Multistability as an indication of chaos in a discharge plasma

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(Received 16 June 1994)

We investigated multistable and related chaotic behavior in plasma discharge and experimentally found a new type of catastrophe in the *I-V* characteristic as well as the crises-induced intermittency. Meanwhile, we emphasized the importance of the spiral behavior of the chaotic attractor and the practicality of multistability as a better physical coordinate in the plasma discharge.

PACS number(s): 52.80.−s, 05.45.+b

The existence of multistable states in discharge plasmas manifests itself as a setting for the occurrence of chaotic behavior [1–5]. It is obvious that multistability in the plasma discharge causes the fold and hysteresis behavior in the current-voltage (*I-V*) characteristic and by these behaviors the negative differential resistance of the plasma is demonstrated to exist. The negative differential resistance is generally considered as an essential condition of producing self-oscillation and even the chaotic oscillation in the plasma or the discharge circuit [6,10]. Multistability in the plasma is also responsible for the stability and change of the topological structure of the chaotic attractor, while the topological properties of the chaotic attractor, together with the metric properties, may provide sufficient discrimination power to allow for the practical separation of the observed physical systems [7]. The well-developed dynamical bifurcation theory and catastrophe theory [8] supply useful tools for us to expound the multistability existing in the plasma and, accordingly, the multistable phenomena in the plasma contribute a real counterpart to these theories. As a foundation of the nonlinear discharge theory, the multistable phenomena found in the plasma can build a solid experimental basis too. Therefore, the experimental researches focused on the multistable phenomena in the plasma discharge, including its occurring conditions, its general principle, and its association with chaos, have been an important experimental area.

Multistable phenomena are accompanied by sudden jumps and hysteresis in the *I-V* characteristic. The finding and initial discussions concerning these phenomena in plasmas date back to the work of Cartier and Merlino [9] and that of Knorr [10] in 1984. In the following years, more investigations were reported by other authors under various experimental circumstances [3,11,12]. In these papers, bistability, which is a special case of multistability, and the hysteresis phenomena were focused on more than the multistability itself and also the connection between the multistability of the plasma and the chaotic attractor lacked detailed investigations, whereas the multistability is very important to illustrate the occurrence, stability, and change of chaos in the plasma. It is worth referring to the work of Chern and co-workers [3]. Under their experimental circumstance, the multistable phenomena occurring in the magnetized plasma were investigated. The physical process in the magnetized plasma is naturally different from that in the gas discharge to acquire the unmagnetized plasma and the magnetized plasma device itself brings complexity. It is our belief that in the investigation of basic plasma properties, if the complicated nonlinear phenomena can already occur in a relatively simple experimental configuration, the intrinsic complexity from the simplicity should first be researched and thus understood. It is also the reason that we chose dc discharge rather than adding an external drive. From the viewpoint of differential equations, the temporal evolution of the experimental signals will be determined by autonomous differential equations.

Our experimental device is a steady-state one as described previously [2]. This kind of device confines the plasma by a multipole field produced by the permanent magnets on its outside surface and is force-free inside. As the source of electron emission, the thermal cathode has the ability of supplying several kilowatts heating power. With fixed filament resistance, the filament current (I_f) becomes a controllable parameter to change the heating power. The working gas is argon and its pressure (P) is controlled by using feedback technique. Another controllable parameter is the discharge voltage, the anode of which, through a low-value resistance, is commonly grounded with the chamber. The observed and recorded signals come from the discharge current (I_d) and the floating potential of the probes (V_p). The typical plasma parameters are the electron density 10^8–10^9 cm^{-3} and the electron temperature 1–3 eV. We employed an *X-Y* recorder to graph the *I-V* characteristic curve and a personal computer, model No. PC286, to sample the oscillating signals from I_d and V_p. It is very interesting that, since the recorder is unable to respond to the higher frequency oscillation signals and actually records the temporal-average effect of the oscillating signals, the curves graphed by it record various steady-state bifurcations such as transcritical, pitchfork, and saddle-node bifurcations. The bifurcation behavior is directly associated with the occurrence of the multistable states of the plasma. On the contrary, the computer (with a 12-bit analog-to-digital converter) records the "motional" bifurcation such as the Hopf bifurcation and period-doubling

1063-651X/95/51(4)/3475(5)/$06.00

bifurcation (as far as the Poincaré map of the flow is concerned) and these kinds of bifurcation are the precursors of the regular periodic and quasiperiodic oscillation as well as the irregular intermittent and chaotic oscillation.

