Acoustic laser with dispersed particles as an analog of a free-electron laser

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The theoretical scheme of an acoustic laser is suggested. A liquid dielectric with dispersed particles is considered as an active medium. Pumping is created by an oscillating electric field that deforms dispersed particle volumes. Phase bunching of initially incoherent emitters is realized by acoustic radiation forces. The time increment of the useful mode and the value of the starting pumping amplitude are calculated. The suggested scheme is analogous to the free-electron laser.

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I. INTRODUCTION

At the present time, there are many types of lasers. Their operation is based on the stimulated emission (or scattering) of light by an active medium that is in optical resonance. Lasers are generators of coherent electromagnetic waves in the range from ultraviolet to submillimeters. But acoustic analogs of such devices have not been created up to now in spite of the progress in laser technology. There are a number of papers [1,2] in which theoretical schemes of acoustic lasers were suggested. The self-synchronization in a system of noncoherent mechanical oscillators (monopoles) and the amplification of an acoustic field was considered by Kobelev *et al.* [1]. In these papers nonlinear effects have been taken into account. Sound oscillations in the Helmholtz resonator filled with supercooled vapor were investigated by Kotusov and Nemtsov [2].

The purpose of this paper is to suggest a very simple scheme of an acoustic laser which can easily be realized experimentally. This scheme is analogous to that of the free-electron laser (FEL). It is well known that in FEL the electromagnetic emission is created by electron beams moving through magnetic periodic systems. These systems are called ondulators or wigglers [3]. Undulators play the role of pumping. Strong laser waves or some electrostatic systems may be used as an alternative to the undulators. Initially the emission of the electrons is not coherent, but they become grouped due to their interaction with the applied electromagnetic wave. As a result, the emission becomes coherent. It leads to the amplification of the electromagnetic field.

Analogous results are obtained for the suggested scheme of an acoustic laser, in which a liquid dielectric with dispersed particles is an active medium and particles are emitters. The pumping is created by an oscillating electric field that deforms particle volumes. The phase bunching of initially incoherent emitters is realized by acoustic radiation forces.

II. SCHEME OF AN ACOUSTIC LASER

Consider an acoustic resonator with a liquid dielectric containing dispersed particles as an active medium. For example, we can use different types of oils or distilled water as a liquid dielectric. It is well known that the distilled water has a high dielectric constant.

Static electric field acting on such a system results in the deformation (electrostriction) of the dielectric [4] and, hence, changes particle volumes. The value of the effective pressure acting on the particle is equal to [4,5]

$$\Delta P=\frac{3}{8\pi}\frac{\varepsilon_l E^2(\varepsilon_l-\varepsilon_p)}{(2\varepsilon_l+\varepsilon_p)} . \quad (1)$$

Here, ε_l and ε_l are the dielectric constants of the liquid and particle, respectively, E is the electric intensity. For example, for air bubbles in water ($\varepsilon_p \approx 1, \varepsilon_l \approx 81$) and at an electric intensity $E=10$ kV/cm the value of ΔP is of the order of 0.5 kPa. Let us suppose that E is a periodic time function: $E=E_0\cos(\Omega t)$. The electromagnetic waves propagate through the medium with the velocity of light (for this medium), which is much greater than the velocity of sound. Consequently, the pumping pressure wave may be considered as being independent on the spatial coordinates,

$$P(t)=P_E\exp(i\omega t) . \quad (2)$$

Here, $\omega=2\Omega$ is the angular frequency. The pressure amplitude of the pumping wave P_E can easily be calculated from (2). The constant term in (2) is omitted.

Under the action of the pumping wave, the particles oscillate and emit sound waves. The initial distribution of the particles is spatially homogeneous. As a result, the waves created by particles are added with the different phases. Consequently, the resulting pressure of the useful wave is equal to zero.

However, for the active medium in the resonator, an acoustic mode can be excited. Then the particles can be bunched due to the acoustic radiation forces. Moreover, it is well known that the state of the medium with the spatially homogeneous bubble distribution is unstable not only for a standing but also for a traveling wave [6]. This leads to self-synchronization of the oscillating particles and the amplification of the mode. The theoretical scheme of an acoustic laser is shown in Fig. 1.

