Modified formula of nonlocal electron transport in a laser-produced plasma

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A nonlocal heat-transport formula for electrons is derived to include the terms associated with the electrostatic potential and $\partial/\partial v(f_0,f_1)$ in the Fokker-Planck (FP) equation. Then the FP equation for a strongly inhomogeneous plasma is solved. It is found that the behavior of the electron thermal conductivity at a large temperature gradient is considerably affected by the electrostatic field, and the thermal conductivity κ/κ_{SH} for electrons scales as $1/k$ in a large temperature gradient k when there exists a non-negligible electrostatic field, where κ_{SH} is the Spitzer-Härm heat coefficient.

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I. INTRODUCTION

It has been found in laser fusion experiments in the region of the steeped temperature and density gradient near the critical surface of the laser produced plasma that the electron thermal conductivity is far less than that predicted by the Spitzer-Härm value obtained from classical theory. Since this phenomenon is found in many experiments measuring properties of a laser-produced plasma, much effort [1–7] has been made to obtain an analytical formula which can give the best approximation of the results given by the numerical simulation. The most successful of these works is that of Albritton *et al.* [2]. They derived a formula (AWBS) for nonlocal electron transport by directly solving the Fokker-Planck (FP) equation in a simplified form. There are also some modified models of AWBS [3–7].

Despite the success of AWBS, there exist some problems that need further study. Comparing the results obtained from the formula of AWBS and the numerical simulation, Epperlein and Short [3] pointed out that the results of AWBS deviate from that of the numerical simulation in the large temperature gradient, and the reason for that deviation is that the theory of AWBS is too delocalized, while that of Spitzer-Härm is too localized.

Neglecting the electrostatic field is one possible way to cause the excess delocalization of AWBS. When the temperature gradient is steep, slow electrons inside the steeped region cannot compensate for the departure of fast electrons outside. In order to satisfy the condition of quasineutrality, an electrostatic field must occur. This electrostatic field directs electrons to the outside of the steeped region, and increases with increasing position and drops to zero in the corona region. The non-negligible electrostatic field associated with the large temperature gradient makes the simplified FP equation of the AWBS model inadequate in the large temperature gradient.

In this paper we will discuss the contribution of the electrostatic field on electron transport in the steeped region in a laser-produced plasma. The electrostatic field is determined self-consistently by the quasineutrality condition. In Sec. II, we give a modified formula of electron distribution function, from which we obtain a modified formula for particle and energy flux. The discussion and conclusion are given in Sec. III.

II. MODIFIED ELECTRON DISTRIBUTION AND ELECTRON THERMAL CONDUCTIVITY

We start with the FP equation for electrons:

$$\frac{\partial f}{\partial t}+\mu v\frac{\partial f}{\partial x}-\frac{eE}{m}\left[\mu\frac{\partial f}{\partial v}+\frac{1-\mu^2}{v}\frac{\partial f}{\partial \mu}\right]=\left[\frac{\partial f}{\partial t}\right]_c , \quad (1)$$

where $\mu=v_x/v$, v_x is the x component of velocity v, and $[\partial f/\partial t]_c$ is the FP collision term

$$\left[\frac{\partial f}{\partial t}\right]_c=\frac{1}{v^2}\frac{\partial}{\partial v}\left\{v^2\left[\frac{D_1}{2}\frac{\partial f}{\partial v}+Cf\right]\right\}+\frac{D_2}{2v^2}\frac{\partial}{\partial \mu}\left[(1-\mu^2)\frac{\partial f}{\partial \mu}\right] , \quad (2)$$

where C is a constant for slowing down, and D_1 and D_2 represent the coefficients for diffusion in velocity space and angular space, respectively. Because energy is transported at predominantly high electron velocities, we use the high-velocity asymptotic form of these coefficients [8].

