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Control of dynamical tunneling in a bichromatically driven pendulum

Miroslaw Latka, Paolo Grigolini, and Bruce J. West
Department of Physics, University of North Texas, P.O. Box 5368, Denton, Texas 76203
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We demonstrate that the avoided level crossing between the Floquet state associated with the chaotic part of classical phase space and a member of the quasidegenerate doublet may lead to the enhancement of the doublet splitting by several orders of magnitude. As a result of this interaction the two-level tunneling dynamics is replaced by the more intricate dynamics involving three states. It is shown that three-level tunneling is much more robust against symmetry breaking perturbations than is ordinary tunneling.

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The recent advances in the generation of laser pulses with specific properties (ultrashort pulses, shaped pulses, pulses with well-defined phase relationship) and high intensity have created novel opportunities for the control of quantum dynamics with laser fields [1]. The possible applications of this emerging technology comprise, among others, active product selection in chemical reactions and the control of tunneling for a diverse class of chemical and physical systems.

The investigation of the latter problem was initiated after Lin and Ballentine [2] discovered that the tunneling rate in the quartic double well potential may be increased by several orders of magnitude by the application of an external monochromatic driving field. Herein we use a broad definition of tunneling which accommodates both conventional penetration of classically insurmountable potential barriers and the quantum motion between parts of classical phase space separated by dynamical barriers, the phenomenon known as dynamical tunneling [3]. Grossman *et al.* [4] demonstrated that with the appropriate choice of frequency and amplitude of the external field it is possible to suppress tunneling altogether, i.e., to permanently localize a wave packet in one of the wells of the potential. Bavli and Metiu [5] found that the suppression of tunneling may also be accomplished with a semi-infinite laser pulse. Holthaus [6] pointed out that when a perturbation is turned on sufficiently slowly the nearly degenerate eigenstates which form a wave packet localized in one of the wells evolve into the connected Floquet states with phase factors determined by the corresponding quasienergies. Consequently, under the influence of a smooth pulse both components of a wave function acquire a relative phase difference which may lead to a tunneling time much shorter than the one resulting from the splitting of the unperturbed doublet or, in the opposite limit, to complete localization of wave function. Thus the shaping of the driving pulse presents another possibility for controlling tunneling.

External driving can significantly affect the structure of the classical phase space of one-dimensional systems. With increasing perturbation strength classical dynamics undergoes a gradual, intricate transition to chaos [7]. It is well established that the signatures of chaos may be found in the spectrum of quantum systems and in the structure of eigenfunctions or quasienergy states [7]. On the other hand, the largely unanswered question remains whether, or to what extent, purely quantum mechanical phenomena such as tunneling between symmetry related regular regions of phase space may be influenced by the stochastic layer which surrounds these regular islands. Utermann, Dittrich, and Hänggi [8] found that the splitting of quasidegenerate doublets increases by several orders of magnitude as the regular tori which originally supported the doublet states dissolve into the chaotic sea.

The behavior of the splitting of quasidegenerate doublets has also been the subject of the recent papers by Bohigas *et al.* [9] and Tomsovic and Ullmo [10]. Using the autonomous system of two coupled quartic oscillators they showed that the tunneling rate may be increased as the result of the interaction between one of the doublet states and a state associated with the chaotic part of the phase space, the latter state facilitating the transport of a wave function across the stochastic layer. In the previous papers [11,12] we established that the influence of the avoided level crossing (Landau-Zener effect) on tunneling goes beyond the enhancement of the doublet splitting. For sufficiently strong coupling between the regular doublet and the third state, a condition which may be satisfied close enough to the center of the avoided crossing, the initial two-level dynamics is replaced by the much more complicated dynamics involving three states. This three-level dynamics can give rise to a variety of different quantum effects. In particular, in the center of the avoided crossing the tunneling rate may be several orders of magnitude greater than the rate far away from the crossing (where the influence of the third state is negligible). This property is interesting for possible applications since it enables one to turn tunneling on and off with only small variation of the driving field.