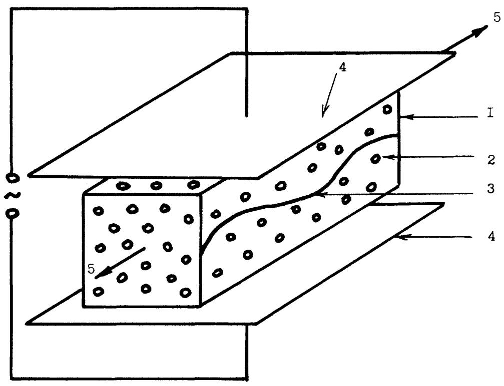

FIG. 1. Scheme of acoustic laser on "free" dispersed particles as an analog of FEL. 1—Acoustic resonator; 2—liquid with dispersed particles; 3—acoustic mode; 4—electromagnetic system which creates the periodic electric field; 5—sound emission.

III. PRINCIPAL EQUATIONS

For simplifications of calculations, we shall suppose that the dispersed particles have the form of a sphere. Their pulsations have been investigated in numerous papers (see, for example, [6–9]). In a monopole approximation, the equation of particle radius pulsation is of the form

$$R_1(t)=-\frac{A}{\rho_l R_0^2\omega^2}[P_E\exp(i\omega t)+P'(\mathbf{r},t)]\ . \tag{3}$$

The right-hand side of this formula contains the resulting pressure acting on a particle: the first term corresponds to the pumping wave (2), the second one describes the pressure created by the oscillations of other particles; A is the scattering amplitude; $\mathbf{r}$ is the position vector of a particle in the liquid; R_0 is the mean particle radius; ρ_l is the liquid density.

The monopole approximation holds true at the condition $k_l R_0 \ll 1$ (k_l is a wave number in liquid). In the case of the liquid with gas bubbles, we have [7]

$$A=\frac{R_0}{(\omega_0/\omega)^2-1+i\delta}\ , \tag{4}$$

where $\omega_0=\omega_0(R_0)$ is the resonance frequency of the bubble, δ is the absorption constant.

The sound pressure wave $P'(\mathbf{r},t)$ is described by the well known equation [6]

$$\Delta P'-\frac{1}{c_l^2}\frac{\partial^2 P'}{\partial t^2}=\rho_l\frac{\partial^2}{\partial t^2}\int_0^\infty 4\pi n(\mathbf{r},R_0,t)R_0^2 R_1(t)dR_0\ , \tag{5}$$

where c_l is the velocity of sound in the pure liquid (without particles), $n(\mathbf{r},R_0,t)$ is the function of particle size distribution (n is equal to the number of the particles per unit liquid volume the mean radii of which range from R_0 to R_0+dR_0).

Let us suppose that at $t=0$ the distribution of the particles is spatially homogeneous, i.e.,

$$n(\mathbf{r},R_0,0)=n_0(R_0)\ . \tag{6}$$

For sound pressure created by the external pumping, one can obtain

$$\Delta P'-\frac{1}{c_l^2}\frac{\partial^2 P'}{\partial t^2}-(\alpha+i\beta)P'=(\alpha+i\beta)P_E\exp(i\omega t)\ , \tag{7}$$

where

$$\alpha=\alpha(\mathbf{r},t)=-4\pi\,\mathrm{Re}\int_0^\infty An(\mathbf{r},R_0,t)dR_0\ , \tag{8}$$

$$\beta=\beta(\mathbf{r},t)=-4\pi\,\mathrm{Im}\int_0^\infty An(\mathbf{r},R_0,t)dR_0\ . \tag{9}$$

In the case of the liquid with gas bubbles, we have

$$\alpha=\alpha(\mathbf{r},t)=4\pi\int_0^\infty\frac{R_0(1-\omega_0^2/\omega^2)}{(1-\omega_0^2/\omega^2)^2+\delta^2}n(\mathbf{r},R_0,t)dR_0\ , \tag{10}$$

$$\beta=\beta(\mathbf{r},t)=4\pi\int_0^\infty\frac{R_0\delta}{(1-\omega_0^2/\omega^2)^2+\delta^2}n(\mathbf{r},R_0,t)dR_0\ . \tag{11}$$

If the spatial distribution is invariable and homogeneous as time passes, then

$$P'(\mathbf{r},t)=P_0'(t)=\frac{(\alpha_0+i\beta_0)P_E\exp(i\omega t)}{k_l^2-(\alpha_0+i\beta_0)}\ . \tag{12}$$

Here, $\alpha_0=\alpha(\mathbf{r},0),\beta_0=\beta(\mathbf{r},0)$ are independent of $\mathbf{r}$, and $k_l=\omega/c_l$. The resulting amplitude is also spatially homogeneous,

$$P_0(t)=P_E\exp(i\omega t)+P_0'(t)=\frac{P_E\exp(i\omega t)}{1-\left[\dfrac{\alpha_0+i\beta_0}{k_l^2}\right]}\ . \tag{13}$$

The appearance of the factor

$$F=\left[1-\frac{\alpha_0+i\beta_0}{k_l^2}\right]^{-1} \tag{14}$$

is caused by the presence of dispersed particles.

