Self-consistent charge dynamics and collective modes in a dusty plasma

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Charges on dust grains immersed in a plasma may vary as a consequence of plasma currents falling onto their surface. Such charge variations might lead to many collective phenomena in the dusty plasma. We have written a set of multifluid equations that describe the charge dynamics of the dust particles self-consistently, i.e., as a consequence of loss in the electron and ion number densities. Various collective phenomena related with dust- and ion-acoustic modes are then studied in the context of these equations. Consistency of the results with previous work and aspects arising due to the present self-consistent charge dynamics are discussed.

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I. INTRODUCTION

In the recent past there has been a growing interest in dusty plasmas. A dusty plasma is a three-component plasma consisting of electrons, ions, and a very tenuous phase of massive solid grains. The mass and charge on the grain vary in the wide range, e.g., $m_d \sim 10^{-2}-10^{-15}$ g and Ze $\sim (10^3-10^6)e$. The typical size of a grain is a few microns. The presence of very massive and charged low density grains in the plasma introduces new scales in the collective behavior leading to novel phenomena. It was realized quite early that the presence of the solid grains in the plasma can influence the overall charge balance in diverse topics like interstellar matter, flames, and rocket exhaust, dense interstellar molecular clouds, and planetary systems [1–4]. However, extensive studies of the collective modes in plasmas due to the presence of massive and charged grains began only in the early 1980s, after the discovery of spokes in Saturn's rings. New waves and instabilities were investigated by several authors [5–10]. In these studies the charge on the dust grains was regarded as constant. These investigations are not qualitatively different from previous studies on the negative ion plasma. But, in general, for a realistic situation the charge on the dust particle may not be constant. This is because when a dust grain is immersed in the quiescent plasma it can acquire charge by collisions with plasma particles, photoionization, etc. When the ambient conditions in the plasma vary, charge on the grain can also fluctuate with it. Thus the grains immersed in a plasma can exhibit charge fluctuations arising due to the wave motion induced oscillations in the plasma currents that flow to the grain. Hence the dust charge becomes a dynamical variable and can give rise to many new collective effects related to the charge fluctuations, which otherwise are absent in the usual three-component models of dusty plasma [5–10].

Study of dusty plasmas with charge fluctuation has begun only recently [11–13]. Using three-component fluid equations together with an equation describing the dust charge dynamics, it was found that in such a plasma the dust-acoustic mode and plasma oscillations are damped due to the dust charge variations [11–13]. Also, the instability of the streaming ion mode with respect to stationary dust was predicted [13].

In earlier work [11–13], charge fluctuations on dust have been treated in an *ad hoc* fashion. On the one hand, dust charge is treated as a dynamical variable which is related to the plasma density, potential, etc., and on the other hand electron and ion continuity equations do not contain any sink term. As a consequence the charge conservation equation is not consistent with the charge variation equation. A proper formulation of the problem would require the presence of the sink term in the continuity equation. Only then will the dynamical equation for charge fluctuation become consistent with the charge continuity equation. We address the dynamics of the charge variation consistently in the above mentioned sense, and find that the coupling leads to damping of the streaming ion-acoustic mode.

This paper is organized in the following sections. In Sec. II we discuss the basic equations. The dispersion relation for dust-acoustic and ion-acoustic modes as well as the streaming instability are discussed in Sec. III. In Sec. IV a brief summary and comparison with previous work is given.

II. BASIC EQUATIONS

We consider a three-component, unmagnetized dusty plasma as consisting of electrons, singly charged ions, and equal radius spherical grains carrying identical charges. We assume that the charging of the grains takes

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place solely due to the attachment of the electrons and ions to the dust grain, and the effects of photoionization, radiation, etc. are ignored. The dust charge fluctuation Q_{d1} is governed by

$$\frac{dQ_{d1}}{dt}=I_{e1}+I_{i1}\ , \tag{1}$$

where I_{e1} and I_{i1} are the perturbations in the plasma currents. The total sum of the equilibrium currents on the dust surface is taken to be zero, i.e., $I_{e0}+I_{i0}=0$, as our interest is in describing fluctuation of the grain charge around its equilibrium value. We shall demonstrate below that this relation can be obtained by using the model fluid equations together with the charge conservation equation.

Basic linearized equations for the number conservation of electron (e), ion (i), and dust (d) fluids are

$$\partial_t n_{e1}+\nabla\cdot(n_{e1}\mathbf{V}_{e0}+n_{e0}\mathbf{V}_{e1})=-\beta_{e0}n_{e1}-\beta_{e1}n_{e0}\ , \tag{2}$$

$$\partial_t n_{i1}+\nabla\cdot(n_{i1}\mathbf{V}_{i0}+n_{i0}\mathbf{V}_{i1})=-\beta_{i0}n_{i1}-\beta_{i1}n_{i0}\ , \tag{3}$$

$$\partial_t n_{d1}+\nabla\cdot(n_{d1}\mathbf{V}_{d0}+n_{d0}\mathbf{V}_{d1})=0\ , \tag{4}$$

where the subscripts 0 and 1 are used to denote equilibrium and perturbed quantities, respectively. n_α and $\mathbf{V}_\alpha$ ($\alpha=e$, i, or d), respectively, are the number density and velocity of the αth specie whereas β_e and β_i denote attachment frequencies of electrons and ions to the dust. The right-hand sides of Eqs. (2) and (3) describe the loss in the electron and ion densities. The attachment frequency of the electrons (ions) may vary due to the change in electron (ion) current falling on the dust grains, and also due to the change in the electron (ion) and dust grain densities, thus the presence of two terms in the right-hand side of Eqs. (2) and (3).

