Nonlinear oblique modulation of ion-acoustic waves in a multicomponent plasma with negative ions

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The stability of oblique modulation of ion-acoustic waves in a plasma consisting of positive ions, electrons, and negative ions is investigated. A nonlinear Schrödinger equation that governs the nonlinear interaction of a quasistatic plasma slow response with ion-acoustic waves for the system is derived. It is found that for a given value of negative-ion concentration (α), there exists a range of obliqueness (θ) corresponding to every value of wave number (k) for which the wave would be modulationally unstable. For a given value of $\alpha<\alpha_c$, and k lying in the range $0<k<k_c$, the wave is unstable for values of $\theta<\theta_c$, whereas, for the values of $k>k_c$, the wave is unstable for values of $\theta>\theta_c$. But for a given value of $\alpha>\alpha_c$, and k lying in the range $0<k<k_c$, the wave would be unstable for values of $\theta>\theta_c$, and for $k>k_c$, the wave is unstable for values of $\theta<\theta_c$.

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I. INTRODUCTION

Studies of nonlinear wave phenomena in plasmas provide a firm base not only for exploring fundamental researches on nonlinear physics, but also for developing practical applications in controlled nuclear fusion technology. The study of wave propagation gives basic information and imports physical understanding about the dynamical behavior of plasma. For proper understanding of the propagation of monochromatic waves in plasmas, consideration of nonlinear effects which can disturb the wave structure is essential. The modulational instability is one of such fundamental effect which needs thorough investigations.

In the past few years, modulational instability has been a topic of significant interest because of its relevance in stable wave propagation, plasma heating [1], plasma beat wave accelerators [2–4], space communication, etc. Several authors have pointed out that modulational instability might have a dramatic effect upon the growth of the beat plasma wave in the context of plasma beat wave accelerator. The modulational instability also plays an important role in optical fibers. The experimental observation of modulational instability in optical fibers have been reported by Tai, Hasegawa, and Tomita [5]. They also indicate that modulational instability may be used to generate a soliton train at a high repetition rate in optical fiber communication.

The modulational instability of ion-acoustic waves due to nonlinear interaction with slow response quasistatic plasma has been studied by several authors [6–10]. Most of these studies are focused on plasmas consisting of singly charged positive ions and electrons. However, in a recent paper [11], we have investigated the modulational instability of ion-acoustic waves in a plasma consisting of positive ions, negative ions, and electrons for parallel modulation. Experimental observations of modulational instability in a plasma consisting of positive ions, electrons, and negative ions have been reported by Tsukabayashi and Nakamura [12] and Bailung and Nakamura [13] for parallel modulation.

The purpose of the present paper is to study the modulational instability of ion-acoustic waves when modulation is allowed in a direction oblique to that of wave propagation vector. Assuming that the wave is propagating in the (x-y) plane making an angle θ with the x axis, which is taken as the direction of modulation. The potential ϕ in this case varies as $\phi(x,t)\exp\{i(k_x x+k_y y-\omega t)\}$. Such a situation provides new additional factors, which contribute to the modulational process significantly. For example, the coefficient of the dispersive term P of the nonlinear Schrödinger equation given by $P=\frac{1}{2}\partial^2\omega/\partial k_x^2$ in this case comes out to be $\frac{1}{2}[\cos^2\theta(\partial^2\omega/\partial k^2)+(1/k)\sin^2\theta(\partial\omega/\partial k)]$. This shows that P consists of two parts: one of them is proportional to $\cos^2\theta$ and depends upon the group velocity dispersion $(\partial^2\omega/\partial k^2)$, and the other one is proportional to $\sin^2\theta$ and depends upon the dispersion of the wave, i.e., group velocity, $\partial\omega/\partial k$. Therefore, in the case of oblique modulation the coefficient P depends not only on the group velocity dispersion but on the dispersion of the wave as well, whereas in the case of parallel modulation it depends only on the group velocity dispersion. It may be noted that $\partial^2\omega/\partial k^2$ is negative for all values of k, whereas $\partial\omega/\partial k$ is always positive. Hence, in this case P has an anisotropic expression and it can in general be different from the case of parallel modulation, i.e., $k_y=0$ case. Therefore, in this case it leads to the possibility that the sign of the product, i.e., PQ becomes positive for certain values of θ and k, leading to modulational instability.

