Size segregation in a two-dimensional sandpile: Convection and arching effects

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We report an experimental study of the ascent of a large disk imbedded in a two-dimensional packing of small beads vertically vibrated. Two distinct mechanisms leading to size segregation are observed *in situ*. At high acceleration, the convection process associated with surface trapping is dominant. At low acceleration, we examine the effect of the size ratio on the dynamics of the segregation which is due to successive formation and destruction of arches.

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Due to their peculiar dynamical behavior and static properties, noncohesive granular materials such as sand, grains, gravel, or powders have recently motivated a widespread interest among physicists ([1], and references therein). So far, few physical properties have been fundamentally elucidated. Among other puzzling features is the segregation of multicomponent mixtures. This features has been known for a long time and is of major importance in numerous industrial processes [2–5]. Also, it is at the source of a large number of experimental works [3–7]. Under agitation, shearing, or flowing, larger particles tend to migrate towards the outer regions of the granular assembly. In particular, the vibration induced size segregation problem, also known as "the brazil-nut problem" [8], has recently been the object of numerical simulations [8,9] and experimental works [6,7]. It is at the focus of a controversy concerning the pertinence of two different processes which have been claimed in the literature. Namely, does the segregation originate solely from a global convection of the particulate and selective surface trapping, as recently evidenced by Knight, Jaeger, and Nagel [6], or is it mainly due to purely geometrical effects in the bulk [8,9]? While a purely convective process naturally leads to a size independent segregation dynamics, geometrical effects in the bulk have been recognized to be sensitive to the size ratio [7–9].

Different approaches have previously identified the geometrical mechanism. On the one hand, there are numerical simulations. Rosato *et al.* [8] proposed a Monte Carlo shaking algorithm and Jullien, Meakin, and Pavlovitch [9] a piling algorithm. The Monte Carlo algorithm showed a continuous ascent as well as intermittent regimes but no size threshold for segregation has been predicted. The piling algorithm displayed the existence of a size threshold for segregation. The existence of such a threshold generated some discussions [10,11]. Furthermore, recent experiments displayed evidence for both an intermittent ascent regime for small sizes and a continuous regime for larger sizes [7]. An explanation was proposed which was based on the stability of arches in the surrounding of the intruder. One of the consequences of this model was to identify the size threshold of Jullien, Meakin, and Pavlovitch [9] with the transition from an intermittent to a continuous regime of ascent and not to a *segregation threshold* [11]. Also, and bolstering the arching effect model, the algorithm of Jullien, Meakin, and Pavlovitch, supplemented with noise, recovered the predicted and observed intermittent regime through their computer simulations [12].

The present work is based on the direct observation of the up motion of a single intruding particle imbedded in a vibrated monodisperse assembly of particles forming a two-dimensional (2D) vertical pile. Up to now, and to our knowledge, no experimental results about the detailed *local dynamics* of a segregation process have ever been reported. In this paper, we point out the fact that the convection mechanism and the arching mechanism *do not occur under the same range of excitation parameters.* Here and extending the works reported in [6,7], we explore the low amplitudes of a vibration regime where convections are unable to advect the intruder upwards. Under these conditions, we still observe an ascent of the intruder and we study the kinetics as a function of the diameter ratio of the intruder and the surrounding particles. Furthermore, we observe for the first time the existence of a new critical diameter ratio separating segregation and no segregation regimes. This threshold is also sensitive to the vibration amplitude, the cell aspect ratio, the position of the intruder in the cell, and the micromechanical parameters (elasticity and dry friction). We analyze this effect in the spirit of our previous arching effect model [7].

The experimental setup we use here (Fig. 1) is built along the lines of our previous experiments described in Refs. [7,13]. The cell is made of two rectangular glass plates (10×15 cm^2) separated by rectangular plastic wedges. The width is adapted to accommodate, as accurately and freely as possible, the monodisperse set of 1.5-mm-diam aluminum beads. The beads we use are oxidized aluminum beads (no segregation nor convection was ever observed for a set of *polished* aluminum beads) which have high solid friction coefficient at the lateral walls and high momentum loss on collision. In this case, convection rolls and heaping show up spontaneously for accelerations larger than $\Gamma_c=(1.03\pm0.03)g$ where g is the gravitational acceleration [13].