To solve the electron kinetic equation, one expands the distribution function into spherical harmonics, i.e.,

$$f(\mathbf{r},\mathbf{v},t)=\sum_{l=0}^{N} f_l(x,v,t)P_l(\mu) ,$$

where P_l is the l-order Legendre polynomial.

Numerical calculation [4] has found that although the high-order terms $(f_2,f_3,\ldots)$ are not negligible, their influence on f_1 (which describes heat flow and current) and f_0 (temperature and density) is small. Hence we take the kinetic equation curtailed in $N=1$, and have

$$\frac{v}{3}\left[\frac{\partial}{\partial x}-\frac{eE}{mv}\left[\frac{\partial}{\partial v}+\frac{2}{v}\right]\right]f_1=\frac{v^2}{2\lambda_e}\frac{\partial}{\partial v}\left[\frac{T}{mv}\frac{\partial f_0}{\partial v}+f_0\right], \tag{3}$$

$$v\left[\frac{\partial}{\partial x}-\frac{eE}{mv}\frac{\partial}{\partial v}\right]f_0=-\frac{2v}{\lambda_{90}}f_1 , \tag{4}$$

where E is the electric field, λ_e is the stopping length of electrons defined as $\lambda_e=T^2/4\pi ne^4(Z+1)^{1/2}\ln\Lambda$, and

$$\lambda_{90}=(mv^2)^2/2\pi ne^4(Z\ln\Lambda_{ei}+\Lambda_{ee})$$

is the scattering mean free path, while Z is the ion charge in the plasma, T is the electron temperature, and Λ the Coulomb logarithm.

In order to derive Eqs. (3) and (4) we have assumed that all temporal variations are slow.

The number of electrons carrying the heat flux is much smaller than that of background electrons. Therefore we approximate the parallel diffusion term in Eq. (3) by the local Maxwellian distribution function $f_0\sim f_{\rm MB}=n(m/2\pi T)^{3/2}\exp[-\epsilon/T]$, where $\epsilon=\frac{1}{2}mv^2+e\phi$ is the electron energy and ϕ is the electrostatic potential, so that Eqs. (3) and (4) can be rewritten as

$$\frac{v}{3}\left[\frac{\partial}{\partial x}-\frac{eE}{mv}\left[\frac{\partial}{\partial v}+\frac{2}{v}\right]\right]f_1=\frac{v^2}{2\lambda_e}\frac{\partial}{\partial v}[f_0-f_{\rm MB}] , \tag{5}$$

$$v\left[\frac{\partial}{\partial x}-\frac{eE}{mv}\frac{\partial}{\partial v}\right]f_0=-\frac{2v}{\lambda_{90}}f_1 . \tag{6}$$

In other works [5,6], the second terms in the left-hand sides of Eqs. (5) and (6) are neglected. Those terms may have an important influence on the state of nonlocal models.

Substituting Eq. (6) into Eq. (5), we have

$$\epsilon^3\left[\frac{\partial^2 f_0}{\partial\xi^2}+\tilde\lambda_s eE\left[\frac{2}{T}-\frac{3}{\epsilon}\right]\frac{\partial f_0}{\partial\xi}\right]+\frac{\partial f_0}{\partial\epsilon}=-\frac{f_{\rm MB}}{T} , \tag{7}$$

where $\xi=x/\tilde\lambda_s$ and $\tilde\lambda=\lambda/(mv^2)^2$, and $\tilde\lambda_s=(2\tilde\lambda_e\tilde\lambda_{90}/3)^{1/2}$. $\epsilon\gg e\phi$ is exploited everywhere by letting $e\phi\to 0$ except under differentiation by ξ.