In the present paper we intend to investigate the modulational instability of ion-acoustic waves in a plasma due to nonlinear interaction with slow response quasistatic plasma, when negative-ion species and obliqueness of modulation contribute simultaneously. We have derived a nonlinear Schrödinger equation which governs the slow modulation of wave amplitude. The presence of negative ion species changes the coefficient of nonlinear term in the nonlinear Schrödinger equation. It is found that in the presence of negative-ion species, the sign of the

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coefficient of nonlinear term (i.e., Q) depends sensitively on the negative ion concentration (α), charge multiplicity ratio (ε_z), and relative mass of the two-ion species. The sign of Q does not depend on θ, whereas the sign of the coefficient of the dispersive term P depends only on θ and is independent of α, ε_z, and μ. Hence, the unstable regions in the (k-θ) plane strongly depend on α, ε_z, and μ.

Although the basic motivation behind the present paper is to examine the modulational instability of ion-acoustic waves, when obliqueness of modulation and the negative-ion species contribute simultaneously, the analysis presented here however remains valid in the presence of both positive and negative ions with arbitrary charges. Therefore, one can directly apply this method to investigate the modulational instability of ion-acoustic waves in dusty plasmas. The dusty plasmas have been frequently observed in different space plasma environments such as asteroid zones, planetary rings, magnetosphere, cometary tails, as well as in the lower part of the Earth's ionosphere [14–16]. Moreover, such plasmas are being increasingly studied in laboratory experiments [17–19]. Recently, ion-acoustic waves have been shown to exist in a dusty plasma by de Angelis, Formisano, and Giordano [20]. Hence, we expect that using the appropriate values of parameters of the dusty plasmas, the study can be applied to investigate the modulational instability of ion-acoustic waves in such plasmas. In addition to this negative-ion plasmas are found in the D region of the Earth's ionosphere [21]. Since, in the present paper as an approximation, we have taken the plasma to be collisionless, we therefore expect that theory can be applied to study the modulational instability of ion-acoustic waves in the upper part of the D region where ion-collisional frequency is expected to be small and can be neglected. However, to provide a more realistic and full description of the entire range of the D region of the Earth's ionosphere, inclusion of collisions is essential.

Present analysis is a bit more general than the earlier one in the sense that the results of the present investigation reduce to those obtained by Shukla [6] in the limit $\alpha=0$ and $\theta=0$ and to those studied by Mishra, Chhabra, and Sharma [11] in the limit $\theta=0$.

The paper is organized as follows. In Sec. II we write the normalized fluid equations for the system. In Sec. III the nonlinear Schrödinger equation describing the amplitude of obliquely modulated ion-acoustic waves is derived. Discussion is given in Sec. IV. Conclusions are summarized in Sec. V.

II. BASIC EQUATIONS

We consider an ion-acoustic wave traveling in the (x-y) plane making an angle θ with the x direction in a collisionless plasma consisting of a mixture of warm positive-ion species and negative-ion species with different charges, masses, and concentrations and hot isothermal electrons. We further assume that the modulated amplitude of the ion-acoustic wave varies in the x direction. The nonlinear interaction of finite amplitude ion-acoustic waves with the background collisionless plasma is governed by the following set of normalized fluid equations:

$$\frac{\partial n_{i1}}{\partial t}+\nabla\cdot(n_{i1}\mathbf{V}_{i1})=0\ , \tag{1}$$

$$\frac{\partial \mathbf{V}_{i1}}{\partial t}+(\mathbf{V}_{i1}\cdot\nabla)\mathbf{V}_{i1}=-\frac{1}{\beta}\nabla\phi-\frac{1}{\beta Z_1}\frac{T_i}{T_e}\frac{1}{n_{i1}}\nabla n_{i1}\ , \tag{2}$$

$$\frac{\partial n_{i2}}{\partial t}+\nabla\cdot(n_{i2}\mathbf{V}_{i2})=0\ , \tag{3}$$