Tunneling is associated with existence of discrete symmetries of the underlying Hamiltonian. Thus one expects that tunneling is suppressed whenever the relevant symmetry is destroyed. Morillo and Cukier in their studies of proton-transfer reactions [13] used a static field to inhibit tunneling. Farrelly and Milligan [14] achieved control of tunneling in the double well potential with the help of two external fields whose frequencies were in a 1:2 ratio. The main field was employed to enhance the doublet splitting while the additional symmetry breaking field was used to arrest a wave packet in one of the wells of the potential. In the same vein, the purpose of the present paper is to investigate the differences between the effects of symmetry breaking perturbations upon the ordinary two-level and three-level dynamical tunneling.

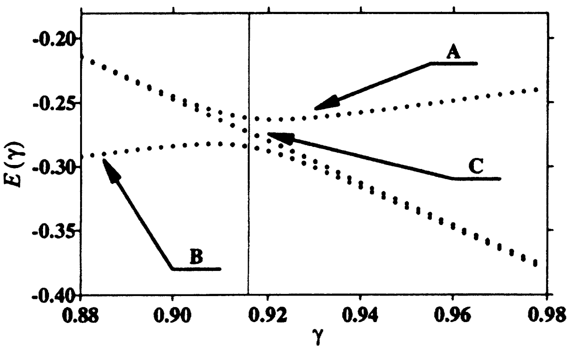

FIG. 1. The avoided crossing between the quasienergy state B and a member of nearly degenerate doublet A. The other doublet state, labeled by C, is not affected by the crossing. The vertical gridline indicates the center of the crossing ($\delta=0$) .

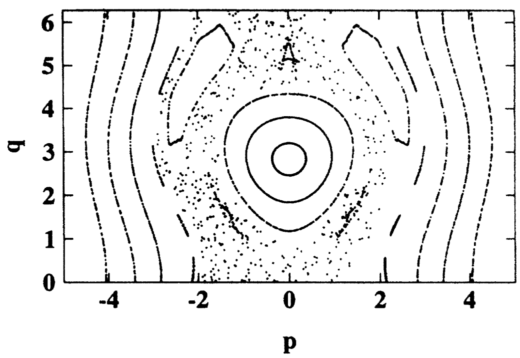

FIG. 2. Poincaré surface of section of classical phase space of the bichromatically driven pendulum with $\gamma=0.90$ and $\delta=0.15$. Momentum was scaled by the parameter μ.

As our model system we choose the bichromatically driven pendulum $H=H_0-\mu\gamma q\cos(\Omega t)-\mu\delta q\cos(2\Omega t+\phi)$, where the unperturbed Hamiltonian H_0 reads $H_0=p^2/2\mu+\mu(1+\cos q)$, γ is the peak amplitude of the main external driving field, and Ω is its frequency. The auxiliary symmetry breaking field has the amplitude δ and frequency 2Ω. Throughout this paper $\Omega=2$, $\mu=5$, and $\phi=\pi/2$. The angle q varies between 0 and 2π.

The periodicity of the above Hamiltonian enables us to describe the evolution of a wave function in terms of the Floquet states $|u_n(t)\rangle$ which are true stationary states of periodically time-dependent quantum systems [7]. The Floquet states are eigenvectors of the operator $\mathscr{H}=\hat{H}-i\hbar\partial/\partial t$ with the periodic boundary condition imposed in time. The corresponding eigenvalues, quasienergies ϵ_n, for bounded systems are real numbers. For times $t=kT$, $k=1,2,\ldots$ the wave function may be written as [7]

$$|\psi(kT)\rangle=\sum_n \exp(-iE_nk)|u_n\rangle\langle u_n|\psi(0)\rangle, \qquad (1)$$

where $E_n=\epsilon_nT$ and $|u_n\rangle=|u_n(0)\rangle$. It is apparent from (1) that only the Floquet states overlapping the initial wave function contribute to its subsequent time evolution.

In the absence of the symmetry breaking field ($\delta=0$) the Hamiltonian H remains invariant under the following dynamical symmetry: $q\rightarrow-q$, $t\rightarrow t+\pi/\Omega$ and the Floquet states may be classified into states of even and odd parity with respect to this generalized parity transformation.