The translational motion of the particle is given by the equation [10],

$$\frac{4\pi}{3}\left[\rho_p+\tfrac{1}{2}\rho_l\right]R_0^3\frac{d\mathbf{U}}{dt}=\mathbf{F}_1+\mathbf{F}_2+\mathbf{D}+\mathbf{F}_r+\mathbf{F}_B\ . \tag{15}$$

The left part of this equation contains the usual mass of particle $m_p=\frac{4}{3}\pi\rho_p R_0^3$ (ρ_p is the density of particles) and apparent mass $m_l=\frac{2}{3}\pi\rho_l R_0^3$ (see, for example, [11]); $\mathbf{F}_1=-4\pi(\rho_p+\frac{1}{2}\rho_l)R_0^2\mathbf{U}(dR_1/dt)$ is the drag force due to the particle volume oscillations (its time average $\langle\mathbf{F}_1\rangle=0$); $\mathbf{F}_2$ is the buoyant force which is small for small particles; $\mathbf{D}$ is the viscous drag force which for small Reynolds number $\mathrm{Re}=2R_0U\rho_l/\mu_l$ (μ_l is the liquid viscosity) is given by Stokes' law,

$$\mathbf{D}=-6\pi\mu_l R_0 \mathbf{U} f_v \ , \tag{16}$$

where f_v is the correcting factor which is given $f_v=1$ for solid particles and $f_v=\frac{2}{3}$ for gas bubbles [11]; $\mathbf{F}_r$ is the time-average acoustic radiation force. The expression for $\mathbf{F}_r$ is very complicated but in the case being considered it can be represented as (see Appendix A)

$$\mathbf{F}_r=-\frac{4\pi}{3}\langle R^3(t)\nabla P(\mathbf{r},t)\rangle f_r \ , \tag{17}$$

where $R(t)=R_0+R_1(t)$ is the current particle radius, $P(\mathbf{r},t)$ is the resulting pressure acting on the particle, the numerical factor f_r has the form as follows:

$$f_r=\frac{\left[1+2\frac{\rho_p}{\rho_l}-3\frac{\rho_p^2 c_p^2}{\rho_l^2 c_l^2}\right]}{\left[1+2\frac{\rho_p}{\rho_l}\right]} \ . \tag{18}$$

$\mathbf{F}_B$ is the so-called secondary Bjerkness force [12] which is due to the interaction between a given particle and others (this force is created by the secondary radiation of the particles and is usually smaller as compared with $\mathbf{F}_r$).

Substitution of all these terms into (15) and the time averaging give the following equation:

$$\gamma\mathbf{U}=-\alpha\nabla|P|^2+i\beta(P^*\nabla P-P\nabla P^*) \ . \tag{19}$$

Here, the functions α and β are given by the formulas (8) and (9), respectively, $\gamma=12\pi\mu_l R_0\rho_l\omega^2 N f_v/f_r$, where $N(\mathbf{r},t)$ is the full number of bubbles per unit volume of the liquid at the point $\mathbf{r}$. To simplify the calculations, all particles are assumed to be equal in radius, i.e.,

$$n(\mathbf{r},\tilde{R}_0,t)=N(\mathbf{r},t)\delta(\tilde{R}_0-R_0) \ , \tag{20}$$

Equations (7) and (19) must be supplemented with the balance equation (we shall neglect the coagulation of particles).

$$\frac{\partial n}{\partial t}+\operatorname{div}(n\mathbf{U})=0 \tag{21}$$

IV. ANALYSIS OF EQUATIONS

We shall study the initial stage of the particle bunching when the deviation $n'(\bar{r},R_0,t)$ of the distribution function from the equilibrium value of $n_0(R_0)$ is small,

$$n(\bar{r},R_0,t)=n_0(R_0)+n'(\bar{r},R_0,t) \ . \tag{22}$$

The resulting pressure is

$$P(\bar{r}t)=P_0(t)+\Psi(\bar{r},t) \tag{23}$$

[correspondingly, $P(\bar{r},t)=P_0(t)+\Psi(\bar{r},t)$], where $\Psi(\bar{r},t)$ is the useful acoustic wave which is much less than $P_0(t)$. The values of $P_0'(t)$ and $P_0(t)$ are given by formulas (12) and (13).