Since $\tilde\lambda_s eE=-\partial e\phi/\partial\xi$ and $\epsilon\gg e\phi$, we can rewrite Eq. (7) as

$$(\epsilon+e\phi)^3\left[\frac{\partial^2 f_0}{\partial\xi^2}+\tilde\lambda_s eE\frac{2}{T}\frac{\partial f_0}{\partial\xi}\right]+\frac{\partial f_0}{\partial\epsilon}=-\frac{f_{\rm MB}}{T} . \tag{8}$$

In order to transform Eq. (8) into a parabolical-type equation, we make a transformation $y=y(\xi)$, and have

$$\frac{\partial f_0}{\partial\xi}=\frac{\partial y}{\partial\xi}\frac{\partial f_0}{\partial y}$$

and

$$\frac{\partial^2 f_0}{\partial x^2}=\frac{\partial^2 y}{\partial\xi^2}\frac{\partial f_0}{\partial y}+\left[\frac{\partial y}{\partial\xi}\right]^2\frac{\partial^2 f_0}{\partial y^2} .$$

Substituting into Eq. (8), we find that when

$$\frac{\partial^2 y}{\partial\xi^2}+\tilde\lambda_s eE\frac{2}{T}\frac{\partial y}{\partial\xi}=0 , \tag{9}$$

Eq. (8) becomes a parabolic-type equation:

$$(\epsilon+e\phi)^3\left[\frac{\partial y}{\partial\xi}\right]^2\frac{\partial^2 f_0}{\partial y^2}+\frac{\partial f_0}{\partial\epsilon}=-\frac{f_{\rm mB}}{T} . \tag{10}$$

Integrating Eq. (9) over ξ, we have

$$\frac{\partial y}{\partial\xi}=C_1\exp\left[-\int d\xi\,\tilde\lambda_s eE\frac{2}{T}\right] ,$$

where C_1 is an integer. When the electric field is negligible, $y\to\xi$ and $\partial y/\partial\xi\to 1$, we have $C_1=1$.

$$\frac{\partial y}{\partial\xi}=\exp\left[-\int d\xi\,\tilde\lambda_s eE\frac{2}{T}\right] , \tag{11}$$

$$y=\int d\xi\exp\left[-\int d\xi\,\tilde\lambda_s eE\frac{2}{T}\right] . \tag{12}$$

From Eq. (11), we find that when modified terms due to the electrostatic field are negligible, Eq. (8) is reduced to the results of Albritton *et al.* [2]. But when the electrostatic field is non-negligible, y will deviate quickly from ξ along with an increase of the electrostatic field E or of the temperature gradient.

Solving Eq. (10), we obtain the formula for the electron distribution function:

$$f_0=\int dy(\xi')\int_\epsilon^\infty d\epsilon'\frac{\exp\left[-\dfrac{[y(\xi)-y(\xi')]^2}{[\epsilon'+e\phi(\xi')]^4\left[\dfrac{\partial y}{\partial\xi'}\right]^2-[\epsilon+e\phi(\xi)]^4\left[\dfrac{\partial y}{\partial\xi}\right]^2}\right]}{\left\{\pi\left[[\epsilon'+e\phi(\xi')]^4\left[\dfrac{\partial y}{\partial\xi'}\right]^2-[\epsilon+e\phi(\xi)]^4\left[\dfrac{\partial y}{\partial\xi}\right]^2\right]\right\}^{1/2}}\frac{f_{\rm MB}(\xi',\epsilon')}{T(\xi')} . \tag{13}$$

The fluxes of the particle and energy are defined as

$$\begin{Bmatrix}\Gamma\\ Q\end{Bmatrix}=-16\pi\int_{-e\phi}^{\infty}d\epsilon\times\begin{Bmatrix}1\\ \epsilon\end{Bmatrix}\times\epsilon^3\frac{\tilde\lambda_{90}}{3m^2}\frac{\partial f_0}{\partial\xi} . \tag{14}$$

