Instability of two-dimensional solitons and vortices in defocusing media

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In the framework of the three-dimensional nonlinear Schrödinger equation the instability of two-dimensional solitons and vortices is demonstrated. The soliton instability can be considered as the analog of the Kadomtsev–Petviashvili instability (Dokl. Akad. Nauk SSSR **192**, 753 (1970) [Sov. Phys. Dokl. **15**, 539 (1970)]) of one-dimensional acoustic solitons in media with positive dispersion. For large distances between the vortices, this instability transforms into the Crow instability [AIAA J. **8**, 2172 (1970)] of two vortex filaments with opposite circulations.

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I. PRELIMINARY REMARKS

This paper is devoted to the stability problem of two-dimensional solitons and vortices, described by the nonlinear Schrödinger equation (NLSE) with repulsion

$$i\psi_t + \frac{1}{2}\nabla^2\psi + (1-|\psi|^2)\psi = 0 . \tag{1}$$

This equation has at least two important applications. The first one relates to nonlinear optics and here Eq. (1) describes the propagation of the electromagnetic wave in defocusing media. Then the refractive index n has a negative nonlinear addition

$$n = n_0(\omega_0, k_0) - \beta|E|^2, \tag{2}$$

where n_0 is the linear refractive index, ω_0 and k_0 are the carrier frequency and the wave number, respectively, $\beta > 0$ is a constant, and E is the complex amplitude of the electric field. When the wave dispersion is positive ($\omega'' = \frac{\partial^2\omega}{\partial k^2} > 0$), the equation for the envelope $E(\mathbf{r}, t)$ reads

$$i(E_t + v_{\rm gr}E_x) + \frac{v_{\rm gr}}{2k_0}\nabla_\perp^2 E + \frac{\omega''}{2}E_{xx} - \omega_0\frac{\beta|E|^2}{n_0}E = 0. \tag{3}$$

After a translation to a system of reference, moving along the x axis with the group velocity $v_{\rm gr}$ and a simple rescaling, Eq. (3) transforms into the NLSE of the form of Eq. (1).

It should be noted that the nonlinear term in Eq. (3) amplifies the linear effects, diffraction and dispersion, by broadening the optical pulse in transverse and longitudinal (relative to the pulse propagation) directions. Thus meaningful nonlinear dynamics is possible only for pulses sufficiently long in time and wide in transverse direction when, for instance, dark solitons are observed. Therefore we will further assume that ψ tends to the constant value, say, to 1, as $|\mathbf{r}| \to \infty$. In such a formulation Eq. (1) is also used as a model for the description of the condensate motion in a weakly imperfect Bose gas, with ψ being the condensate wave function. For the Bose gas this equation was first derived by Gross [3] and Pitaevsky [4] and therefore it is sometimes called the Gross-Pitaevsky equation.

Equation (1) can be interpreted as a model of dispersive hydrodynamics if one rewrites it in terms of density N and phase ϕ ($\psi = \sqrt{N}e^{i\phi}$),

$$N_t + {\rm div}N\nabla\phi = 0, \tag{4}$$

$$\phi_t + \frac{1}{2}(\nabla\phi)^2 + N - 1 = \frac{\Delta\sqrt{N}}{2\sqrt{N}}, \tag{5}$$

where the pressure $p = N^2/2$ is positive. Depending on the spatial dimensions of the problem, the NLSE (1) gives rise to different nonlinear behaviors. As is well known, in the one-dimensional case the equation can be integrated by the inverse scattering transform [5]. One of the main results of this theory is a stability proof of one-dimensional solitons,

$$\psi_0 = \nu\tanh\nu(x - \kappa t - x_0) + i\kappa, \quad \kappa^2 + \nu^2 = 1. \tag{6}$$

In optics such objects are called gray solitons; they represent the propagation of density (or intensity) wells with the velocity κ. In the particular case $\kappa = 0$ the solution (6) corresponds to the so-called dark solitons.

In two and three dimensions soliton solutions cannot been found explicitly in all range of parameters (except for some limited cases), but only numerically. Multi-dimensional solitons have been studied in detail in several papers, mainly in the context of the dynamics of the Bose condensate. Among these we would like to distinguish a series of five papers by Roberts *et al.* [6–10] and the paper of Iordanskii and Smirnov [11].