In Fig. 1 we present a small portion of the Floquet spectrum of the driven pendulum ($\delta=0$). One can see from this figure that a characteristic double cone structure of the avoided level crossing originates as a result of the interaction between a member of nearly degenerate doublet A and quasienergy state B, both having the same symmetry. The other member of the doublet, labeled in Fig. 1 by C, has a dynamical symmetry opposite to that of states A and B and is not affected by the interaction. A more extensive discussion of the properties of the quasienergy states from Fig. 1 may be found in Ref. [12]. Far away from the center of the crossing, where the influence of the state B is small, the Husimi distribution of the doublet states is localized in the region of quantum phase space where classically there are two prominent nonlinear resonances immersed in the stochastic sea (cf. Fig. 2); see, for example, [12], for a discussion of the Husimi distribution in this context. By taking a symmetric or antisymmetric combination of the doublet states one obtains a wave packet localized on one of the islands which in time tunnels to the other one. The application of the symmetry breaking perturbation may significantly change this two-level dynamics.

In Fig. 2 we show the Poincaré surface of section of the bichromatically driven pendulum for $\gamma=0.90$ and $\delta=0.15$. The classical motion was strobed at $t=kT$, $k=0,1,2\ldots$. The large symmetric Kolmogorov-Arnold-Moser (KAM) islands, immersed in the stochastic sea, for $\delta=0$ underlie the structure of the doublet states. Interestingly enough, the modifications of the surface of section produced by the auxiliary field are hardly noticeable. However, the quantum effects of the symmetry breaking are much more strongly pronounced.

In Fig. 3 we contrast the structure of Floquet states A, B, and C calculated for $\gamma=0.90$ (top panel) with those of the corresponding states in the presence of the symmetry breaking field with amplitude $\delta=0.15$ (bottom panel). The states were expanded in the eigenbasis of angular momentum operator $\{|\phi_n\rangle\}$, $|\phi_n\rangle=\exp(inq)/\sqrt{2\pi}$. In all the graphs in Fig. 3 the quantum number n varies between -20 and 20, the range of occupation probability is $[0,0.15]$. It is apparent that the perturbation destroys the double hump form of states

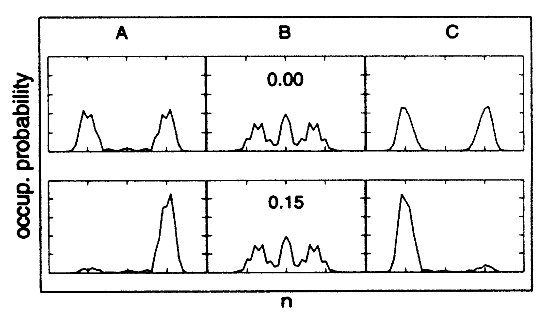

FIG. 3. Expansion of Floquet states A, B, and C in the basis of the eigenfunctions of angular momentum operator plotted for two values of the amplitude δ of the symmetry breaking perturbation ($\gamma=0.90$).

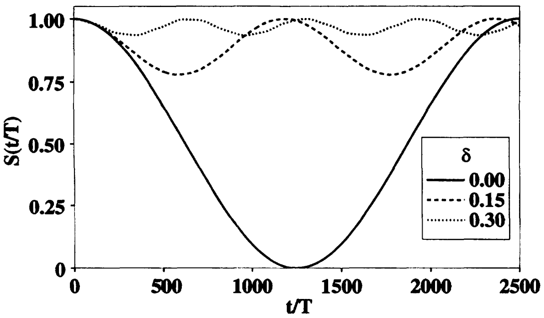

FIG. 4. Time evolution of the survival probability of the wave packet P for different values of the amplitude δ of the symmetry breaking perturbation ($\gamma=0.90$).

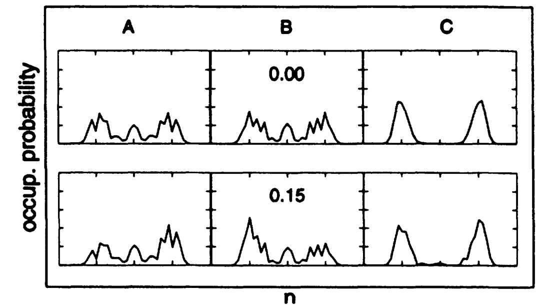

FIG. 5. Expansion of Floquet states A, B, and C in the basis of the eigenfunctions of angular momentum operator plotted for two values of the symmetry breaking amplitude δ ($\gamma=0.9159$).