Substituting Eq. (13) into Eq. (14), and we write $\partial f_0/\partial\xi$ as

$$\frac{\partial f_0}{\partial \xi}=\int dy(\xi')\int_{\epsilon}^{\infty} d\epsilon' K(\xi,\xi',\epsilon,\epsilon')\frac{f_{\mathrm{MB}}(\xi',\epsilon')}{T(\xi')}\frac{\partial}{\partial \xi'}\frac{\exp\left[-\dfrac{[y(\xi)-y(\xi')]^2}{\epsilon'^4\left[\dfrac{\partial y}{\partial \xi'}\right]^2-\epsilon^4\left[\dfrac{\partial y}{\partial \xi}\right]^2}\right]}{\left\{\pi\left[\epsilon'^4\left[\dfrac{\partial y}{\partial \xi'}\right]^2-\epsilon^4\left[\dfrac{\partial y}{\partial \xi}\right]^2\right]\right\}^{1/2}},$$

where

$$K(\xi,\xi',\epsilon,\epsilon')=\frac{\left[2[y(\xi)-y(\xi')]+\epsilon^4\left[1-2\dfrac{[y(\xi)-y(\xi')]^2}{\left[\epsilon'^4\left[\dfrac{\partial y}{\partial \xi'}\right]^2-\epsilon^4\left[\dfrac{\partial y}{\partial \xi}\right]^2\right]^2}y''(\xi)\right]\right]y'(\xi)}{\left[-2[y(\xi)-y(\xi')]+\epsilon'^4\left[1-2\dfrac{[y(\xi)-y(\xi')]^2}{\left[\epsilon'^4\left[\dfrac{\partial y}{\partial \xi'}\right]^2-\epsilon^4\left[\dfrac{\partial y}{\partial \xi}\right]^2\right]^2}y''(\xi')\right]\right]y'(\xi')},$$

where y' and y'' are the first and second differentials of y. When $E\to 0$, $K(\xi,\xi',\epsilon,\epsilon')\to -1$. And when $\epsilon \gg e\phi$, the $e\phi$ only brings in a second-order modification. Thus we replace $\epsilon+e\phi$ with ϵ in the expressions above and below, and integrate by parts in ξ' to cast the spatial derivative to $f_{\mathrm{MB}}(\xi',\epsilon')/T(\xi')$. Thus the particle and heat flux Γ, Q are expressed as

$$\begin{aligned}\begin{Bmatrix}\Gamma\\ Q\end{Bmatrix}=&-16\pi\int_{-e\phi}^{\infty}d\epsilon\times\begin{Bmatrix}1\\ \epsilon\end{Bmatrix}\times\epsilon^3\frac{\tilde{\lambda}_{90}}{3m^2}\\ &\times\int dy(\xi')\int_{\epsilon}^{\infty}d\epsilon'\frac{\exp\left[-\dfrac{[y(\xi)-y(\xi')]^2}{\left[\epsilon'^4\left[\dfrac{\partial y}{\partial \xi'}\right]^2-\epsilon^4\left[\dfrac{\partial y}{\partial \xi}\right]^2\right]}\right]}{\left\{\pi\left[\epsilon'^4\left[\dfrac{\partial y}{\partial \xi'}\right]^2-\epsilon^4\left[\dfrac{\partial y}{\partial \xi}\right]^2\right]\right\}^{1/2}}\\ &\times\frac{f_{\mathrm{MB}}(\xi',\epsilon')}{T^2(\xi')}\left[\frac{\epsilon}{T}\frac{\partial T}{\partial x}-\frac{\partial e\phi}{\partial x}-\frac{5}{2}\frac{\partial T}{\partial x}+\frac{T}{n}\frac{\partial n}{\partial x}+\frac{\partial}{\partial x}K(\xi,\xi',\epsilon,\epsilon')\right],\end{aligned}\tag{15}$$