The shape of the soliton solution in the form $\psi = \psi_0(x - vt, r_\perp)$ is determined by integration of the equation

$$-iv\frac{\partial\psi_0}{\partial x} + \frac{1}{2}\nabla^2\psi_0 + (1-|\psi_0|^2)\psi_0 = 0. \tag{7}$$

Here v is the velocity of the soliton and $\psi \to 1$ for all directions as $r \to \infty$. The possible range of soliton velocities is defined from the form of the spectrum of small oscillations on the background of constant density, $N = 1$, for (1) (the Bogolyubov spectrum),

$$\omega = k(1 + k^2/4)^{1/2}. \quad (8)$$

This means that the range of velocities has to lie in the interval $0 \leq v \leq C_s$ $(= 1)$. Here C_s is the minimal phase velocity $v_{\rm ph} = \omega/k$, the sound velocity. The soliton velocity cannot exceed the minimal phase velocity because then the Cherenkov-like radiation will become possible and, as a result, such a localized structure cannot be stationary; it will lose its energy and finally disappear. Therefore, close to the threshold of the Cherenkov radiation, but for $v < C_s$, the amplitude of the soliton will be small and vanish for $v = C_s$. Near the threshold the nonlinearity, being weak on the soliton solution, is compensated by the (positive) dispersion, which also has to be weak for this reason. In this velocity region in the two-dimensional (2D) case the soliton solutions are close to the 2D acoustic solitons of the Kadomtsev-Petviashvili (KP I) equation, the so-called lumps. These lumps were found explicitly for the 2D KP equation by means of the inverse scattering transform [12]. For the NLSE similar soliton solutions were later found numerically by Jones and Roberts [9]. In that work the whole family of two-dimensional solitons was also found numerically. According to these results, the density well at the center of the soliton becomes deeper and deeper when the velocity decreases. There exists a velocity $v_{\rm cr}$ for which the density well reaches the "bottom," i.e., N becomes equal to zero. For smaller velocities this zero bifurcates; it splits into two separate zeros in the direction transverse to the direction of the soliton propagation. These zeros correspond to two vortices with opposite circulations and look like a vortex dipole. The reduction of the soliton velocity results in a growth of the distance between the two vortices so that in the small velocity limit the dipole vortex pair is described, with a good accuracy, by the Euler equation for incompressible fluids. Thus, in one limit we have the KP solitons and, the KP equation, respectively, and in the other limit for small velocities we get two parallel vortex filaments with opposite circulations, which are similar to the vortex solutions of the 2D Euler equation.

The main purpose of the present paper is to investigate the stability of the whole family of two-dimensional soliton solutions. We assume that these solutions, representing stationary points of the Hamiltonian H for fixed momentum $\mathbf{P}$, should be stable in the 2D case because both the KP and the Euler limits indicate their stability. In the first limit, the KP soliton realizes the minimum of the Hamiltonian for fixed $\mathbf{P}$ and therefore it is stable in accordance with the Lyapunov theorem [13]. For the Euler equation the fact of stability of two point vortex distribution is well known (see, for instance [14]). We show that such solitons are unstable with respect to three-dimensional perturbations. We demonstrate the instability in the long-wavelength approximation, when the perturbation wavelength is larger than the soliton size.

For the study of the stability problem we use the approach developed for the related topics, e.g., for one- and two-dimensional acoustic solitons in media with positive dispersion [13] and for the two-dimensional stability of gray solitons in nonlinear optics [15]. The present paper represents the natural development of the latter. The instability that we found turns out to be of the self-focusing type, analogous to the instability of 1D (gray) solitons against transverse perturbations [15]. In the long-wavelength limit the symmetric (relative to the soliton) perturbations are unstable and antisymmetric perturbations are stable. This is in agreement with the stability analysis of Crow for two line vortex filaments in the fluid case [2] as well as that for the KP instability of two-dimensional acoustic solitons [13].