In a previous work [2], Qin *et al.* reported on the bifurcation behavior in the same experimental device. Due to the different choice of filament resistance, the stochastic bifurcation was reproduced under our experimental conditions with $I_f \sim 55$ A [12]. In Ref. [13] we classified the stochastic bifurcation as the pitchfork bifurcation based on the bifurcation theory. In the same paper we reported the bistable phenomena occurring in the relatively low regime of gas pressure ($P \sim 10^{-2}$ Pa) and a wide range of $I_f \sim 70-90$ A and concluded that there exists an approximately linear relationship between the width of the hysteresis loop of the *I-V* characteristic and the gas pressure P. Thus we were able to provide a three-dimensional (3D) Whitney curved surface to signify the relationship between the state variant I_d and the parameters P and V_d (Fig. 1). As the usual description, the bistability of the *I-V* characteristic only consists of two discharge modes, respectively, in the lower and the upper branch of the *I-V* characteristic curve [Fig. 3(a)]. It was thought that the lower branch belongs to the anode-glow (AG) mode and the upper branch to the temperature-limited (TL) mode. The transient process from the AG to the TL mode (sudden upward jump) occurs through the ball-of-fire mode and the Langmuir mode [5]. Under such conditions of $P \sim 10^{-2}$ Pa, the chaos only occurs in the lower branch and no oscillation was found in the upper branch. From a visual observation, the lower branch undergoes the dark discharge course in contrast to the intensive glow discharge in the upper branch. Since the plasma potential is positive with respect to the anode in the time of glow discharge, our result is consistent with that of Cheung, Donovan, and Wong [1].

From experimental observation we found that the higher the values the discharge parameters P and I_f reach, the more complex behavior the discharge process shows. Within the controllable limits of discharge parameters we raised the filament current I_f up to 90 A and controlled the gas pressure P in $\sim 10^{-1}-10^{0}$ Pa. The multistable phenomena and abundant nonlinear oscillation accordingly occur under these conditions. It needs to be pointed out that although the multistability does not lose its dependence on the upper and lower main branches of the discharge hysteresis loop, it may not coexist in the both branches. For instance, at $P=1.5\times10^{-1}$ Pa and low I_f (77 A), only in the lower branch does the multistable phenomena exist (Fig. 2). When the discharge current begins to be different from zero, transcritical bifurcation occurs. At the upward-jump and downward-jump points, the saddle-node bifurcation occurs and the corresponding catastrophe is said to be cusp catastrophe [8]. Further, the chaotic signals from I_d and V_p are shown in all the lower subbranches and the upper branch for their dark discharge process (the plasma potential is negative with regard to the anode). With the increase of I_f up to 90 A, the forms of the multistability regularly evolve with the increase of gas pressure P [Figs. 3(a)–3(f)]. At $P=5.5\times10^{-2}$ Pa, the typical bistability is shown. The spike at point S denotes the beginning of the glow discharge. At $P=8.5\times10^{-2}$ Pa, the multistability begins in the lower branch. It is important that a seemingly anomalous catastrophe occurs and this kind of catastrophe continues to exist through $P\sim10^{-1}$ Pa [Figs. 3(b) and 3(c)]. By a voltage scan around the jump points, we find that there exist small hysteresis loops near these points [Fig. 3(c)]. Thus, according to catastrophe theory, a swallow tail-like catastrophe surface (Fig. 4) shows the discharge states in two-parameter and one-variant space. From the schematic view of Fig. 4, we can judge that under the low pressure condition the lower branch of the *I-V* characteristic should be smooth just as the experiment demonstrated. The physical mechanism of producing this catastrophe in the plasma discharge is yet unclear, although it is consistent with catastrophe theory. If the instability from the plasma sheaths is mostly responsible for the discharge process, the disappearance of the sheaths has several