$$\frac{\partial \mathbf{V}_{i2}}{\partial t}+(\mathbf{V}_{i2}\cdot\nabla)\mathbf{V}_{i2}=\left[\frac{\mu\varepsilon_z}{\beta}\right]\nabla\phi-\left[\frac{\mu}{\beta Z_1}\right]\frac{T_i}{T_e}\frac{1}{n_{i2}}\nabla n_{i2}\ , \tag{4}$$

$$\nabla\phi=\frac{1}{n_e}\nabla n_e\ , \tag{5}$$

$$\nabla^2\phi=n_e-\frac{n_{i1}}{(1-\alpha\varepsilon_z)}+\frac{\alpha\varepsilon_z}{(1-\alpha\varepsilon_z)}n_{i2}\ , \tag{6}$$

where $\mathbf{V}_{ij}=(V_{ijx},V_{ijy},0)$, $\nabla=(\partial/\partial x,\partial/\partial y,0)$, $\beta=(1+\alpha\mu\varepsilon_z^2)/(1-\alpha\varepsilon_z)$, $\varepsilon_z=Z_2/Z_1$, $\mu=M_1/M_2$, and $\alpha=n_{i2}^{(0)}/n_{i1}^{(0)}$.

In the above equations, $n_{i1},\mathbf{V}_{i1}$ and $n_{i2},\mathbf{V}_{i2}$ are the density and fluid velocity of the positive ion and negative ion, respectively. n_e is the electron density, ϕ is the electrostatic potential, μ is the mass ratio of the positive ions to the negative ions, α is the density ratio of negative ions to positive ions, and ε_z is the charge multiplicity ratio of negative ions to positive ions. In Eq. (5), we have neglected the electron inertia. The quantities $\mathbf{V}$, ϕ, t, and x and y are normalized with respect to ion-acoustic wave speed in the mixture $C_s=(T_e\beta Z_1/M_1)^{1/2}$, thermal potential (T_e/e), inverse of the ion plasma frequency in the mixture $\omega_{pi}^{-1}=(4\pi n_e^{(0)}e^2Z_1\beta/M_1)^{1/2}$, and Debye length λ_D, respectively. Densities n_{i1}, n_{i2}, and n_e are normalized with their corresponding equilibrium densities, i.e., $n_{i1}^{(0)}$, $n_{i2}^{(0)}$, and $n_e^{(0)}$, respectively.

III. DERIVATION OF THE NONLINEAR SCHRÖDINGER EQUATION

In order to examine the slow quasistatic plasma response to the ion-acoustic waves, we write the field quantities in normalized form as follows:

$$n_j=1+n_j^h+n_j^l\ , \tag{7}$$

$$\mathbf{V}_j=\mathbf{V}_j^h+\mathbf{V}_j^l\ , \tag{8}$$

$$\phi=\phi^h+\phi^l\ , \tag{9}$$

where $n_j^{h(l)}\ll 1$. The superscripts h and l represent the corresponding quantities associated with the ion-wave (high frequency) and with the quasistatic plasma slow motion (low frequency), respectively.

On substituting Eqs. (7)–(9) in Eq. (5), the electron density perturbation associated with the ion-acoustic waves in the presence of plasma slow motion is given by

$$n_e^h=(1+n_e^l)\phi^h . \tag{10}$$

Now we combine Eqs. (1) and (2), and (3) and (4). Then introducing Eqs. (6)–(9), we obtain the following nonlinear equation for the ion-acoustic waves in the presence of the plasma slow response

$$\left[\left[1-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right]\frac{\partial^2}{\partial t^2}-\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\right]\phi^h+\left[\frac{\partial^2}{\partial t^2}-\frac{1}{\beta}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\right]n_e^l\phi^h=0 . \tag{11}$$

In deriving Eq. (11), ions are assumed to be much colder than electrons, i.e., $T_i/T_e \ll 1$. We have used the quasineutral and quasistatic behavior of the plasma towards slow response, i.e.,

$$\left[\frac{n_{i1}^l}{(1-\alpha\varepsilon_z)}-\frac{\alpha\varepsilon_z}{(1-\alpha\varepsilon_z)}n_{i2}^l\right]=n_e^l \text{ and } \mathbf{V}_i^l=\mathbf{V}_e^l\simeq 0 .$$