The experiments have been made using an excitation frequency of 15 Hz; the excitation amplitude [$a(t)=A\sin\omega t$] is indicated as usually via the reduced

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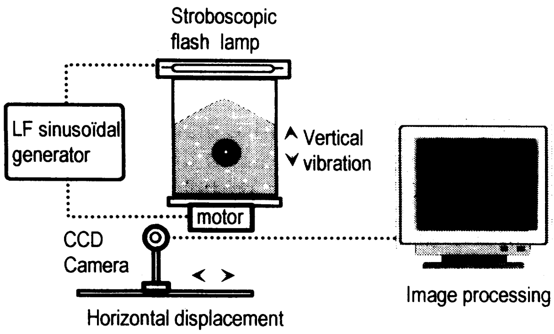

FIG. 1. Sketch of the experimental setup.

acceleration parameter $\Gamma = A\omega^2/g$. At the beginning of the experiment, the intruding aluminum disk (1 mm thick, radius R) is inserted among the assembly of small beads at 1.5 cm above the bottom of the container. In order to start from well defined conditions, the pile is prepared to match as much as possible a compact triangular lattice, except around the intruder which induces local perturbations in the array [Fig. 2(a)]. In order to provide a quasi-2D free motion of the intruder between the two frontal windows of the cell, three equidistant steel 1.5-mm-diam beads are pinned in the outer ring of the circular disc. The intruder is painted in black with a white dot decorating its center. All experiments are characterized by a diameter ratio $\Phi = R/r$. For the present report, we performed two different sets of experiments.

The first set is a direct visualization of the trajectories of the small beads and of the larger disk. Prior to vibration, we dispersed test beads among the assembly of corrugated aluminum beads. These test beads are about 1% in number. They are made of steel with the same diameter, 1.5 mm, and look more shiny compared to the aluminum ones. Using an image processing device, we selectively record the test beads and the intruder positions. Accumulation of such pictures is a so-called computer posed photograph (CPP). Note that the stroboscopic flashing light is locked on the vibration so that positions are recorded for the same height of the container. Examples of CPP are displayed in Figs. 2(b)–2(d).