A and C whose Husimi distributions, as indicated by the basis expansion, are predominantly localized on the right and left KAM torus, respectively (cf. Fig. 2). On the other hand, for $\delta=0.15$ the state B remains essentially unaffected by the symmetry breaking perturbation which may be explained by taking into account the low value of the dipole moment of this delocalized state. We have shown previously that the Husimi representation of this state while not fully chaotic is spread out in the stochastic part of the phase space [12]. The change of the structure of the doublet states under the influence of the symmetry breaking field is reflected in the quantum dynamics of wave packets centered on the KAM islands. For simplicity let us form such a wave packet by taking the symmetric superposition of the doublet states from the top panel of Fig. 3 ($\gamma=0.9$ and $\delta=0$). This wave packet, in further discussion referred to as P, is localized on the right KAM island shown in Fig. 2. Note that the doublet states are obtained directly from the numerical calculations, therefore wave packets centered on either of the symmetric islands may be obtained from a suitable combination of these states. In the absence of the auxiliary field the wave packet P tunnels back and forth between the islands. In this case using (1) one obtains the following expression for the survival probability $S(t)=\langle\Psi(t)|\Psi(0)\rangle$ at the integer multiples of the period:

$$S(kT)=\tfrac{1}{2}+\tfrac{1}{2}\cos[(E_1-E_3)k], \tag{2}$$

where E_1 and E_3 are the scaled quasienergies of states A and C, respectively. The solid line in Fig. 4 corresponds to (2). With the growing amplitude of the symmetry breaking field the nearly degenerate states lose their double hump structure and the state A dominates the expansion of the wave packet P (in the two-dimensional basis made up of states A and C) which explains its trapping on the right island. This trapping is illustrated by the time evolution of the survival probability depicted in Fig. 4 for $\delta=0.15$ and $\delta=0.30$. Note that the increased frequency of the oscillations of the survival probability in the presence of the auxiliary field is caused by the enhanced splitting of quasienergies E_1 and E_3. This effect may be accounted for with the help of a simple two-level model.

If in the absence of the symmetry breaking we again use the wave packet P as an initial condition but we increase the amplitude γ of the main field, then the quantum dynamics gets more involved [12]. Due to the repulsion between states A and B (cf. Fig. 1) the wave packet becomes the linear superposition of all three quasienergy states A, B, and C. This is associated with the exchange of the structure of interacting states, a generic property of the Landau-Zener effect [15]. The first panel in Fig. 5 shows all three states at the center of the avoided crossing ($\gamma=0.9159$) where the mixing of states A and B is most strongly pronounced. It is clear that state A no longer resembles its doublet counterpart C. Both states A and B are now appreciably delocalized and their dipole moments are significantly smaller than that of state C which maintains its double hump structure. Consequently, it is not surprising that the effect of the symmetry breaking on these states (cf. the bottom panel in Fig. 5) is very limited compared to the change of the doublet states in Fig. 3. The symmetry breaking operator $\mu\delta\hat{q}\cos(2\Omega t)$ is odd with respect to the previously mentioned generalized parity transformation. States A and B have the same dynamical symmetry so that considering the weak interaction, in the first approximation, this perturbation does not directly mix these states. On the other hand, state C is coupled to both states A and B. However, as can be seen from Fig. 5 the modification of state C is rather minor. Note that for the same value of the amplitude δ the doublet states in Fig. 3 have already lost the tunneling–double hump structure.

We have already pointed out that despite the change of the structure of one of the quasidegenerate states, in the absence of the auxiliary field, the tunneling rate is maximal at the center of the avoided crossing. In this case the time evolution of survival probability of the wave packet P is given by [12]

$$S(kT)=\tfrac{1}{2}\{1+\cos[(E_1-E_3)k]\}-4d_1d_2\sin^2[(E_1-E_3)k], \tag{3}$$

where d_1 and d_2 are the square moduli of the overlap of the packet with the quasienergy states A and B, respectively. In Fig. 6 we plotted the above function along with the survival probability for two values of the amplitude δ of the symmetry breaking perturbation used in Fig. 4. We can see that even for $\delta=0.30$, the strength of the field almost completely suppresses the two-level tunneling, but the three-level tunneling persists and the quantum dynamics remains very similar to that given by Eq. (3). The increase of the frequency of oscillations in Fig. 6 is significantly smaller than that of the two-level dynamics in Fig. 4. This small change is another manifestation of the weak influence of the symmetry breaking field on the Floquet states and their quasienergies.