In the absence of nonlinear interaction, linearization of (11) yields the following dispersion relation:

$$\omega^2=\frac{k^2}{(1+k^2)} , \tag{12}$$

where $k^2=k_x^2+k_y^2$, k_x and k_y being the x and y components of the wave vector $\mathbf{k}$ of the ion-acoustic wave. The modulation group velocity (i.e., the velocity with which the modulation propagates) of the wave is given by

$$V_g=\frac{\partial\omega}{\partial k_x}=\frac{\omega^3}{k^3}\cos\theta , \tag{13}$$

which is the component of the group velocity $(\partial\omega/\partial k)$ along the direction of modulation, where θ is the angle between the wave vector of the ion-acoustic wave and the x axis, the direction in which the modulation of the wave amplitude propagates.

Now we calculate the electron density perturbation n_e^1 associated with the quasistatic plasma slow motion. Taking the x component of the momentum balance equations for ions and electrons, i.e., Eqs. (2), (4), and (5) and using Eqs. (7)–(9), then averaging over the ion-acoustic wave period, we get

$$\frac{1}{2}\frac{\partial}{\partial x}\langle|(V_{i1x}^h)^2|\rangle=-\frac{1}{\beta}\frac{\partial\phi^1}{\partial x}-\frac{1}{\beta Z_1}\frac{T_i}{T_e}\frac{\partial n_{i1}^l}{\partial x} , \tag{14}$$

$$\frac{1}{2}\frac{\partial}{\partial x}\langle|(V_{i2x}^h)^2|\rangle=\left[\frac{\mu\varepsilon_z}{\beta}\right]\frac{\partial\phi^l}{\partial x}-\left[\frac{\mu}{\beta Z_1}\right]\frac{T_i}{T_e}\frac{\partial n_{i2}^l}{\partial x} , \tag{15}$$

$$\frac{\partial\phi^l}{\partial x}=\frac{\partial n_e^l}{\partial x} , \tag{16}$$

where we have assumed that the phase velocity of the modulation is much smaller than the electron and ion thermal velocities. In the above equations angular brackets denote averaging over the ion-acoustic wave period. The left-hand sides of Eqs. (14) and (15) represent the ion ponderomotive force. With the help of Eqs. (14)–(16), and using quasineutral behavior of plasma towards slow response, i.e.,

$$\left[\frac{n_{i1}^l}{(1-\alpha\varepsilon_z)}-\frac{\alpha\varepsilon_z}{(1-\alpha\varepsilon_z)}n_{i2}^l\right]=n_e^l ,$$

we get

$$\frac{\beta Z_1}{2(1-\alpha\varepsilon_z)}\frac{\partial}{\partial x}\langle|(V_{i1x}^h)^2|\rangle-\frac{\beta}{2}\frac{Z_1}{\mu}\frac{\alpha\varepsilon_z}{(1-\alpha\varepsilon_z)}\frac{\partial}{\partial x}\langle|(V_{i2x}^h)^2|\rangle=-\left[\frac{Z_1(1+\alpha\varepsilon_z^2)}{(1-\alpha\varepsilon_z)}+\frac{T_i}{T_e}\right]\frac{\partial n_e^l}{\partial x} . \tag{17}$$

Now from the ion-momentum equations, i.e., Eqs. (2) and (4), we get

$$V_{i1x}^h=\left[\frac{k_x}{\beta\omega}\right]\phi^h \tag{18}$$

and

$$V_{i2x}^h=-\left[\frac{\mu\varepsilon_z}{\beta}\right]\frac{k_x}{\omega}\phi^h . \tag{19}$$

