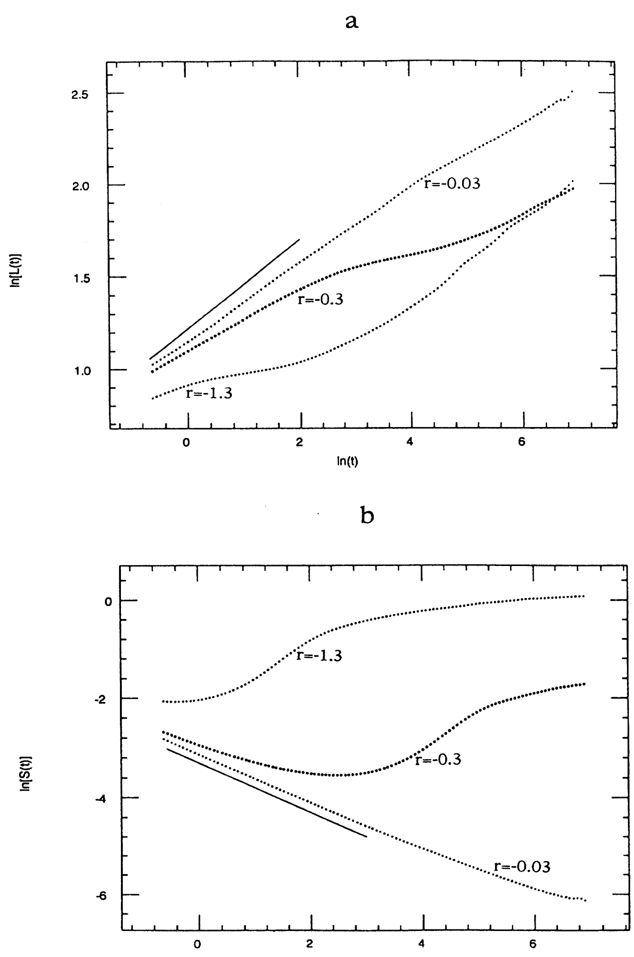

FIG. 5. Behavior of (a) $L(t)$ and (b) $S(t)$ for a conserved system quenched inside the phase consistence region, with $g = 1$ and different values of r ($r = -0.03$, $r = -0.3$, and $r = -1.3$). The continuous lines represent the trivial scaling behavior with $z = 4$ and $\alpha = 2$. The best fit of the data with $r = -0.03$ yields $z = 4.1 \pm 0.1$ and $\alpha/z = 0.49 \pm 0.05$.

by an approximately constant behavior of $L(t)$ (or, at least, by a lower growth rate), which characterizes the intermediate time regime. Later on, when $S(t)$ saturates to its asymptotic value, the system enters the late stage, dominated by the nontrivial fixed point with $\alpha = 0$ and $z = 3$ (this is not shown in Fig. 5 since the z exponent reaches its asymptotic value for much longer times).

Finally, we consider the effect of performing off-critical quenches, in the case of a conserved order parameter, by varying $m = \langle \Phi(\vec{x}, t) \rangle$ through the region $m^2 < -\frac{r}{g}$ (see Fig. 6). The asymptotic scaling behavior inside this region is still controlled [9] by the same nontrivial fixed point of the symmetric quenches, with $z = 3$ and $\alpha = 0$. The domain of attraction of this fixed point is therefore the entire region $m^2 < -\frac{r}{g}$ where phase separation occurs. At the intersection of the coexistence curve with the $T = 0$ manifold (i.e., at the very edge of this region, with $m^2 = -\frac{r}{g}$), no phase separation occurs since the field reaches a uniform configuration. In this case the amplitude of $G(|\vec{x} - \vec{x}'|, t)$, namely, $S(t)$, vanishes asymptotically approaching equilibrium, as for a critical quench on the $r = 0$ axis. More precisely, at this point of the phase diagram the long time behavior of the model is no longer controlled by the same fixed point as for critical quenches, which on the contrary requires $S(t) \to$ const, and we expect to observe a dynamics reminiscent of the $r = 0$, $m = 0$ case, where also $S(t) \to 0$. Therefore it is of considerable interest to understand the nature of the quenches close to $m^2 = -\frac{r}{g}$ and to study the crossover phenomena induced on the dynamics of the system for $m^2 < -\frac{r}{g}$.