The functions $\alpha(\bar{r},t)$ and $\beta(\bar{r},t)$ take the form

$$\alpha(\bar{r},t)=\alpha_0+\alpha'(\bar{r},t)=\alpha_0-4\pi\,\mathrm{Re}\int_0^\infty An'(\bar{r},R_0,t)dR_0 \ , \tag{24}$$

$$\beta(\bar{r},t)=\beta_0+\beta'(\bar{r},t)=\beta_0-4\pi\,\mathrm{Im}\int_0^\infty An'(\bar{r},R_0,t)dR_0 \ . \tag{25}$$

After the substitution of (19) into (21) and the linearization in $\Psi(\mathbf{r},t)$ and $n'(\mathbf{r},R_0,t)$ of Eqs. (21) and (7), we can obtain the self-consistent set of two equations:

$$\left[\Delta-\frac{1}{c_l^2}\frac{\partial^2}{\partial t^2}-(\alpha_0+i\beta_0)\right]\Psi(\mathbf{r},t) =(\alpha'+i\beta')FP_E\exp(i\omega t) \ , \tag{26}$$

$$\gamma\frac{\partial n'}{\partial t}-n_0[(\alpha_0-i\beta_0)P_0^*\Delta\Psi+(\alpha_0+i\beta_0)P_0\Delta\Psi^*]=0 \ . \tag{27}$$

We restrict our consideration to the simplest case when $\Psi(\mathbf{r},t)$ *and* $n'(\mathbf{r},R_0,t)$ are functions of one spatial coordinate only, z, for example. In order to solve Eqs. (26) and (27), we use the substitution

$$\Psi(\mathbf{r},t)=-\frac{\partial Z(\mathbf{r},t)}{\partial t}\exp(i\omega t) \ . \tag{28}$$

We take the spatial dependence of $Z(\mathbf{r},t)$ and $n'(\mathbf{r},R_0,t)$ on z in the form of a standing wave,

$$Z(\mathbf{r},t)\sim\sin(k_{\text{eff}}z), \quad n'(\mathbf{r},R_0,t)\sim\sin(k_{\text{eff}}z) \ ,$$

where the effective wave number is

$$k_{\text{eff}}=\sqrt{k_l^2-\alpha_0} \ .$$

As a result, we have a differential equation of the third order for Z:

$$\frac{d^3Z}{dt^3}+2i\omega\frac{d^2Z}{dt^2}+i\beta_0 c_0^2\frac{dZ}{dt} =\frac{FP_E^2c_l^2k_{\text{eff}}^2}{\gamma}(\alpha_0+i\beta_0)\{(\alpha_0-i\beta_0)F^*Z +(\alpha_0+i\beta_0)FZ^*\} \ . \tag{29}$$

For the following analysis, we suggest some simplifications. Let us assume that the particle content is small, i.e., $|\alpha_0|\ll k_l^2$, and the particles are far from the resonance. Then, $\beta_0\ll|\alpha_0|$, $k_{\text{eff}}\approx k_l$.

We put $Z=a+ib$, where a and b are real functions. This substitution allows us to obtain two equations for the functions a and b as follows:

$$\begin{aligned}&\frac{d^3a}{dt^3}-2\omega\frac{d^2b}{dt^2}-\beta_0c_l^2\frac{db}{dt}\approx\frac{2\alpha_0^2P_E^2\omega^2}{\gamma}a \ ,\\ &\frac{d^3b}{dt^3}+2\omega\frac{d^2a}{dt^2}+\beta_0c_l^2\frac{da}{dt}\approx0 \ .\end{aligned} \tag{30}$$

We will seek a solution to these equations as $a,b\sim\exp(Gt)$, where G is the time increment of the useful mode. Then one can obtain

$$G\left[G^3-\frac{2\alpha_0^2P_E^2\omega^2}{\gamma}\right]+(2\omega G+\beta_0 c_l^2)^2\approx 0\ . \quad (31)$$