On substituting the values of V_{i1x}^h and V_{i2x}^h from Eqs. (18) and (19) in Eq. (17), the electron density perturbation n_e^l associated with the quasistatic plasma slow motion is given by

$$n_e^l=-\frac{Z_1(1+k^2)}{2\beta(1-\alpha\varepsilon_z)}\frac{(1-\alpha\mu\varepsilon_z^3)}{\left[\frac{Z_1(1+\alpha\varepsilon_z^2)}{(1-\alpha\varepsilon_z)}+\gamma\right]}\cos^2\theta|\phi^h|^2 , \tag{20}$$

where $\gamma=T_i/T_e$ is the ratio of the ion to electron temperature. Substituting Eq. (20) in Eq. (11), we get

$$\left[\left[1-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right]\frac{\partial^2}{\partial t^2}-\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\right]\phi^h -\frac{Z_1(1+k^2)}{2\beta(1-\alpha\varepsilon_z)}(1-\alpha\mu\varepsilon_z^3)\left[\frac{Z_1(1+\alpha\varepsilon_z^2)}{(1-\alpha\varepsilon_z)}+\gamma\right]^{-1}\left[\frac{\partial^2}{\partial t^2}-\frac{1}{\beta}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\right]\cos^2\theta|\phi^h|^2\phi^h=0 . \tag{21}$$

We assume that the nonlinear interaction of the quasistatic plasma slow response with the ion-acoustic waves gives rise to an envelope of wave whose amplitude varies on time and space scales much more slowly than those of the ion-acoustic oscillations; accordingly we let

$$\phi^h=\varepsilon^{1/2}\phi^h(\xi,\tau)\exp(-i\omega t+ik_x x+ik_y y)+\text{c.c.} \,, \tag{22}$$

where ε indicates the magnitude of small but finite amplitude ϕ^h, ξ, and τ are defined such that

$$\xi=\varepsilon^{1/2}(x-V_g t) \,, \tag{23a}$$

and

$$\tau=\varepsilon t \,. \tag{23b}$$

Substituting Eqs. (22) and (23) in Eq. (21) and using Eqs. (12) and (13), we get to $O(\varepsilon^{3/2})$, the following nonlinear Schrödinger equation:

$$i\frac{\partial\phi^h}{\partial\tau}+P\frac{\partial^2\phi^h}{\partial\xi^2}+Q|\phi^h|^2\phi^h=0 \,. \tag{24}$$

In the above equation (24), P and Q are the coefficients of the dispersive and nonlinear terms, respectively. The coefficient of the dispersive term P is half the component of the modulation group velocity dispersion $(\partial V_g/\partial k)$ along the direction of modulation, i.e.,

$$P=\frac{1}{2}\frac{\partial V_g}{\partial k}\cos\theta$$

$$=\frac{1}{2}\frac{\partial V_g}{\partial k_x}=\frac{1}{2}\frac{\partial^2\omega}{\partial k_x^2}=\frac{1}{2}\left[\cos^2\theta\frac{\partial^2\omega}{\partial k^2}+\sin^2\theta\frac{1}{k}\frac{\partial\omega}{\partial k}\right]$$

or

$$P=\frac{1}{2k}\frac{1}{(1+k^2)^{5/2}}[(1+k^2)-(1+4k^2)\cos^2\theta] \tag{25}$$

and

$$Q=\frac{Z_1k^3}{4(1+k^2)^{1/2}}\frac{F_2F_3}{F_1}\cos^2\theta \,, \tag{26}$$

where

$$F_1=\left[\frac{Z_1(1+\alpha\varepsilon_z^2)}{(1-\alpha\varepsilon_z)}+\gamma\right] , \tag{27a}$$

$$F_2=\left[1-\frac{\alpha\mu\varepsilon_z^2(1+\varepsilon_z)}{(1+\alpha\mu\varepsilon_z^2)}\right] , \tag{27b}$$

$$F_3=\left[1-\frac{\alpha\varepsilon_z(1+\mu\varepsilon_z)}{(1+\mu\alpha\varepsilon_z^2)}\frac{(1+k^2)}{k^2}\right] . \tag{27c}$$

It may be noted that our expressions for P and Q reduce to those obtained by Shukla [6] in the limit $\alpha=0$ and $\theta=0$ with $\mu=1$ and $\varepsilon_z=1$, and for Mishra, Chhabra, and Sharma [11] in the limit of $\theta=0$. It may be pointed out that for a given plasma with Z_1, Z_2, $n_e^{(0)}$, and $n_{i1}^{(0)}$ fixed, α is given by

$$\alpha=\frac{1}{Z_2}\left[Z_1-\frac{n_e^{(0)}}{n_{i1}^{(0)}}\right] , \tag{28}$$

which shows that only for the case of $Z_1=Z_2=1$ can α be equal to 1 but this corresponds to $n_e^{(0)}=0$. Hence, in this situation plasma composes of positive ions and negative ions only and ion-acoustic waves are no longer possible. However, in all other cases α is less than unity.