A second series of experiments is designed to follow

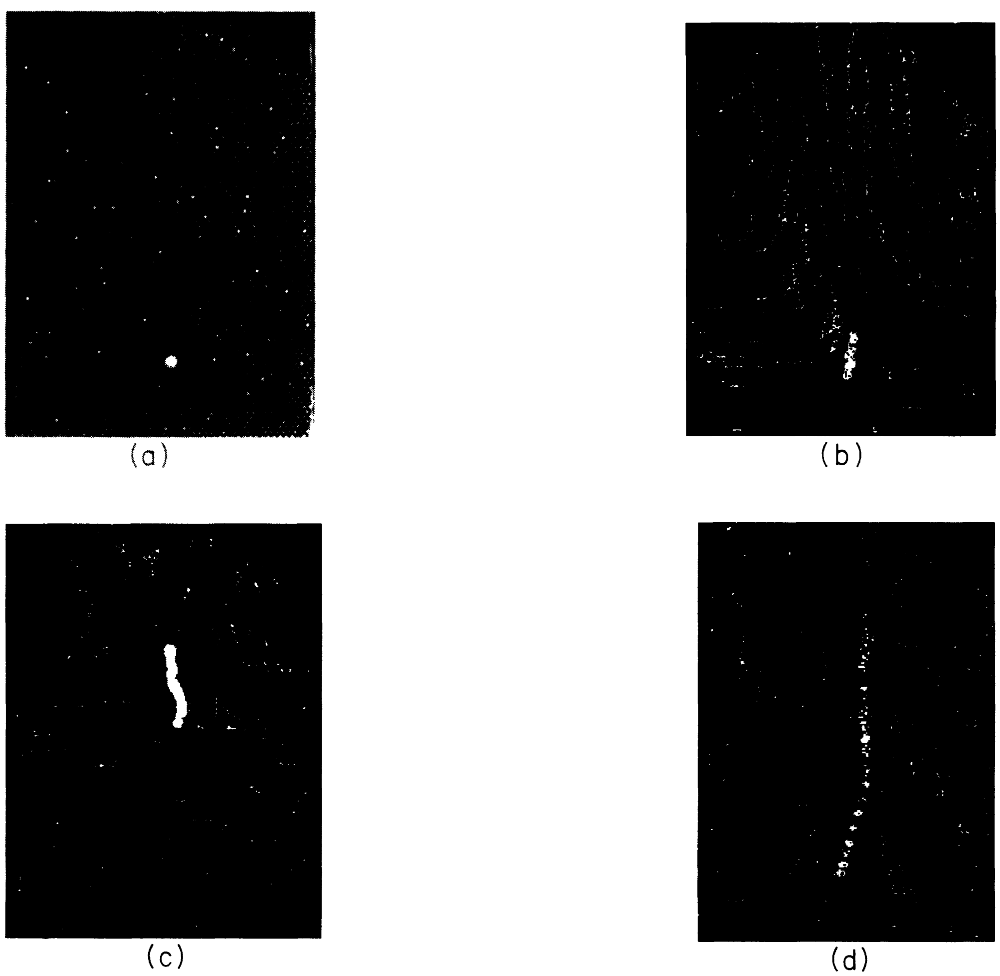

FIG. 2. (a) Snapshot of a 2D cell at the beginning of an experiment. Here, the diameter ratio is $\Phi = 12.9$. The width of the cell is 7.5 cm, (d) CPP of the 2D vibrated cell in the *convective regime* ($\Phi = 5.3$ and $\Gamma = 2$ at 15 Hz and $A = 2.2$ mm); (b) and (c) CPP of the 2D vibrated cell in the *vault regime* at two different moments during the ascent ($\Phi = 12.9$ and $\Gamma = 1.25$ at 15 Hz and $A = 1.4$ mm). The total time of ascent is about 10 min.

and record the vertical position of the center of the intruding disk as a function of time: $h(t)$. We horizontally move the charge-coupled device (CCD) camera at a constant speed in parallel to the front windows of the container (see Fig. 1). The cell is illuminated by a stroboscopic flashing light synchronized to the vibration. The image of the white central dot on the intruder is selected, recorded, and accumulated to the previous ones. We checked that the rise of the intruder, when it occurs, is vertical, which means a symmetrically and correctly vibrated cell. Thus we obtain directly on the screen of our image processing computer the curve $h(t)$ as reported in Fig. 3.

In Fig. 2 [from (b) to (d)] we observe that exciting the cell at an acceleration larger than the gravitational acceleration induces internal motions in the pile. We recall that in the absence of intruder, the convection rolls spring up at the upper lateral corners of the granulate and, when the excitation acceleration Γ is increased, they expand in size towards the center and the bottom of the cell [13,14] thereby destroying the initially prepared triangular regular geometry of the packing. Compared to our earlier observations, the introduction of a larger disk among an assembly of monodisperse beads produces several new and different characteristic features. We observe that, depending on the excitation amplitude, two different regimes are clearly distinguished: At relatively large accelerations, i.e., when the convection rolls are of the size of the container (in our case, when $\Gamma > 1.5$) and in agreement with recently reported experiments [6] and simulations [15], we observe global convective motions of all the particles in the cell. The CPP of such an experiment at $\Phi = 9$ is displayed in Fig. 2(d). Note that the trajectories of the small beads evidence convection rolls taking place *below* the intruding disk and carrying upwards the bigger particle at the same speed. In agreement with [6], we observed that the probability of being reconvected downwards at one of the lateral walls, when the surface is reached, strongly depends on the diameter ratio [16].

Now let us consider Figs. 2(b) and 2(c) which are the CPP of an experiment performed at a lower acceleration ($\Gamma = 1.25$) and for a diameter ratio $\Phi = 13$. Without intruder, the convection rolls would be confined in the upper left and right corners of the cell. Figures 2(b) and 2(c) stand for two different snapshots taken in the course of the same experiment. In Fig. 2(b) the CPP is taken for the first 60 s while in Fig. 2(c), the CPP is resumed after a break of 2 min. Two important features should be noticed. First, we observe that the upward ascent of the intruder is accompanied by several *horizontal* bead rearrangements which are directly visualized via the *horizontal tracks* of the markers. Notice that both in Figs. 2(b) and 2(c), the intruder is not carried upward in a convective motion of the surrounding particles, since no vertical trail is present in the underlying environment. Second, by comparing Figs. 2(b) and 2(c), we note an important feature: once the intruder has left a lower region, no internal motion is any longer evidenced: these displacements start just above the intruder, and vanish when the particle has climbed up.