For small G, the solution of this equation takes the form

$$G_{1,2}\approx\frac{\left[\frac{\alpha_0^2P_E^2\omega^2}{\gamma}-2\omega\beta_0c_l^2\right]\pm\sqrt{[(\alpha_0^2P_E^2\omega^2/\gamma)-2\omega\beta_0c_l^2]^2-4\omega^2(\beta_0c_l^2)^2}}{4\omega^2}\ . \quad (32)$$

Note that $G_{1,2}\geq 0$, if the condition

$$P_E\geq P_{\rm st}=\frac{2c_l}{|\alpha_0|}\sqrt{(\beta_0\gamma/\omega)}$$
$$=\frac{2c_l}{|\mathrm{Re}A|}\sqrt{(-3\,\mathrm{Im}A)\mu_l\rho_lR_0\omega f_v/f_r} \quad (33)$$

is valid.

The quantity $P_{\rm st}$ corresponds to the starting current in FEL [3]. It follows from (33) that $P_{\rm st}\sim\mu_l^{1/2}$ and $P_{\rm st}\sim\beta_0^{1/2}$. The physical meaning of these dependences is clear. The viscosity prevents us from dispersed particles bunching. Therefore, the increase of the viscosity leads to the increase of the quantity $P_{\rm st}$. The increase of β_0 results in the increase of the energy absorption. Correspondingly, the starting pumping amplitude increases as well.

For a liquid dielectric with gas bubbles at the limit $\omega\ll\omega_0$, we can obtain

$$P_{\rm st}\approx 2c_l\sqrt{2\mu_l\rho_l\omega\delta}\ . \quad (34)$$

In the case of air bubbles with the radii $R_0=15$ μm in water and at the frequency $f=1.5$ kHz, one can obtain $P_{\rm st}=4$ kPa ($E_0\approx 30$ kV/cm). Analogous values can be found for solid dispersed particles with high compressibility.

V. CONCLUSIONS

Thus, there is a strong analogy between FEL and the suggested scheme of the acoustic laser with dispersed particles. In both cases, we have the same integral parts, such as an active medium, the phase bunching, and the starting pressure (or current). But for the acoustic laser in the case of the liquid with gas bubbles, we have an additional problem of the bubble coalescence (under the action of the second Bjerkness forces). However, as it is shown in previous papers, [13–15] bubbles may not only coalesce but also form bound states (in which the distance between them is kept constant). Such bubble clusters were previously observed experimentally by Kobelev, Ostrovsky, and Sutin [16]. Moreover, as it has been shown above, solid particles with high compressibility can be used as an alternative to bubbles.

APPENDIX A

The expression for the acoustic radiation force acting a the small spherical particle is given by

$$\mathbf{F}=-\int_{S(t)}p\,d\mathbf{S}$$
$$=\rho_l\int_{S(t)}\left[\frac{\partial\varphi}{\partial t}+\frac{V^2}{2}-\frac{1}{2c_l^2}\left[\frac{\partial\varphi}{\partial t}\right]^2\right]d\mathbf{S}\ , \quad (\mathrm{A1})$$

where $S(t)$ is the position of the sphere surface, p is the change of pressure, φ is the velocity potential, $\mathbf{V}=\nabla\varphi$ is the liquid velocity.

Using the equality

$$\frac{d}{dt}\int_{S(t)}\varphi\,d\mathbf{S}=\int_{S(t)}\frac{\partial\varphi}{\partial t}d\mathbf{S}+\int_{S(t)}\nabla\varphi(\nabla\varphi\cdot\mathbf{n})dS\ , \quad (\mathrm{A2})$$

one obtains (with accuracy up to the second-order terms in the amplitude of the pressure wave) the expression for the acoustic radiation force as

$$\langle\mathbf{F}\rangle=-\rho_l\langle\int_{S_0}\nabla\varphi(\mathbf{V}\cdot d\mathbf{S})\rangle$$
$$+\rho_l\left\langle\int_{S_0}\left[\frac{V^2}{2}-\frac{1}{2c_l^2}\left[\frac{\partial\varphi}{\partial t}\right]^2\right]d\mathbf{S}\right\rangle\ , \quad (\mathrm{A3})$$

where S_0 is the unperturbed surface.