IV. DISCUSSION

The amplitude of the modulated ion-acoustic wave, defined by the nonlinear Schrödinger equation, i.e., Eq. (24), will be modulationally stable or unstable according as $PQ<0$ or $PQ>0$ [22].

Hence, the wave will be modulationally unstable in the following two situations: (i) when $P>0$ and $Q>0$ and (ii) when $P<0$ and $Q<0$.

From Eqs. (25) and (26), we note that for a given value of k, the sign of P depends on θ, whereas that of Q depends upon α. To locate the regions of modulational instability, we plot two sets of curves for plasma consisting of Cs^+, F^- ions, and electrons: (i) a curve for $P=0$ on a polar $(k\text{-}\theta)$ graph separating regions of $P>0$ and $P<0$ (Fig. 1) and (ii) curves $F_2=0$ and $F_3=0$ on a $(\alpha\text{-}k)$ graph indicating regions of $Q>0$ and $Q<0$ (Fig. 2).

From these figures, it is clear that the wave will be modulationally unstable when the values of (α,k,θ) are such that they lie either (a) in region I of Fig. 1 and in regions I and III of Fig. 2 or (b) in region II of Fig. 1 and in regions II and IV of Fig. 2. Thus we have $PQ>0$ (i) for $\alpha<\alpha_c$ when (a) $k<k_c$ and $\theta<\theta_c$, (b) $k>k_c$ and $\theta>\theta_c$, and (ii) for $\alpha>\alpha_c$ when (a) $k<k_c$ and $\theta>\theta_c$, (b) $k>k_c$ and $\theta<\theta_c$, where, from Eqs. (25) and (26),

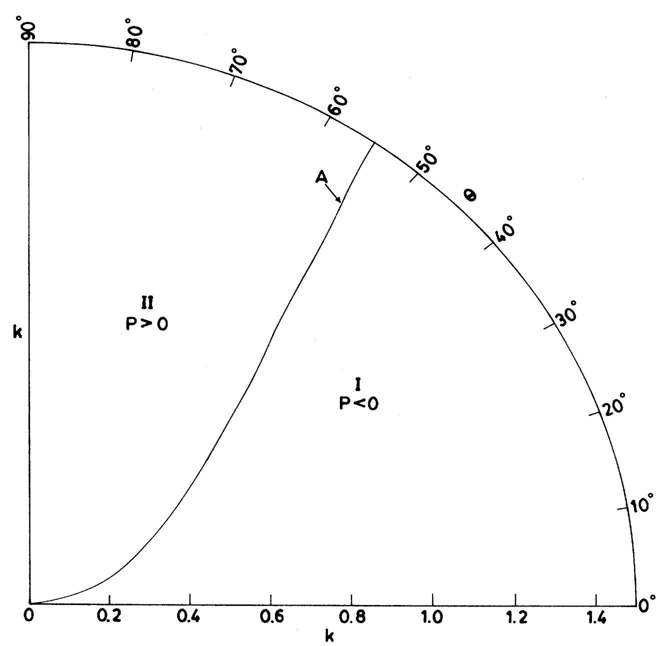

FIG. 1. Plot of $P=0$ in the $(k\text{-}\theta)$ plane for the plasma having Cs^+, F^- ions and electrons with $Z_1=Z_2=1$, $\varepsilon_z=1$, and $\mu=6.99$. Region I corresponds to $P<0$ and region II corresponds to $P>0$.