where

$$\begin{aligned}\frac{\partial}{\partial x}K(\xi,\xi',\epsilon,\epsilon')=&\frac{2y'(\xi)y'(\xi')+\dfrac{4\epsilon^4[y(\xi)-y(\xi')]y'(\xi)y'(\xi')y''(\xi)}{-[\epsilon^4y'(\xi)^2]+\epsilon'^4y'(\xi')^2}+\dfrac{4\epsilon^4\epsilon'^4[y(\xi)-y(\xi')]^2y'(\xi)y'(\xi')y''(\xi)y''(\xi')}{[-(\epsilon^4y'(\xi)^2)+\epsilon'^4y'(\xi')^2]^2}}{-2[y(\xi)-y(\xi')]y'(\xi')+\epsilon'^4y'(\xi')y''(\xi')-\dfrac{2\epsilon'^4[y(\xi)-y(\xi')]^2y'(\xi')y''(\xi')}{-(\epsilon^4y'(\xi)^2)-\epsilon'^4y'(\xi')^2}}\\ &+\left[-2[y(\xi)-y(\xi')]y'(\xi)+\epsilon^4y'(\xi)y''(\xi)-\frac{2\epsilon^4[y(\xi)-y(\xi')]^2y'(\xi)y''(\xi)}{-(\epsilon^4y'(\xi)^2)+\epsilon'^4y'(\xi')^2}\right]\\ &\times\left[\frac{2y'(\xi')^2-2[y(\xi)-y(\xi')]y''(\xi')+\dfrac{4\epsilon'^4[y(\xi)-y(\xi')]y'(\xi')^2y''(\xi')}{-(\epsilon^4y'(\xi)^2)-\epsilon'^4y'(\xi')^2}+\epsilon'^4y''(\xi')^2}{\left[-2[y(\xi)-y(\xi')]y'(\xi')+\epsilon'^4y'(\xi')y''(\xi')-\dfrac{2\epsilon'^4[y(\xi)-y(\xi')]^2y'(\xi')y''(\xi')}{-(\epsilon^4y'(\xi)^2)-\epsilon'^4y'(\xi')^2}\right]^2}\right.\\ &+\frac{-\dfrac{4\epsilon'^8(y(\xi)-y(\xi'))^2y'(\xi')^2y''(\xi')^2}{[-(\epsilon^4y'(\xi)^2)-\epsilon'^4y'(\xi')^2]^2}-\dfrac{2\epsilon'^4(y(\xi)-y(\xi'))^2y''(\xi')^2}{-(\epsilon^4y'(\xi)^2)-\epsilon'^4y'(\xi')^2}}{\left[-2[y(\xi)-y(\xi')]y'(\xi')+\epsilon'^4y'(\xi')y''(\xi')-\dfrac{2\epsilon'^4[y(\xi)-y(\xi')]^2y'(\xi')y''(\xi')}{-(\epsilon^4y'(\xi)^2)-\epsilon'^4y'(x')^2}\right]^2}\\ &\left.+\frac{+\epsilon'^4y'(\xi')y^{(3)}(\xi')-\dfrac{2\epsilon'^4[y(\xi)-y(\xi')]^2y'(\xi')y^{(3)}(\xi')}{-(\epsilon^4y'(\xi)^2)-\epsilon'^4y'(\xi')^2}}{\left[-2[y(\xi)-y(\xi')]y'(\xi')+\epsilon'^4y'(\xi')y''(\xi')-\dfrac{2\epsilon'^4[y(\xi)-y(\xi')]^2y'(\xi')y''(\xi')}{-(\epsilon^4y'(\xi)^2)-\epsilon'^4y'(\xi')^2}\right]^2}\right].\end{aligned}\tag{16}$$