The course of the instability is qualitatively the same as for 1D acoustic solitons [1]. The soliton amplitude decreases while the soliton velocity increases. Therefore, by the transverse modulation of the soliton the regions with smaller amplitude (shallow wells) will overtake those with higher amplitudes (deep density wells). This gives an instability of the self-focusing type. In the nonlinear stage such a tendency would provide the division of 2D solitons or dipole vortices into separate cavities. For vortex filaments these cavities look like vortex rings. Such an assumption means that the process of the cavity formation in this limit should be accompanied by the reconnection of vortex filaments. If initially the soliton distribution has no zeros, this instability can be assumed to lead to the cavitation, i.e., to the appearance of zero in the density profile, and, probably, at the later stages to the birth of the vortex rings. In that connection we should note that the reconnection of vortex lines has recently been investigated by numerical solutions of Eq. (1) in three dimensions [16]. The main result was that vortex filaments of opposite "circulation" would reconnect whenever they come within a distance of a few core radii of one another. Further support for the nonlinear stage of the instability, conjectured above, is obtained from the following observations. First of all is the collapse of acoustic waves, which can be considered as the nonlinear stage of the KP instability of solitons. The acoustic collapse, studied in detail both theoretically and numerically [17,18], demonstrates the tendency of the catastrophic decreasing in the density profile for solitons of small amplitude. Besides, recent experimental observations and numerical study of the nonlinear development of the dark soliton instability showed the formation of a point vortex street [19,20], familiar to the von Karman street in fluids. It should be noted that the instability we consider here represents the unification of two, at first glance, different kinds of instabilities: the KP instability of acoustic solitons for media with positive dispersion [1] and the Crow instability of two vortex filaments [2].

II. STATIONARY SOLITARY WAVES

Let us consider an axisymmetric two-dimensional solution of Eq. (7),

$$\psi(x', y) = \psi_0(x', -y) \quad (x' = x - vt).$$

It is easy to see that this solution (as well all other stationary ones) can be obtained from the variational problem

$$\delta(H - vP_x) = 0, \tag{9}$$

where

$$H = \frac{1}{2}\int[|\nabla\psi|^2 + (|\psi|^2 - 1)^2]d\mathbf{r} \tag{10}$$

and

$$\mathbf{P} = \frac{i}{2}\int[\psi\nabla\psi^* - \psi^*\nabla\psi]d\mathbf{r} \tag{11}$$

are the Hamiltonian and the momentum, respectively. Equation (9) says that the soliton solution represents the stationary point of the Hamiltonian for fixed momentum. The Lagrange multiplier v in (9) coincides with the soliton velocity in (7). Hence, in particular, it follows that on the soliton family the velocity v can be defined as

$$v = \frac{\partial\varepsilon}{\partial P}, \tag{12}$$

where ε is the soliton energy and $P = P_x$ is the x component of its momentum.

However, the momentum $\mathbf{P}$, given by Eq. (11), diverges logarithmically at infinity on the 2D soliton, as shown in Ref. [9]. This follows by considering the asymptotics of ψ at the infinity,

$$\psi(x,y) \sim 1 + im\frac{x}{x^2 + (1-v^2)y^2}, \tag{13}$$

where m is real. Thus $\mathbf{P}$ in Eq. (11) needs to be redefined. If the momentum is defined as (see [9])

$$\mathbf{P} = \frac{i}{2}\int[(\psi - 1)\nabla\psi^* - (\psi^* - 1)\nabla\psi]d\mathbf{r}, \tag{14}$$

then it will converge and also remain conservative. In terms of the density fluctuation $n = N - 1$ and the velocity $\mathbf{U} = \nabla\phi$, the momentum (14) can be rewritten as

$$\mathbf{P} = \int n\mathbf{U}d\mathbf{r}. \tag{15}$$

The definition (14) guarantees now that $\mathbf{P}$ remains bounded. It allows us to characterize the whole soliton family by the velocity v or by P (15). For this family it is possible to find some integral relations (for more details, see [10]). Here we present one of them. Let us perform two independent scaling transformations along the x and y axis:

$$\psi_0(x,y) \to \psi_0(ax,y), \quad \psi_0(x,y) \to \psi_0(x,by),$$

where a and b are scaling parameters. Inserting this transformation into the variational problem (9), it is evident that the following two integral relations hold on the soliton solutions:

$$\frac{\partial}{\partial a}(H - vP)|_{a=1} = \frac{\partial}{\partial b}(H - vP)|_{b=1} = 0\,.$$