Now we explore the segregation dynamics at a given low acceleration $\Gamma = 1.25$ but using a set of intruders with different diameter ratios. We report in Fig. 3 the experimental curves as they have been plotted on the screen of the image processing computer and put together on the graphics. This figure displays several important and novel features. First of all, the ascent velocity strongly depends on the diameter ratio Φ. The larger the diameter of the intruder, the faster is the ascent. Also, and as confirmation of [7], we observe that for $\Phi = 5.3$, the small intruder undergoes an intermittent ascent. Then, the function $h(t)$ looks like a staircase with steps of the order of a small bead diameter which implies long waiting times between successive upwards and discretized motions. For larger Φ we observe a mixture of continuous ascents and long waiting time distribution. Yet unexplained but noteworthy is the repeatedly observed fact that the curves $h(t)$ display a smaller upwards velocity at the beginning of the ascent. It is also remarkable that, under the same experimental conditions, intruders smaller than $\Phi = 3.3$ do not exhibit any upwards motion. Then we observe that if we increase slightly the reduced acceleration Γ or if we start from a higher position in the cell, they can start rising up. In reverse, the ascent of a larger particle can be stopped if the acceleration Γ is slightly decreased. We plotted in Fig. 4 the typical ascent velocities as a function of the diameter ratio Φ. Ascent velocities are determined as the typical slopes measured at the upper part of the ascent curves of Fig. 3. Within experimental errors, this plot shows up a quasilinear dependence starting from a critical value $\Phi = 3.3$ below which no up motion could be observed during a waiting time longer than one hour. Furthermore, we made the following and crucial observation. When we light the sample from behind, we clearly observe the presence of density microfluctuations inside the bulk which occur at

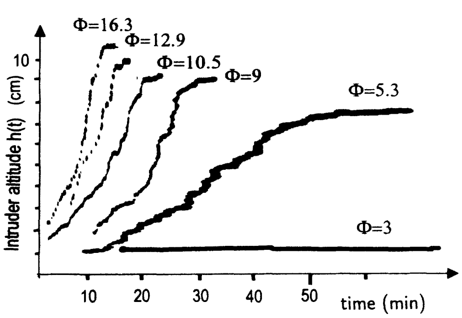

FIG. 3. Ascent diagrams $h(t)$ at $\Gamma = 1.25$ for a set of different diameter ratios Φ.

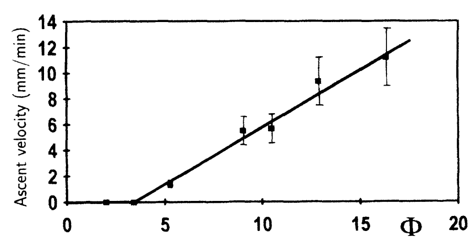

FIG. 4. Ascent velocity of intruders as a function of diameter ratios Φ at $\Gamma = 1.25$.

every period of excitation and are effective during one-quarter of the period (phase between $\pi/2$ and π). These microscopic fluctuations occur in a number of reversible cracks which occasionally result in irreversible long duration macroscopic distortions of the piling such as horizontal slip lines. A more detailed analysis of these density fluctuations, which turn out to be quite general in vibrated beds of particles, will be reported elsewhere [14]. However, and at the present time, it is clear that in the regions where microfluctuation are present, the upwards motion of the intruding disk is governed by the intricate interplay between statistics of this microfluctuations and local geometrical effects around the intruder. Starting from this observation and in the context of the arching effect model proposed in Ref. [7], we give a tentative interpretation for the existence of a *size threshold for segregation*. Under given experimental conditions (excitation amplitude, container geometry, etc.) and for a given height in the piling, a set of relative vertical fluctuations with a length scale δ is picked up by the system. Now, we refer to the ascent diagram of Fig. 3 in Ref. [7] which shows the continuous sets of stable arches holding the intruder. For smaller size ratios, the continuous sets are connected within each other by discrete sets of vertical jumps. Thus, for a given Φ, and if δ is too small, there is no possibility for the intruder to reach the next stable position or the next set of stable vaults and the segregation is stopped. Thus the interpretation of Φ_c is that for a given fluctuation scale δ, Φ_c is the minimum diameter ratio which allows a passage between all the sets of stable positions or next set of arches. The assessment of the correct time dependence depends on the difficult question of the microfluctuation spectrum which, at the present time, may be seen as playing a key role in the understanding of the dynamic of the sandpile.

In this paper, we present *a direct experimental* observation of a size dependent dynamics for segregation and a segregation size threshold measurement. Our experiments clearly evidence the presence of a dominating arching mechanism when the transport by convection is marginal (slightly above the acceleration threshold $\Gamma \approx 1$). Under our experimental conditions, we found a size-ratio threshold for segregation to be much smaller than the size-ratio threshold separating the continuous and intermittent regimes and we realized that its exact value depends on the aspect ratio of the cell, the excitation amplitude, the micromechanical conditions of the beads and of the walls, and also the initial depth of the intruder in the bulk. These results are currently restricted to a bidimensional geometry and to a rather regular piling of beads. The question of whether it is transportable in 3D is still open. However, and whatever their generic meaning, these experiments provide a firm benchmark to test an increasing number of 2D numerical simulations being currently performed on granular materials. In the same spirit, the direct visualization of density fluctuations, appearing periodically in time in the bulk, provides a bridge with earlier computer simulation approaches [8,12], namely, Monte Carlo brazil-nut and piling algorithms with noise, since these algorithmic procedures take into account, in an *ad hoc* manner, fluctuations is position and include geometrical effects. The assessment of the spectrum of microdensity fluctuations with the external parameters of vibration and the aspect ratio of the cell is now under investigation in our laboratory.