The calculation of the acoustic radiation force reduces to the problem of the scattering of sound wave by spherical particle [9]. As has been shown in [9], the radial pulsations of spherical particle may be represented as

$$R_1(t)=-\frac{1}{3}\frac{k_p^2R_0B}{\omega}\ , \quad (\mathrm{A4})$$

where

$$B=\varphi(\mathbf{r},t)\frac{1+(ik_lR_0)^3/3}{\rho_p/\rho_l+(ik_lR_0)^2(1+ik_lR_0)^3/3}\ . \quad (\mathrm{A5})$$

Here, $k_p=\omega/c_p$, $\varphi(\mathbf{r},t)$ is the velocity potential of the external wave. The resulting acoustic radiation force can be written as

$$\langle F_\alpha\rangle=\tfrac{2}{9}\pi\rho_lR_0^3\left[k_p^2\left[1+2\frac{\rho_p}{\rho_l}-3\frac{\rho_p^2c_p^2}{\rho_l^2c_l^2}\right]\mathrm{Re}(BB_\alpha)\right.$$
$$+\frac{1}{5}\left[3\frac{\rho_p}{\rho_l}+2\right]\left[\frac{\rho_p}{\rho_l}-1\right]$$
$$\left.\times\mathrm{Re}(3B_\gamma B^*_{\alpha\gamma}-B_\alpha B^*_{\gamma\gamma})\right]\ , \quad (\mathrm{A6})$$

where

$$B_\alpha = \left\{ \frac{\partial \varphi(\mathbf{r},t)}{\partial x_\alpha} \right\} \frac{3\rho_l}{\rho_l + 2\rho_p} \left[1 + \frac{(ik_l R_0)^3}{3} \frac{\rho_l - \rho_p}{\rho_l + 2\rho_p} \right] , \tag{A7}$$

$$B_{\alpha\beta} = \left\{ \frac{\partial^2 \varphi(\mathbf{r},t)}{\partial x_\alpha \partial x_\beta} \right\} \frac{5\rho_l}{2\rho_l + \rho_p} \left[1 + \frac{2ik_l R_0}{45} \frac{\rho_p - \rho_l}{3\rho_p + 2\rho_l} \right] . \tag{A8}$$

The external pressure is the sum (23). In the first approximation in $\Psi(\mathbf{r},t)$, we can take into account only the first term in (A6) because $\partial P_0 / \partial x_\alpha = \partial^2 P_0 / \partial x_\alpha \partial x_\beta = 0$.

Using (A4)–(A7), one obtains

$$\mathbf{F}_r = -4\pi R_0^2 \langle R_1(t) \nabla \Psi(\mathbf{r},t) \rangle f_r ,$$

where f_r is given by (18). The last expression is analogous to (17).

[1] Yu. A. Kobelev, L. A. Ostrovsky, and I. A. Soustova, Izv. Vuzov. Radiofiz. **29**, 1129 (1986).

[2] A. N. Kotusov and B. E. Nemtsov, Akust. Zh. **37**, 123 (1991) [Sov. Phys. Acoust. **37**, 62 (1991)].

[3] C. Marshall, *Free-Electron Laser* (MacMillan, New York, 1985).

[4] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Homogeneous Mediums* (Nauka, Moscow, 1976).

[5] S. T. Zavtrak and E. V. Korobko, Akust. Zh. **37**, 944 (1991) [Sov. Phys. Acoust. **37**, 491 (1991)].

[6] K. A. Naugol'nykh and L. A. Ostrovsky, *Nonlinear Processes in Acoustics* (Nauka, Moscow, 1990).

[7] K. Klei and G. Medvin, *Acoustic Oceanography* (Mir, Moscow, 1980).

[8] K. Yosioka and Y. Kavasima, Acustika **3**, 167 (1955).

[9] V. N. Alekseev, Akust. Zh. **29**, 129 (1983) [Sov. Phys. Acoust. **29**, 77 (1983)].

[10] Yu. Levkovsky, *Structure of Cavity Current* (Sudostroenie, Leningrand, 1978).

[11] L. D. Landau and E. M. Lifshitz, *Hydrodynamics* (Nauka, Moscow, 1990).

[12] L. A. Crum, J. Acoust. Soc. Am. **53**, 1163 (1975).

[13] E. A. Zabolotskaya, Tr. Inst. Obsch. Fiz. Ak. Nauk. SSSR **18**, 121 (1989).

[14] A. A. Doinikov and S. T. Zavtrak, Akust. Zh. **36**, 429 (1990) [Sov. Phys. Acoust **36**, 238 (1990)].

[15] S. T. Zavtrak, J. Phys. A **23**, 1493 (1990).

[16] Yu. A. Kobelev, L. A. Ostrovsky, and A. M. Sutin, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 423 (1979) [JETP Lett. **30**, 395 (1979)].