Simple algebra gives

$$\int\left|\frac{\partial\psi}{\partial x}\right|^2 d\mathbf{r} = \varepsilon, \tag{16}$$

$$\int\left|\frac{\partial\psi}{\partial y}\right|^2 d\mathbf{r} = \varepsilon - vP, \tag{17}$$

$$\int(|\psi|^2 - 1)^2 d\mathbf{r} = vP. \tag{18}$$

Hence, with the help of (12) we get the inequality

$$\frac{\varepsilon}{P} > \frac{\partial\varepsilon}{\partial P}. \tag{19}$$

In the two limiting cases, i.e., in the limit $v \to C_s$ and in the limit for small velocities, the 2D soliton solutions can be found in analytical form. In the first case, as may be seen from the asymptotic form (13), the gradient of ψ along x is larger than the corresponding gradient along the y direction,

$$\frac{\partial}{\partial x} \sim (1 - v^2)^{1/2}\frac{\partial}{\partial y}.$$

Moreover, it is observed that the density fluctuation n, as well as the characteristic inverse soliton size along the x axis, tends to zero as the soliton velocity approaches C_s. These properties permit us to make a reduction of the NLSE (1) to the KPI equation, describing the propagation of acoustic waves of small amplitude with a narrow angular distribution and possessing positive dispersion. The regular procedure of such a reduction [21] consists of the introduction of both slow time and slow coordinates

$$t' = \epsilon^3 t, \quad x' = \epsilon(x - C_s t), \quad y' = \epsilon^2 y, \quad z' = \epsilon^2 z$$

and the representation of n in the form of series in powers of the small parameter ϵ

$$N = 1 + \sum_{k=1}^{\infty}\epsilon^{2k}n_k(x', y', z', t').$$

For stationary solitary waves $\epsilon = \sqrt{1 - v}$.

The KP equation appears in third order ($\sim \epsilon^3$),

$$\frac{\partial}{\partial x}\left(n_t - \frac{1}{8}n_{xxx} + \frac{3}{2}nn_x\right) = -\frac{1}{2}\nabla_\perp^2 n, \tag{20}$$

where $\nabla_\perp^2 = \partial_y^2 + \partial_z^2$ and primes are omitted. The momentum P in this case can be expressed through the density fluctuation n

$$P = \int n^2 d\mathbf{r} > 0, \tag{21}$$

and the energy ε coincides with the leading order with P. The Hamiltonian for the KP equation appears in the next order of the perturbation theory,

$$H_{\rm KP}=\frac{1}{2}\int\left[\frac{1}{4}n_x^2+n^3+(\nabla_\perp\phi)^2\right]d\mathbf{r}\quad(\phi_x=n),\quad(22)$$

so that the Hamiltonian for Eq. (1) in this limit can be written approximately as

$$H\simeq P+H_{\rm KP}.\qquad(23)$$

As the NLSE the KP equation (20) belongs to the Hamiltonian equations, and it can be represented as

$$2\frac{\partial n}{\partial t}=-\frac{\partial}{\partial x}\frac{\delta H_{\rm KP}}{\delta n}.\qquad(24)$$

In full accordance with (9) the soliton solutions for the KP equation represent stationary points of the KP Hamiltonian (22) for fixed momentum (21),

$$\delta[H_{\rm KP}-v'P]=0,\qquad(25)$$

where $v'=1-v$.

The solution of this variational problem can be found explicitly in the form of a two-dimensional soliton, the so-called lump [12],

$$n=-6\frac{8v'+6v'y^2-3(x-v't)^2}{[8v'+6v'y^2+3(x-v't)^2]^2}.\qquad(26)$$

The momentum P on the lump is proportional to $\sqrt{v'}$ so that

$$\frac{\partial P}{\partial v}=-\frac{A}{\sqrt{1-v}}<0\ ,\qquad(27)$$

where

$$A=\frac{P}{2\sqrt{1-v}}$$

is a positive quantity. The soliton solution (26) realizes the minimum of the Hamiltonian $H_{\rm KP}$ for fixed momentum P and is therefore stable with respect to two-dimensional perturbations [15], in accordance with the Lyapunov theorem in the framework of the KP equation. In the other limit of small velocities v, the solution of (7) represents the vortex dipole pair as noted in Sec. I. In this case the distance between vortices grows inversely proportional to the velocity, v,

$$L=1/v,$$

as $v\to0$. The density fluctuations n for such scales are unessential with respect to the phase variations. The density vanishes at the centers of each vortex and saturates sufficiently rapidly at the distances of the core radius $a\sim1$ (the so-called healing length). Thus the flow outside the core regions can be considered incompressible with a good accuracy (see, for instance, recent papers [22,23], devoted to this subject)

$$\mathrm{div}\mathbf{U}=\nabla^2\phi=0.\qquad(28)$$

The solution of this equation, as $v\to0$, can be written in the form

$$\phi(w)=\arg(w-iL/2)+\arg(w+iL/2)\ ,$$

where $w=x-vt+iy$. The main contribution to the energy in this limit is connected with this incompressible flow,

$$\varepsilon\simeq2\pi\ln(1/v).\qquad(29)$$

Using relation (12) we can write

$$\frac{\partial\varepsilon}{\partial v}\frac{\partial v}{\partial P}=v.\qquad(30)$$