In order to address these questions we have simulated asymmetric quenches by preparing the initial configurations with different values of m^2 in the range $(0, -\frac{r}{g})$. First of all, it must be stressed that, for quenches inside the metastability region, i.e., for m^2 sufficiently close but different from $-\frac{r}{g}$ (see Fig. 6), the influence of the ordering fixed point with $\alpha = 0$ and $z = 3$ is hardly observed because the system relaxes into a metastable state inside a single minimum of the local potential (the one of the majority phase). This can be avoided by increasing the variance Δ of the initial condition. When stable equilibrium is reached in the final state, the behavior of $S(t)$ reveals again the presence of three distinct dynamical regimes. In Fig. 7 this quantity is shown for quenches to $r = -1$ and $g = 1$ with different values of m. In the early regime, whose duration increases indefinitely approaching the coexistence curve, we again find the trivial scaling behavior with $\alpha = d$ and $z = 4$ as for $r = 0$ and $m = 0$. Later on the system enters the intermediate and eventually the asymptotic regime, with the same features described in the case of critical quenches. In the renormalization group language this means that, by increasing m from 0 toward the coexistence line, the trajectories start closer to the domain of attraction of the trivial fixed point $r = 0$, $m = 0$. This is analogous to the amplification of the early regime in the critical quench by decreasing the value of r. However, asymptotically, the nontrivial fixed point prevails for $m^2 < -\frac{r}{g}$. Therefore, in the space of the parameters (r, g, m), the region $m^2 = -\frac{r}{g}$ and the axis $r = 0$ axis are the domain of attraction of the trivial fixed point.

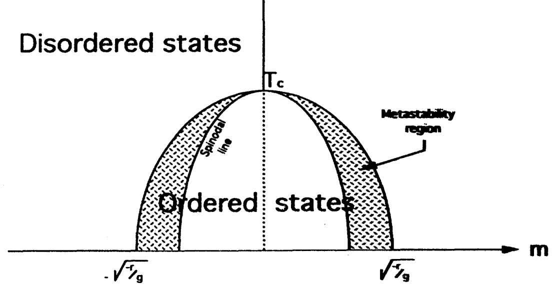

FIG. 6. Phase diagram in the (m, T) plane. The phase coexistence region and the spinodal line, above which metastability occurs, are shown.

We conclude this section observing that, as far as the role of the parameter R in Eq. (7) is concerned, by rescaling one can easily show that the effect of increasing R corresponds to a magnification of the space and time scales. Since R controls the range of the interaction, it is conceivable that the early stage scaling regime is more clearly observable in systems with sufficiently long range interactions.

IV. COMPARISON WITH THE LARGE-N MODEL

In this section we compare the results of our simulations with the solution of the large-N model [7]. When the order parameter is an N-component vector field $\vec{\Phi}(\vec{x}) = (\Phi_1(\vec{x}), ..., \Phi_N(\vec{x}))$ in the limit of an infinite number of components, Eq. (7) is linearized

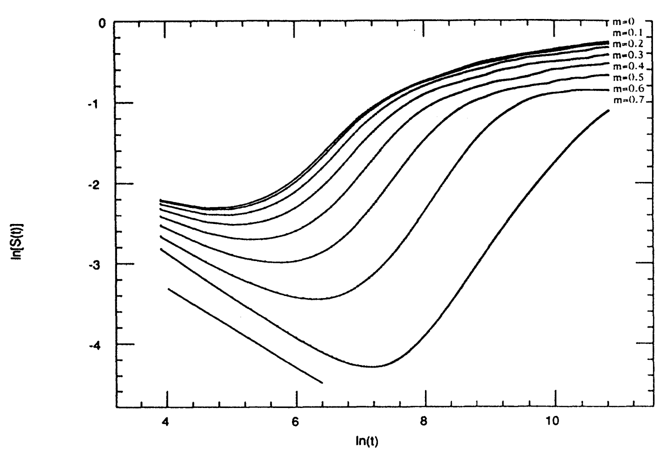

FIG. 7. Behavior of $S(t)$ for a conserved system with $r = -1$, $g = 1$, and m ranging between 0 and 0.7 (from top to bottom, $m = 0$, $m = 0.1$, $m = 0.2$, $m = 0.3$, $m = 0.4$, $m = 0.5$, $m = 0.6$, and $m = 0.7$). The lower straight line represents trivial scaling with $\alpha/z = 0.5$.