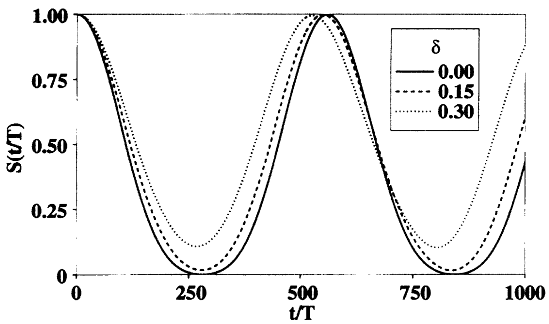

FIG. 6. Time evolution of the survival probability of the wave packet P for different values of the amplitude δ of the symmetry breaking perturbation ($\gamma=0.9159$).

We attributed to delocalization of the quasienergy state A, stemming from the interaction with the state B, the insensitivity of three-level dynamics in Fig. 6 to symmetry breaking perturbation. This interpretation is corroborated by the additional numerical simulations which show that away from the center of the avoided crossing, where the influence of state B is weaker, the quantum dynamics is more responsive to the auxiliary driving.

The various methods of controlling quantum dynamics of bistable systems [4–6] proposed so far have relied on purely quantum mechanical effects. Herein we have demonstrated that control of tunneling may, in principle, be based upon the phenomenon of avoided level crossings between one of the quasidegenerate states and a third state associated with the chaotic part of the phase space. Unlike the previous approaches, this method is intimately related to the metamorphosis of the structure of classical phase space induced by the external driving field. We emphasize that the avoided crossing between a tunneling doublet and a regular state [e.g., one associated with the large island around the elliptic fixed point (0,0) in Fig. 2] is quite conceivable. However, our analysis of the Floquet spectrum of the driven pendulum shows that this type of crossing is significantly less prevalent than the one involving a third state which is "chaotic" and consequently is less important with respect to possible applications. Moreover, if the third state is sufficiently delocalized (characteristic feature of Floquet states associated with the stochastic layer) then three-level tunneling may be significantly more robust against symmetry breaking perturbations than is the ordinary two-level tunneling. This property might be useful in discriminating between two and three level tunneling, for example, in molecular systems with anharmonic bonds.

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[1] S. A. Rice, Science **258**, 412 (1992).
[2] W. A. Lin and L. E. Ballentine, Phys. Rev. Lett. **65**, 2927 (1990).
[3] M. Davis and E. J. Heller, J. Chem. Phys. **75**, 1 (1981).
[4] F. Grossmann, T. Dittrich, P. Jung, and P. Hänggi, Phys. Rev. Lett. **67**, 516 (1991).
[5] R. Bavli and H. Metiu, Phys. Rev. Lett. **69**, 1986 (1992).
[6] M. Holthaus, Phys. Rev. Lett. **69**, 1596 (1992).
[7] L. E. Reichl, *The Transition to Chaos* (Springer-Verlag, New York, 1991).
[8] R. Utermann, T. Dittrich, and P. Hänggi, Phys. Rev. E **49**, 273 (1994).
[9] O. Bohigas, S. Tomsovic, and D. Ullmo, Phys. Rep. **223**, 43 (1993).
[10] S. Tomsovic and D. Ullmo, Phys. Rev. E **50**, 145 (1994).
[11] M. Latka, P. Grigolini, and B. J. West, Phys. Rev. E **50**, 596 (1994).
[12] M. Latka, P. Grigolini, and B. J. West, Phys. Rev. A **50**, 1071 (1994).
[13] M. Morillo and R. I. Cukier, J. Chem. Phys. **98**, 4548 (1993).
[14] D. Farrelly and J. A. Milligan, Phys. Rev. E **47**, R2225 (1993).
[15] C. Cohen-Tannoudji, B. Diu, and F. Laloe, *Quantum Mechanics* (Hermann, Paris, 1977).