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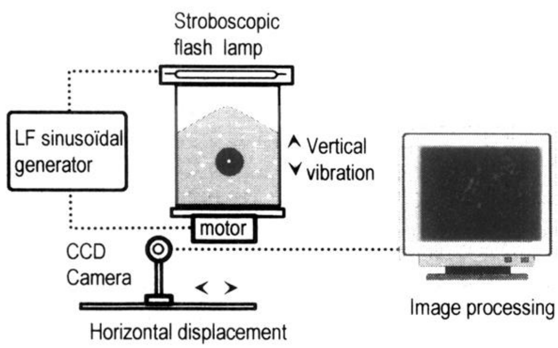

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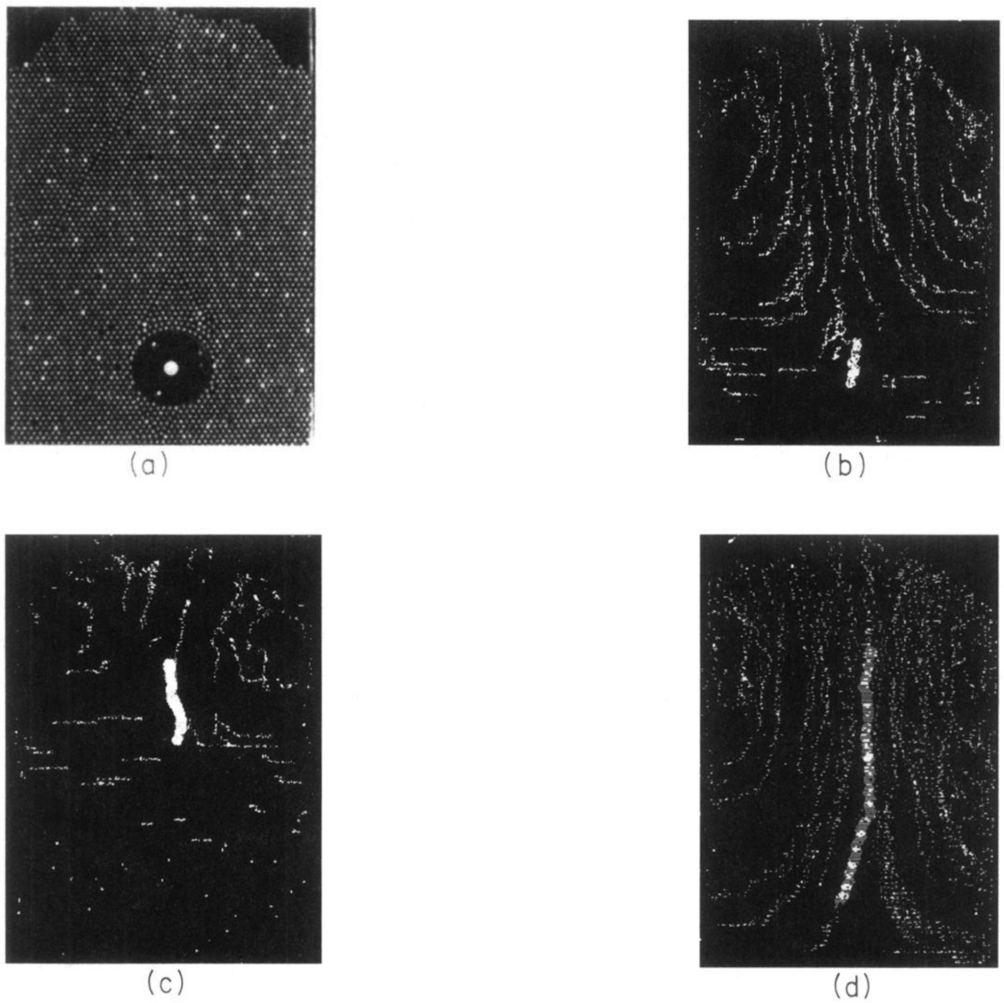

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