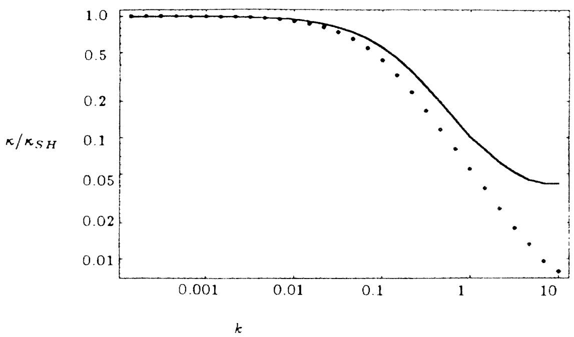

FIG. 1. The electron thermal conductivity $\kappa/\kappa_{\text{SH}}$ vs k with a temperature form $T_c\exp[k(x/L-1)]$ at the point $x=L$, when $R_t=0.001$. The dots and the line correspond to the modified formula and that given by Albritton *et al.*

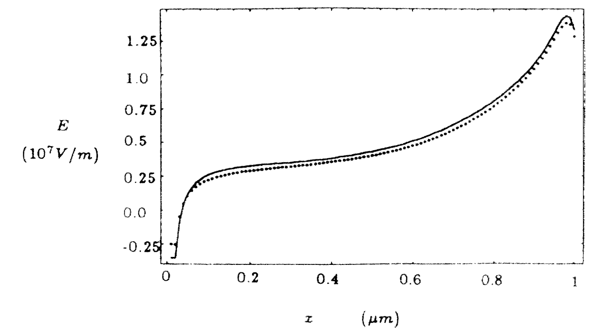

FIG. 3. The electrostatic field E vs position x with a temperature gradient $k\sim 10$ at the point $x=L$ calculated with the modified formula. The dots and the line correspond to the formula of Albritton *et al.* and our modified formula, respectively.

Equation (15) expresses Γ and Q as implicit functions of electric field E. The electric field is determined by the requirement of quasineutrality, $n=NZ$, where N is the ion density. In this case, the electric field is determined by the vanishing of the particle flux $\Gamma(E)=0$. However, there are two unknown parameters, electron density $n(x)$ and temperature $T(x)$, in Eq. (15). In principle, $n(x), T(x)$ can be determined from an implicit equation

$$\left\{\begin{matrix} n \\ T \end{matrix}\right\}=\int_{-e\phi}^{\infty} d\epsilon \times \left\{\begin{matrix} 1 \\ \epsilon \end{matrix}\right\}\times \epsilon^{1/2} f_0 \ .$$

III. RESULTS AND CONCLUSION

The electrostatic field determined from the quasineutrality condition in a laser-produced plasma can be written as $eE=-T\partial[\ln(xT^{\gamma})]/\partial x$ [2]; here γ is a parameter that is determined by total configuration and temperature gradient. When nT^{γ} is not a constant, but varies with position, an electrostatic field occurs.

First we discuss the effect of the electrostatic field on nonlocal electron transport in different temperature gradients by assuming the form of temperature $T(x)$ and density $n(x)$. Without loss generality, we assume that the plasma temperature has a form of $T_c\exp[k(x/L-1)]$, where T_c is the critical temperature, L is a scale length of temperature gradient, and $nT^{\gamma}=C_2T^{R_t}$. Here C_2 and R_t are constants that will be given later. Thence the relation of electrostatic field to $\partial T/\partial x$ is $E=-(R_t/e)\partial T/\partial x$, and Eq. (12) becomes

$$y=-\frac{L}{2k\tilde{\lambda}_s R_t}\exp\left[-\frac{2kxR_t}{L}\right] \ . \tag{17}$$

We take $L=100$ μm in our calculation. Substituting the above assumptions into Eq. (15), using the condition $\Gamma(E)=0$, we determine the profile of electron thermal conductivity.

In the calculation we find that $\partial K/\partial x$ is a small modification that can be neglected.