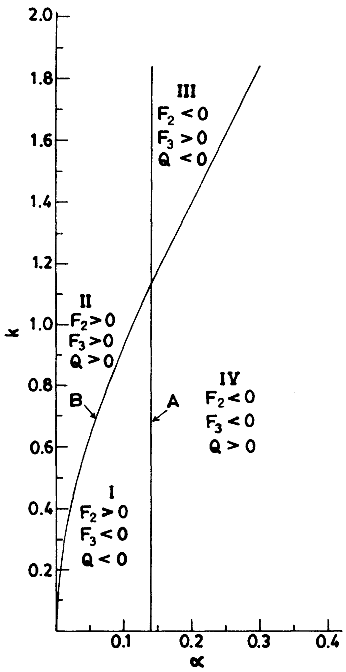

FIG. 2. Plot of $F_2=0$ and $F_3=0$ in the (α-k) plane for the plasma having Cs^+, F^- ions and electrons with $Z_1=Z_2=1$, $\varepsilon_z=1$, and $\mu=6.99$. Curve A refers to $F_2=0$ and B refers to $F_3=0$. In regions I and III $Q<0$, while in regions II and IV $Q>0$.

$$\alpha_c=\frac{1}{\mu\varepsilon_z^3}=\left[\frac{M_2Z_1^3}{M_1Z_2^3}\right], \tag{29a}$$

$$k_c=\left[\frac{\alpha\varepsilon_z(1+\mu\varepsilon_z)}{(1-\alpha\varepsilon_z)}\right]^{1/2}, \tag{29b}$$

$$\theta_c=\cos^{-1}\left[\frac{1+k^2}{1+4k^2}\right]^{1/2}. \tag{30}$$

Thus for every given value of negative-ion concentration, i.e., α, we have domains in the (k-θ) plane over which the wave is modulationally unstable. These domains are shown in Figs. 3(a) and 3(b) for $\alpha<\alpha_c$ and $\alpha>\alpha_c$, respectively.

The dependence of regions of modulational instability on the various parameters related to negative ions, i.e., their fractional concentration, mass ratio, etc., is expressed by the expression for α_c, k_c along with these curves.

A comparison with the earlier study of parallel modulation [11] shows that the results are drastically affected due to the obliqueness of modulation, and obliqueness of modulation provides a new domain of instability. The main new findings in comparison to the earlier study are as follows: (a) In the case of oblique modulation it is found that for a given value of negative-ion concentration (α), there exists a range of obliqueness (θ) corresponding to every value of wave number (k) for which the wave