Introducing (29) we obtain

$$\frac{\partial P}{\partial v}=-\frac{2\pi}{v^2}<0\qquad(31)$$

and consequently

$$P\simeq\frac{2\pi}{v}.$$

Thus, in both limits the derivative $\partial P/\partial v$ is negative. If one assumes that the function $P(v)$ is monotonic, then by applying the inequality (19) it is readily seen that the derivative $\partial P/\partial v$ will be negative in the whole range of velocities v. Numerical integration of Eq. (7) confirms this assumption completely [9].

III. STABILITY ANALYSIS

In this section we consider the linear stability of two-dimensional solitons with respect to three-dimensional perturbations. Let us seek the solution of Eq. (1) in the form

$$\psi(\mathbf{r},t)=\psi_0(x',y)+\delta\psi(x',y,z,t)\ ,\qquad(32)$$

where the soliton solution $\psi_0(x',y)$ obeys Eq. (7), $\delta\psi(x',y,z,t)$ is a small perturbation, and $x'=x-vt$. Let the perturbation depend on t and z in the following way:

$$\begin{pmatrix}\delta\psi\\ \delta\psi^*\end{pmatrix}=\begin{pmatrix}u_1\\ u_2\end{pmatrix}\exp(-i\omega t+ikz).$$

Then after linearization of Eq. (1) on the background of ψ_0, we arrive at the spectral problem

$$\omega\sigma_3u-\frac{1}{2}k^2u+Lu=0\ .\qquad(33)$$

Here

$$L=-iv\sigma_3\frac{\partial}{\partial x}+\frac{1}{2}\nabla_\perp^2-\begin{pmatrix}2|\psi_0|^2-1 & \psi_0^2\\ \psi_0^{*2} & 2|\psi_0|^2-1\end{pmatrix}$$

is a Hermitian operator and

$$\sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}.$$

It is hardly possible to solve this spectral problem exactly; therefore we shall restrict ourselves by considering only the problem in the long-wavelength limit, where k is small compared to the inverse soliton size $1/L$, i.e., we introduce a smallness parameter $\epsilon = kL \ll 1$. This means that the solution of the system (33) may be found in the form of a series in the small parameter ϵ:

$$u = u_0 + \epsilon u_1 + \epsilon^2 u_2 + \cdots, \quad \omega = \epsilon\omega_1 + \epsilon^2\omega_2 + \cdots . \tag{34}$$

To the leading order,

$$Lu_0 = 0, \tag{35}$$

which shows that u_0 are neutral modes. Among them there are two modes corresponding two independent infinitesimal translations of the soliton as a whole,

$$u_{01} = \frac{\partial}{\partial x}\begin{pmatrix} \psi_0 \\ \psi_0^* \end{pmatrix} \tag{36}$$

and

$$u_{02} = \frac{\partial}{\partial y}\begin{pmatrix} \psi_0 \\ \psi_0^* \end{pmatrix}. \tag{37}$$

Both modes are localized and belong to the bound states. These modes have different parities with respect to x and y. The function u_{01} is symmetric with respect to y and u_{02} is antisymmetric. The neutral modes generate two independent branches with different parities, which allows us to consider each branch separately.

The kernel of the operator L contains also an eigenfunction with a zeroth eigenvalue; this is a neutral mode

$$u_{03} = \begin{pmatrix} \psi_0 \\ -\psi_0^* \end{pmatrix},$$

corresponding to a small gauge transformation. This mode belongs to the continuous spectrum and therefore it is not interesting from the point of view of possible instability. From first principles it follows that unstable modes should be bounded. Modes that have a constant amplitude at infinity will evidently be stable. It should be noted that in the case of one-dimensional solitons there are only two functions, connected to translation and gauge in the kernel. It is therefore natural to assume that in the 2D case there will be the three functions presented above in the kernel of L.