Figure 1 shows that an electrostatic field will modify the formula of Albritton *et al.* at the large temperature gradient. Fitting the curve, the $\kappa/\kappa_{\text{SH}}$ profile satisfies the relation

$$\kappa/\kappa_{\text{SH}}=\frac{1}{1+16k} \ ,$$

where $\kappa_{\text{SH}}=640\sqrt{2\pi}\varepsilon_0^2k(kT)^{5/2}\varepsilon_e\delta_e/Ze^4\sqrt{m}\ \ln\Lambda_e$ is the Spitzer-Härm heat coefficient [9], and ε_e, δ_e are two

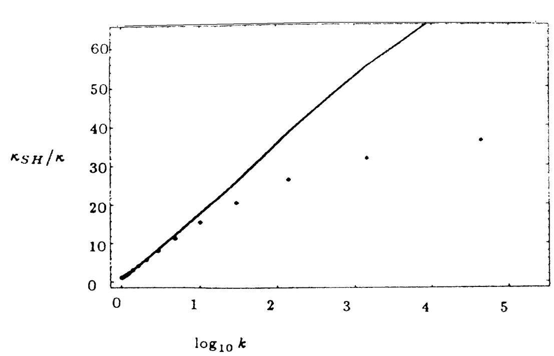

FIG. 2. The electron thermal conductivity $\kappa_{\text{SH}}/\kappa$ vs k with a temperature form $T_c\exp[k(x/L-1)]$ at the point $x=L$ calculated with the modified formula, when $R_t=0.001$ or 0.000 01, respectively. The dots and the line correspond to $R_t=0.000\,001$ or 0.001.

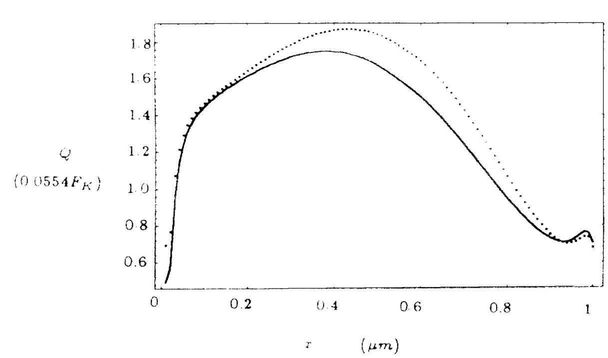

FIG. 4. The heat flux Q vs position x with a temperature gradient $k\sim 10$ at the point $x=L$ calculated with the modified formula. The dots and the line correspond to the formula of Albritton *et al.* and our modified formula, respectively. Here $F_K=4.29\times 10^{-25}n/m^{0.5}$ J m/s.

modified factors given by Spitzer, while the κ/κ_{SH} profile obtained from the formula given by Albritton *et al.* shows it to be a quadratic function of k.

It can be seen from Fig. 2 that, when the electrostatic field is non-negligible ($R_t = 0.0001$), κ/κ_{SH} scales as $1/k$ at large k, and when the electrostatic field is negligible ($R_t = 0.00001$), κ/κ_{SH} scales as $1/k^2$ at large k.

Second, we determine the electrostatic field self-consistently from quasineutrality $\Gamma(E)=0$, and

$$\begin{Bmatrix} n \\ T \end{Bmatrix} = \int_{-i\phi}^{\infty} d\epsilon \times \begin{Bmatrix} 1 \\ \epsilon \end{Bmatrix} \times \epsilon^{1/2} f_0 \ .$$

Then we use the value of E obtained to calculate the heat flux. Figure 3 shows the electrostatic field versus x, and Fig. 4 the heat flux inside the steeped region, when the initial test temperature gradient is $k \sim 10$. It can be seen from Figs. 3 and 4 that although the electrostatic field obtained from our formula is a little different from that of Albritton *et al.* the heat flux has a non-negligible modification.

Finally, we summarize our results. By including terms associated with an electric field in the FP equation, we have a modified formula of nonlocal electron transport. The effect of the electrostatic field on the electron thermal conductivity in various temperature gradients is calculated. We find that the electron thermal conductivity scales as $1/k$ at large k when a small electrostatic field exists, and hence the electrostatic field will have a non-negligible modification on the formula for nonlocal electron transport in a laser-produced plasma.

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