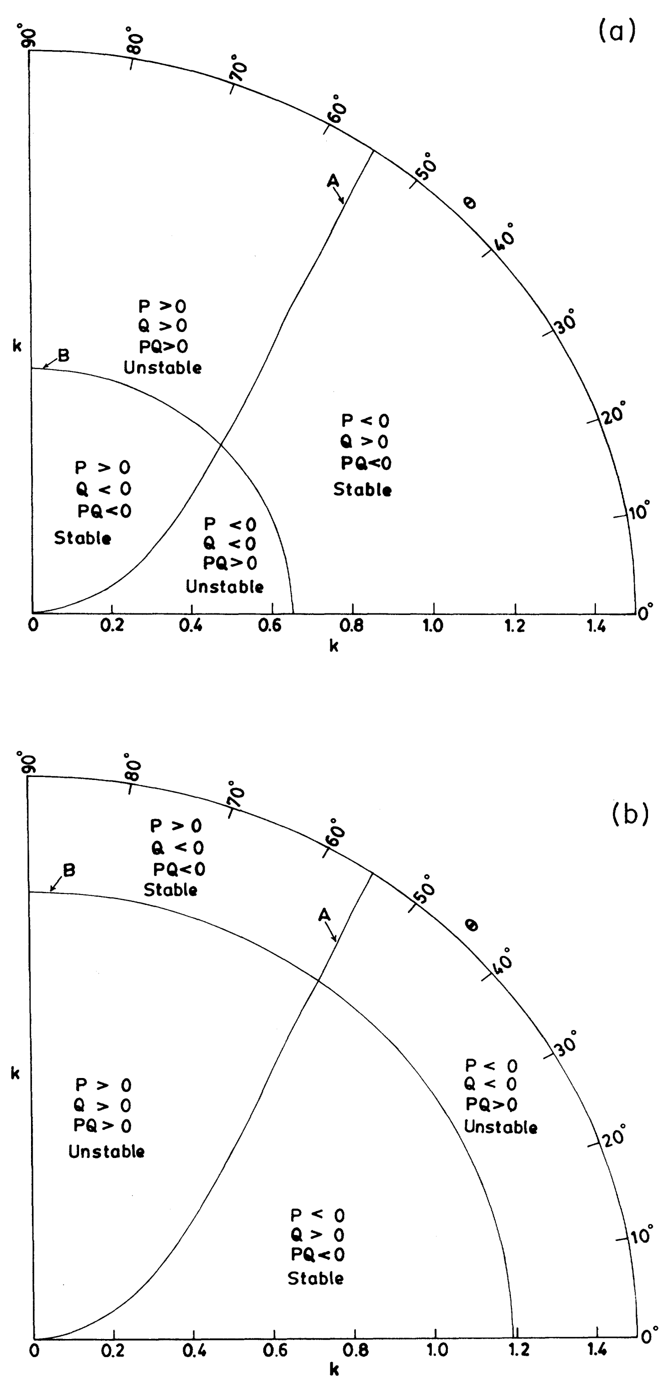

FIG. 3. (a) Plot of $P=0$ and $Q=0$ in the (k-θ) plane for Cs^+ plasma containing F^- ions and electrons with $Z_1=Z_2=1$, $\varepsilon_z=1$, $\mu=6.99$, and $\alpha_c=0.14$. Curve A refers to $P=0$ and curve B refers to $Q=0$ for $\alpha=0.025$ ($<\alpha_c$). The domains (i) lying below curve A and curve B and (ii) lying above curve A and curve B represent the modulationally unstable domains. (b) Plot of $P=0$ and $Q=0$ in the (k-θ) plane for Cs^+ plasma containing F^- ions and electrons with $Z_1=Z_2=1$, $\varepsilon_z=1$, $\mu=6.99$, and $\alpha_c=0.14$. Curve A refers to $P=0$ and curve B refers to $Q=0$ for $\alpha=0.15$ ($>\alpha_c$). The domains (i) lying below curve A and above curve B and (ii) lying above the curve A and below curve B represent the modulationally unstable domains.

would be modulationally unstable. Whereas, in the case of parallel modulation, the wave was found to be unstable only for a range of k values. (b) In the case of parallel modulation, it was found that for a given value of negative-ion concentration α, such that $0<\alpha<\alpha_c$, there exists an upper bound on k, k_{ch}, below which the waves remain modulationally unstable. Whereas, in the case of oblique modulation, it is found that the waves would be unstable even for $k>k_{ch}$ for certain range of obliqueness. (c) For $\alpha>\alpha_c$, it was found that there exists a lower bound on k, k_{cl}, above which the waves remain modulationally unstable in the case of parallel modulation. On the other hand, in the case of oblique modulation, the waves may be unstable even for $k<k_{cl}$ for a certain range of obliqueness.

Since the analysis presented here remains valid in the presence of both positive and negative ions with arbitrary charges, one can directly apply this method to investigate the modulational instability of ion-acoustic waves in dusty plasmas. In addition to this the theory can be used to study the modulational instability of ion-acoustic waves in planetary atmospheres.

It is also expected that the ion-acoustic wave will form an envelope soliton for obliqueness of modulation, corresponding to $PQ>0$, i.e., in the region in which the wave is modulationally unstable.

V. CONCLUSIONS

Our main conclusions are as follows:

(i) It is found that obliqueness of modulation drastically affects the instability and provides a new domain of instability.

(ii) It is found that for a given value of negative-ion concentration, there exists a range of obliqueness (θ) corresponding to every value of wave number k, for which the wave would be modulationally unstable.

(iii) For a given value of $\alpha<\alpha_c$ and k lying in the range $0<k<k_c$, the wave is modulationally unstable for values of θ below a critical value of θ_c. However, for $k>k_c$, the wave would be modulationally unstable for values of $\theta>\theta_c$.

(iv) For values of $\alpha>\alpha_c$ and k lying in the range $0<k<k_c$, the wave is modulationally unstable for values of θ greater than the critical value of θ_c. But for $k>k_c$, the wave would be modulationally unstable for the values of $\theta<\theta_c$.

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