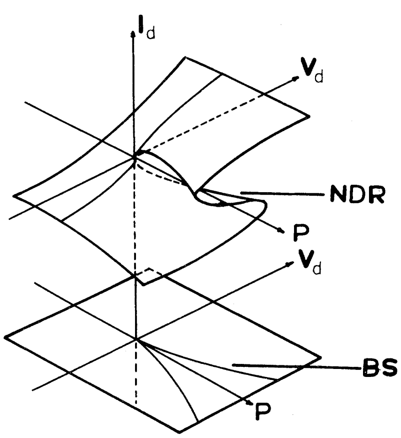

FIG. 1. Whitney curved surface indicating the plasma discharge states. The region with negative differential resistance is located in NDR. The two bifurcation sets (BS) in the regime of $P\sim10^{-2}$ Pa show the approximately linear relationship between the width of the hysteresis loop and the gas pressure.

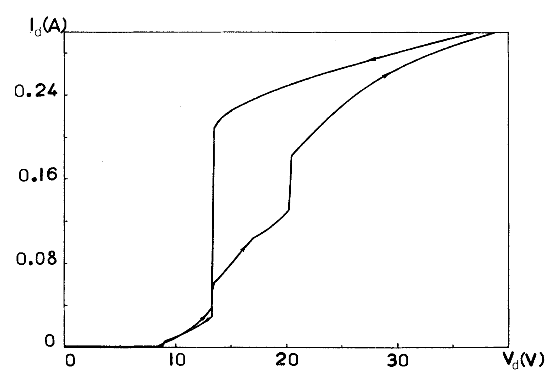

FIG. 2. *I-V* characteristic curve with $P=1.5\times10^{-1}$ Pa and $I_f=77$ A.

stages, i.e., steps. With a further increase of the gas pressure up to $P=1.5$ Pa, the multistability in the lower branch temporarily disappears, while the upper branch transforms, producing multistability. The period-doubling route to chaos occurs in the lower branch, in the intermediate course of which there exists a short duration of intermittent behavior and a period-three window. These courses are much like those described by the logistic map. In regard to the period-doubling route, Qin *et al.* investigated it in detail [2]. The oscillation also occurs in the upper branch and particularly the chaos occurs near the catastrophe points. Ding *et al.* reported a kind of quasiperiodic route (the Ruelle-Takens-Newhouse route) to chaos concerning this multistability in a similar steady-state device [4]. We notice that the glow discharge was less ready to occur with a further increase of the gas pressure, except for increasing the filament current again, so that the spike S was only accompanied by a faint blue glow near the cathode. Whether there is a quantitative relation about this or not has not been determined.

When the gas pressure continues to increase to $P=3.4$ Pa, the multistability begins to simultaneously exist in the both main branches, and in each subbranch, the duration of chaotic behavior elongates. It is more interesting that the spiral ways of the chaotic trajectories are different from each other in the different subbranches. For example, by employing the signals of the discharge current I_d