$$\frac{\partial \vec{\Phi}(\vec{x},t)}{\partial t} = -(i\nabla)^p \left[-\nabla^2 + r + gS(t)\right] \vec{\Phi}(\vec{x},t), \quad (10)$$

where $S(t)$ must be computed self-consistently through $S(t) = (1/N)\langle \vec{\Phi}^2(\vec{x},t)\rangle$. From the exact solution of this model one finds that the asymptotic scaling properties depend, as in the scalar case, on the pair of coupling constants r and g. More specifically, one finds that there is a universality class, under each heading NCOP or COP, for each of the following three regions in the (r,g) space: $\mu_1 \equiv (r=0, g=0)$ (trivial critical state), $\mu_2 \equiv (r=0, g>0)$ (nontrivial critical states), and $\mu_3 \equiv (r<0, g>0)$ (phase ordering region).

In the renormalization group language this means that there are three fixed points and the extension of the universality classes depends on the relative stability of these fixed points. Actually, for quenches to μ_2 there is a critical dimensionality d_c, which depends on the initial condition, above which the nonlinearity in the problem becomes irrelevant. For an initial condition of the type

$$C(\vec{k}, t=0) = \frac{\Delta}{k^\theta}, \quad (11)$$

one finds $d_c = d_l + \theta$, where d_l is the lower critical dimensionality of the static problem. With $\theta = 0$ one has $d_c = d_l = 2$. Therefore, solving the model for $d > 2$, one finds the following asymptotic behaviors for the NCOP:

$$C(\vec{k},t) \sim F(x) \text{ for } \mu_1, \quad (12)$$

$$C(\vec{k},t) \sim F(x) \text{ for } \mu_2, \quad (13)$$

$$C(\vec{k},t) \sim L^d(t) F(x) \text{ for } \mu_3, \quad (14)$$

with $L(t) \sim t^{1/2}$ and $F(x) = e^{-x^2}$.

Let us briefly comment on these results. First of all, we have $z=2$ everywhere. For quenches to the trivial state μ_1 the large-N model and the scalar model coincide since the local potential is absent. The same expression (9) and (12) is found for the structure factor. For quenches to μ_2, the long time behavior is the same as for the quench to μ_1, up to corrections to scaling, as expected since $d > d_c$. The scaling behavior is characterized by $\alpha = d$ and $z = 2$, both for quenches to μ_1 and μ_2.

Different is the case of quenches inside the region of coexisting phases, where Eq. (14) is asymptotically obeyed in any dimension, showing that no upper critical dimensionality exists and therefore that the nonlinearity of the problem is always relevant. In this case scaling is characterized by $\alpha = 0$ and $z = 2$. Furthermore, computing the early time behavior for $|r|$ sufficiently small, a scaling regime is found that is identical to the one found for quenches to μ_1. This behavior is separated from the asymptotic scaling regime described above by an intermediate regime of exponential growth.

In the large-N model the variety of asymptotic properties is more complex when quenches with the COP are considered, due to the existence of multiscaling. The following behaviors are found:

$$C(\vec{k},t) \sim F(x) \text{ for } \mu_1, \quad (15)$$

$$C(\vec{k},t) \sim F(x) \text{ for } \mu_2, \quad (16)$$

$$C(\vec{k},t) \sim L(t)^{\alpha(x)} \text{ for } \mu_3, \quad (17)$$

where $L(t) \sim t^{1/4}$, $F(x) = e^{-(x^4+cx^2)}$, and $\alpha(x) = d[1-(x^2-1)^2]$ is a generalized x-dependent exponent. The general discussion of fixed points and their relative stability goes along the same lines as for the NCOP.

Now we make a qualitative comparison with the results for the scalar case presented in Sec. III. The first observation concerns the existence of an early stage scaling regime that is observed in both models for quenches in the phase ordering region, with the same exponents α and z. This was to be expected since the very nature of this behavior is due to the trivial fixed point with $r = g = 0$, where the two models coincide. We emphasize that the occurrence of early scaling is particularly interesting for the conserved scalar order parameter since in this case, different from the large-N limit, a crossover from $z=4$ to $z=3$ exists in the growth law. Second, we comment on the existence of a critical dimensionality for quenches to μ_2. In the large-N model $d_c = d_l = 2$. In the scalar case $d_l = 1$. However, we do not know whether d_c is still equal to d_l. The results of Sec. III indicate that $d=2$ is above the critical dimensionality. For what concerns the late stage regime, the two models are quite similar for the NCOP, since scaling is obeyed with the same exponents. With the COP, on the contrary, there is less of an analogy since the vector model exhibits multiscaling.