A. Symmetric perturbations

In the next order of the perturbation expansion we obtain

$$\omega\sigma_3 u_0 + Lu_1 = 0 . \tag{38}$$

For symmetric perturbations this equation can easily be solved. Let us consider Eq. (7) for the stationary soliton and its complex conjugate. Differentiation of these equations with respect to v gives

$$-i\sigma_3 u_{01} + L\frac{\partial}{\partial v}\begin{pmatrix} \psi_0 \\ \psi_0^* \end{pmatrix} = 0,$$

which coincides up to the constant factor $i\omega$ with Eq. (38) for $u_0 = u_{01}$. Hence we have

$$u_{11} = i\omega\frac{\partial}{\partial v}\begin{pmatrix} \psi_0 \\ \psi_0^* \end{pmatrix}. \tag{39}$$

The equation for second order reads

$$\omega\sigma_3 u_1 - \frac{1}{2}k^2 u_0 = -Lu_2. \tag{40}$$

The solvability condition for this equation is the orthogonality of its left-hand side to all functions from the kernel of L. For the given case, due to the parity, a nontrivial relation appears only for the function u_{01}. As a result, we have

$$\omega\langle u_{01}|\sigma_3|u_{11}\rangle = \frac{1}{2}k^2\langle u_{01}|u_{01}\rangle. \tag{41}$$

Inserting Eq. (39) into this expression, integrating by parts, and using relation (16), we arrive at the dispersion relation

$$\omega^2 = \frac{\varepsilon}{\partial P/\partial v}k^2 < 0. \tag{42}$$

We recall that $\partial P/\partial v < 0$ as shown in Sec. II. Thus the considered perturbation turns out to be unstable with the growth rate (Im ω) given by Eq. (42). In the limit $v \to C_s$ this growth rate translates to that for the instability of two-dimensional acoustic solitons in media with positive dispersion [13]

$$\omega^2 = \frac{P}{\partial P/\partial v}k^2 = -2(1-v)k^2. \tag{43}$$

For the case of small $v \ll C_s$ the growth rate (42) is also simplified by means of (29) and (31),

$$\omega^2 = -(kv)^2\ln(1/v). \tag{44}$$

The instability governed by Eq. (43) represents the prolongation of the KP instability of 1D acoustic solitons, while instability (44) corresponds to the Crow instability for two parallel vortex filaments in ideal fluids [2]. In spite of the difference between these two physical situations, the reasons for both instabilities are the same. As stated in Sec. I, if the soliton velocity decreases when its amplitude increases, one should expect instability with respect to transverse perturbations. It is important to note that this instability is of the self-focusing type and it is expected that the instability saturates at a level sufficiently larger than the initial amplitude, if it saturates. In the acoustic region the instability initiates in the nonlinear stage the collapse of acoustic waves [17,18]. For vortices this instability represents the first stage of the cardinal reconstruction of the flow topology, i.e., of the vortex reconnection [16]. It is also interesting to note that the general expression for the growth rate (42) does not contain the logarithmic dependence on k, as follows from the results of Crow [2] for filaments with zeroth width.

B. Antisymmetric perturbations

Let us find the dispersion relation for antisymmetric perturbations. To find ω to leading order it is necessary to solve Eq. (38), where instead of u_0 we should substitute u_{02} from Eq. (37). For this case the solution can also be found. Note that if one considers a soliton propagating under a small angle to the x axis, then the following relation may be derived:

$$-i\sigma_3 u_{02} + L\frac{\partial}{\partial v_y}\begin{pmatrix}\psi_0\\ \psi_0^*\end{pmatrix}\Bigg|_{v_y=0} = 0. \qquad (45)$$

The derivatives with respect to v_y are easily expressed through the generator of the infinitesimal rotation

$$\frac{\partial\psi_0}{\partial v_y}\Bigg|_{v_y=0} = -\frac{1}{v}[\mathbf{r}\times\boldsymbol{\nabla}]\psi_0. \qquad (46)$$

As a result, the solution has the form

$$u_{12} = -\frac{i\omega}{v}[\mathbf{r}\times\boldsymbol{\nabla}]\begin{pmatrix}\psi_0\\ \psi_0^*\end{pmatrix}. \qquad (47)$$

Next we replace u_{01} by u_{02} from (37) and u_{11} by u_{12} from (47) in (41) and integrate by parts. Using relations (17) and (18), we obtain the following dispersion relation for the antisymmetric perturbation:

$$\omega^2 = (kv)^2\frac{\int|\psi_y|^2 d\mathbf{r}}{Pv} = (kv)^2\frac{\varepsilon - Pv}{Pv} > 0. \qquad (48)$$

Thus the antisymmetric long-wavelength perturbations are stable in the whole range of soliton velocities including both limits, i.e., for vortex filaments and for the 2D KP solitons. The frequencies for both limits transform into those obtained in [2] and [13].

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