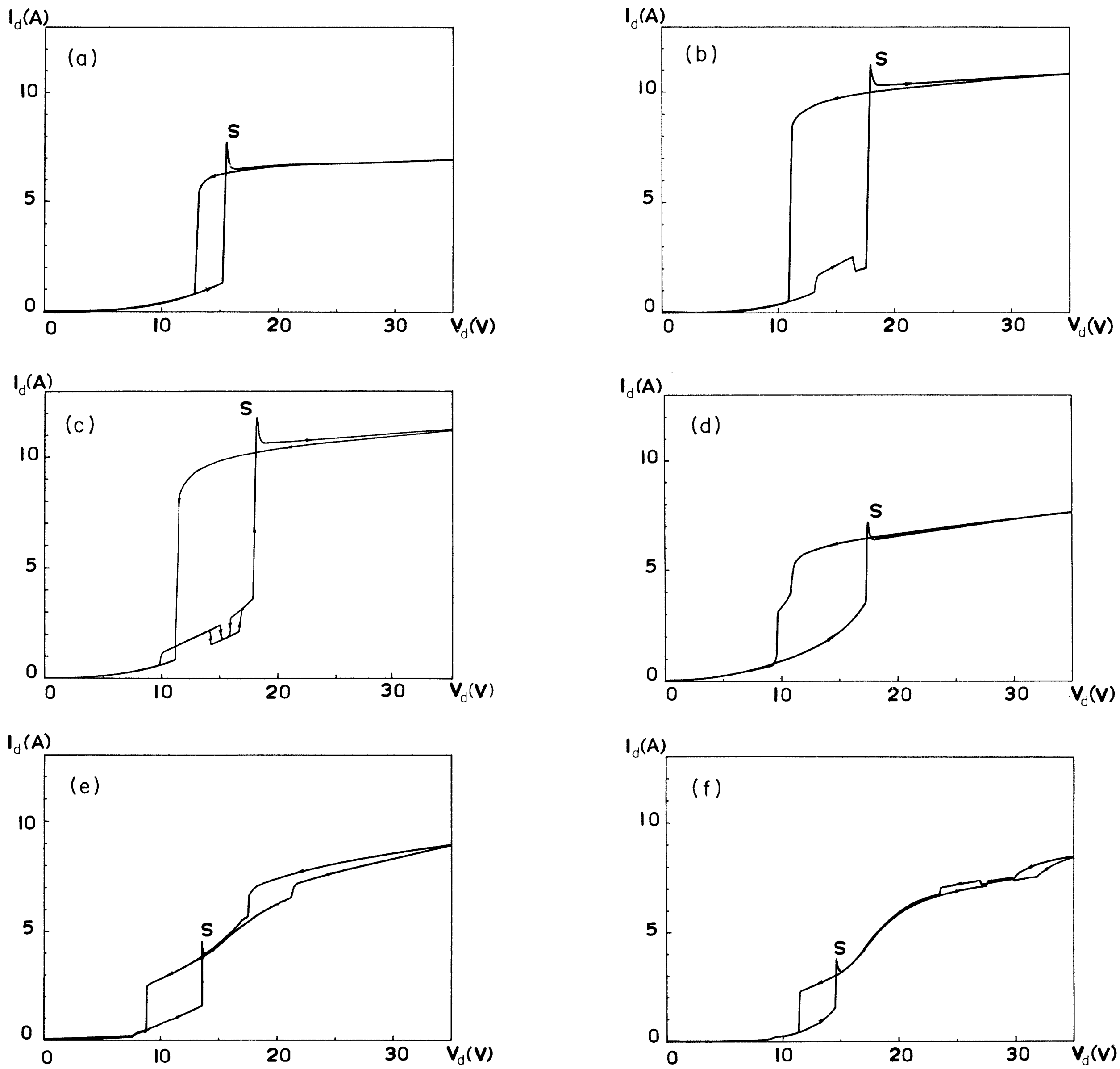

FIG. 3. Evolution of the multistability with the changes of gas pressure $I_f=90$ A: (a) $P=5.5\times10^{-2}$ Pa; (b) $P=8.5\times10^{-2}$ Pa; (c) $P=2.2\times10^{-1}$ Pa; (d) $P=1.5$ Pa; (e) $P=3.4$ Pa; (f) $P=4.7$ Pa.