V. CONCLUSIONS

In this paper we have analyzed the quench to zero temperature of a system with a scalar order parameter through the numerical solution of the time-dependent Ginzburg-Landau equation, both with conserved and nonconserved order parameters. We have paid particular attention to the early stage of the quench. According to the values of the parameters r and g, which characterize the final equilibrium state, an early stage scaling regime associated with pure diffusive behavior may be observed before the usual late stage scaling regime sets in. This is a crossover pattern of the same type found in the analytical solution of the large-N model. In that case the pure diffusive behavior is associated with the trivial fixed point $(r=0, g=0)$, while the late stage scaling behavior is associated with the nontrivial fixed point $(r<0,\ g>0)$ of phase ordering. The competition between the two fixed points determines the crossover pattern. The question of the observability of this phenomenon is clearly related to the time span of the early stage. In this paper we have mentioned long range forces and off-critical quenches toward the coexistence curve as means of amplification of the early stage.

[1] J.D. Gunton, M. San Miguel, and P.S. Sahni, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J.L. Lebowitz (Academic, New York, 1983), Vol. 8, p. 267.

[2] K. Binder and D. Stauffer, Phys. Rev. Lett. **33**, 1006 (1974); J. Marro, J.L. Lebowitz, and M.H. Kalos, *ibid.* **43**, 282 (1979); H. Furukawa, Prog. Theor. Phys. **59**, 1072 (1978); Phys. Rev. Lett. **43**, 136 (1979).

[3] Y.C. Chou and W.I. Goldburg, Phys. Rev. A **23**, 858 (1981); S. Katano and M. Iizumi, Phys. Rev. Lett. **52**, 835 (1984); S. Komura, K. Osamura, H. Fujii, and T. Takeda, Phys. Rev. B **31**, 1278 (1985); B.D. Gaulin, S. Spooner, and Y. Mori, Phys. Rev. Lett. **59**, 668 (1987).

[4] O.T. Valls and G.F. Mazenko, Phys. Rev. B **34**, 7941 (1986); Y. Oono and S. Puri, Phys. Rev. A **38**, 434 (1988); **38**, 1542 (1988); J. Amar, F. Sullivan, and R. Mountain, Phys. Rev. B **37**, 196 (1988); T.M. Rogers, K.R. Elder, and R.C. Desai, *ibid.* **37**, 9638 (1988); R. Toral, A. Chakrabarti, and J.D. Gunton, *ibid.* **39**, 4386 (1989); M. Mondello and N. Goldenfeld, Phys. Rev. A **42**, 5865 (1990); Phys. Rev. E **47**, 2384 (1993); A. Shinozaki and Y. Oono, Phys. Rev. Lett. **66**, 173 (1991); Phys. Rev. E **48**, 2622 (1993).

[5] T. Nagai and K. Kawasaki, Physica A **134**, 483 (1986); A. Coniglio and M. Zannetti, Europhys. Lett. **10**, 575 (1989); A.J. Bray, J. Phys. A **22**, L67 (1990); J.G. Amar and F. Family, Phys. Rev. A **41**, 3258 (1990).

[6] W. Klein, Phys. Rev. Lett. **65**, 1462 (1990); M. Laradji, M. Grant, M.J. Zuckermann, and W. Klein, Phys. Rev. B **41**, 4646 (1990); M. Grant, M. San Miguel, J. Viñals, and J.D. Gunton, *ibid.* **31**, 3027 (1985).

[7] A. Coniglio, P. Ruggiero, and M. Zannetti, Phys. Rev. E **50**, 1046 (1994).

[8] J.W. Cahn and J.E. Hilliard, J. Chem. Phys. **31**, 688 (1959); J.W. Cahn, Acta Metall. **9**, 795 (1970).

[9] A. Chakrabarti, R. Toral, and J.D. Gunton, Phys. Rev. E **47**, 3025 (1993); G.F. Mazenko and R.A. Wickham (unpublished).