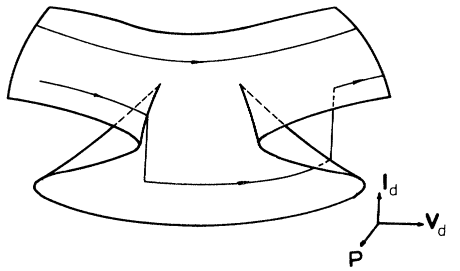

FIG. 4. Schematic of the Whitney curved surface indicating the swallow tail-like catastrophe according to the *I-V* characteristic of Fig. 3(c).

and the time-delay method, we project the chaotic attractors on the 2D planes in the same way and then the chaotic trajectories can be classified as clockwise and counterclockwise (one can notice this by computer graphing, Fig. 5). In the 3D view of the phase space, the motion of the phase "particles" can be specified by the direction of their average angular momentum. (In general, the chaotic attractor in our experimental circumstance is determined geometrically by its correlation dimension 2–3 and dynamically by one positive Lyapunov exponent.) Because the signals from the discharge current I_d and the floating potential V_f exhibit mutual independence when chaos occurs, the plot of I_d versus V_f also recovers the chaotic attractors and the result is the same as that from a scalar variant I_d. This property of the chaotic attractor suggests the specific physical content: Under different multistable conditions, the discharge circuit may have different characteristic, either inductivity or capacitivity, based on different phase relationships. In addition, the chaotic attractor found in the plasma is always of a single vortex, much like the Rössler attractor [17]. According to the consideration of Rössler, this kind of attractor can be caused only by one nonlinear term in the ordinary differential equations (extracted from the partial differential equations with practically physical contents). If there is severe mathematical

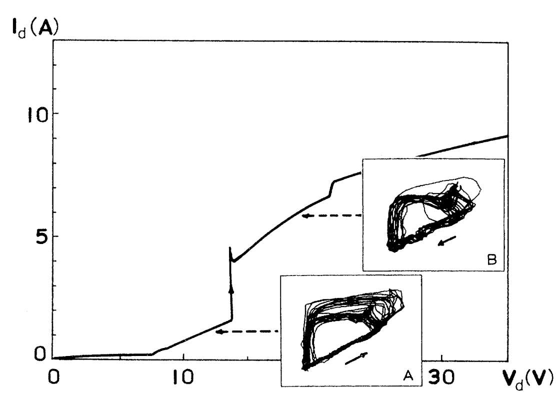

FIG. 5. Examples showing the spiral ways of the chaotic attractors with $I_f=90$ A, $P=3.4$ Pa, and (a) $V_d=12.2$ V and (b) $V_d=19.0$ V. The choice of delay time is based on the average mutual information from the experimental data (see Ref. [7]).

discussion about these phenomena, the topological properties of the plasma chaos will be understood and also make it convenient for one to model the plasma chaos.

With a further increase of the gas pressure to $P\sim5$ Pa, the *I-V* characteristic curve becomes slender and impressive. Such a result of multistability stems from the gradual approach of the main branches with the increase of the gas pressure, while the swallow tail-like catastrophe

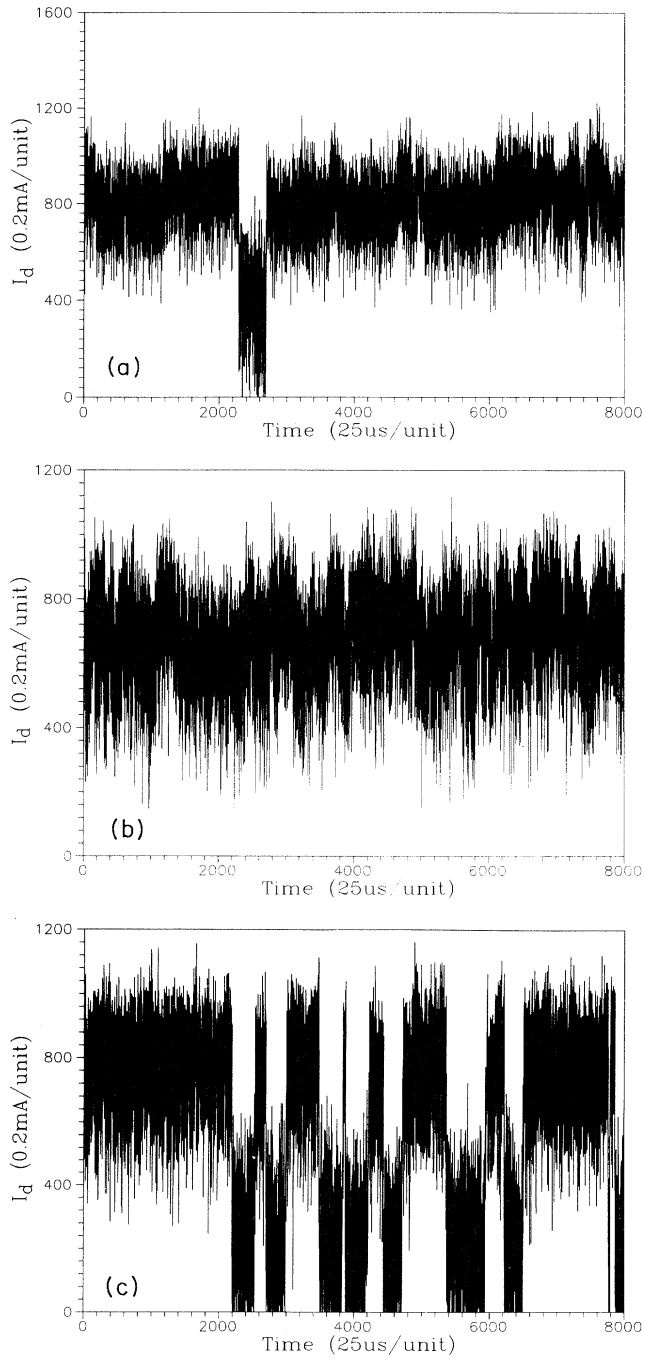

FIG. 6. Crises-induced intermittency occurring at (a) $V_d=26.1$ V, (b) $V_d=29.1$ V, and (c) $V_d=34.2$ V with $I_f=90$ A and $P=4.7$ Pa.

on each branch makes a contribution to the appearance of two small loops. Owing to this change of *I-V* characteristic, a new type of intermittent behavior was found, namely, crises-induced intermittency [14] (Fig. 6). This kind of intermittency has enough obvious characteristics to differ itself from the others [15,16] and, to our knowledge, it is the first reported in the plasma area. Grebogi *et al.* made a scenario of this intermittency as $(\text{chaos})_1 \rightarrow (\text{chaos})_2 \rightarrow (\text{chaos})_1 \rightarrow (\text{chaos})_2 \rightarrow$ *ad infinitum* and classified it as the attractor merging. Two of the examples of the crises-induced intermittency are the forced damped pendulum model and the forced double-well Duffing equation. In the *I-V* characteristic, the intermittency occurs during the upper hysteresis loops, so it is considered that there exists some competitive mechanism between the upper and lower discharge modes when the two stable modes are close to each other. In terms of the forced double-well Duffing equation, the temporal evolution of the discharge current represents intermittent switching between the two plasma potential wells. Like other kinds of intermittency, the crises-induced intermittency also has an exponent law which can quantitatively differ itself from other kinds [16].

To summarize, we report an investigation focused on multistability in a plasma discharge and its association with plasma chaos. A new type of catastrophe and intermittency was found under different multistable conditions. A viewpoint concerning the spiral behavior of the chaotic attractor is presented and briefly discussed. According to our experimental experience, the wall condition of the chamber was difficult to control so it might change more or less in different experimental turns and days. The difference will make it difficult to capture some sensitive experimental phenomena in specified parametric regimes. However, once some kind of multistability occurs, we will most possibly be able to predict the corresponding chaotic behavior.

We would like to acknowledge helpful discussions with Professor S. B. Zheng and X. Z. Yang and also the enthusiastic assistance from Professor Z. M. Li and Dr. Y. H. Zhang. This work was supported by Chinese National Science Foundation and National Basic Research Project "Nonlinear Science."

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[17] In our plasma device, at least four types of intermittency have been found under different discharge conditions, including type I, type X, crises-induced intermittency as well as a kind of unknown (such as on-off) intermittency. We are investigating the critical exponents of them and expect to report them in detail.