Using Eqs. (2)–(4) and the charge conservation equation

$$\partial_t\rho+\nabla\cdot\mathbf{j}=0\ , \tag{5}$$

where $\rho=\Sigma_\alpha q_\alpha n_\alpha$ and $\mathbf{j}=\Sigma_\alpha q_\alpha n_\alpha\mathbf{V}_\alpha$, one can obtain

$$\partial_t Q_{d1}+\mathbf{V}_{d0}\cdot\nabla Q_{d1}=-\frac{e}{n_{d0}}[(\beta_{e0}n_{e1}+\beta_{e1}n_{e0})-(\beta_{i0}n_{i1}+\beta_{i1}n_{i0})]\ . \tag{6}$$

We define the attachment frequencies of the electron and ion as

$$\beta_a=\frac{I_a n_d}{q_a n_a}\ , \tag{7}$$

where $a=e$ or i. Using the above definition together with the constraint $I_{e0}+I_{i0}=0$, one finds $\beta_{a0}=(|I_{e0}|n_{d0}/q_a n_{a0})$ and thus $\beta_{i0}=\beta_{e0}(n_{e0}/n_{i0})$. The expression for β_{a1} would read

$$\beta_{a1}=\beta_{a0}\left[\frac{I_{a1}}{I_{a0}}+\frac{n_{d1}}{n_{d0}}-\frac{n_{a1}}{n_{a0}}\right]\ . \tag{8}$$

Substituting for β_{a0} and β_{a1} in Eq. (5) one obtains

$$\frac{dQ_{d1}}{dt}=I_{e1}+I_{i1}\ , \tag{9}$$

which is the same as Eq. (1). Note that we have derived this equation by considering attachment frequencies in the electron and ion number conservation equations and using the charge conservation equation. Thus we are introducing charge dynamics in a self-consistent manner. It can be shown that when there is no loss term in the continuity equations of the electron and ion, the charge conservation equation gives $dQ_{d1}/dt=0$, i.e., charge on the dust grain is constant.

Next we use expressions for I_{e1} and I_{i1} from Ref. [13] to relate the charge dynamic equation with the density fluctuations in ion and electron fluids:

$$I_{e1}=I_{Ie0}\left[\frac{n_{e1}}{n_{e0}}+\frac{e\phi_{f1}}{kT_e}\right]\ , \tag{10}$$

$$I_{i1}=I_{i0}\left[\frac{n_{i1}}{n_{i0}}-\frac{e\phi_{f1}}{w_0}\right]\ , \tag{11}$$

where T_e is the electron temperature and ϕ_{f1} the fluctuation in the dust grain floating potential. The latter may be expressed in terms of grain charge fluctuation by $\phi_{f1}=Q_{d1}/C$, where C is the capacitance of dust grain and $w_0=T_i-e\phi_{f0}$, with T_i the ion temperature and ϕ_{f0} the equilibrium floating potential. Substituting I_{e1} and I_{i1}, thus defined, into Eq. (9), one obtains

$$\frac{dQ_{d1}}{dt}+\eta Q_{d1}=|I_{e0}|\left[\frac{n_{i1}}{n_{i0}}-\frac{n_{e1}}{n_{e0}}\right]\ , \tag{12}$$

where $\eta=(e|I_{e0}|/C)[(1/T_e)+(1/w_0)]$. Equation (12) describes [13] dust charge dynamics driven by the difference in the fluctuations of electron and ion densities with a natural decay rate η. Our analysis reveals that the ηQ_{d1} term in the above equation is derived from terms containing β_{a1} from the right-hand side of Eq. (6). Thus the charging frequency of dust grains [9,13] becomes intimately connected with the perturbed attachment frequencies of electrons and ions [Eq. (8)]. The terms with β_{a0} give rise to the right-hand side of Eq. (12).

The equations of motion for electron, ion, and dust fluids are

$$m_a n_a\frac{d\mathbf{V}_a}{dt}=-\nabla P_a+q_a n_a\mathbf{E}-m_a n_a\beta_a(\mathbf{V}_a-\mathbf{V}_d)\ , \tag{13}$$

$$m_d n_d\frac{d\mathbf{V}_d}{dt}=Q_d n_d\mathbf{E}+\Sigma_a m_a n_a\beta_a(\mathbf{V}_a-\mathbf{V}_d)\ , \tag{14}$$

where m_a ($a=e$ and i) is the mass of the species a. Poisson's equation is

$$\nabla^2\phi=-4\pi e(n_i-n_e)-4\pi Q_d n_d\ . \tag{15}$$

Equations (11)–(14) are then applied to study various collective phenomena in the dusty plasma.

III. DISPERSION RELATION

First we study the dust-acoustic mode using Eqs. (12)–(15). We assume that the dust grains are cold, i.e.,

$kV_d \ll \omega \ll kV_i, kV_e$, and there is no equilibrium flow ($\mathbf{V}_0 = \mathbf{0}$). Moreover, the electron and ion temperatures are assumed to be equal, i.e., $T_e = T_i = T$. Taking the spatial and temporal dependence of all the perturbed quantities as $\exp[-i(\omega t - kx)]$, one can obtain the following dispersion relation:

$$1+\frac{1}{k^2\lambda_e^2}\left[1+\frac{i\beta}{\omega+i\eta}\right]+\frac{1}{k^2\lambda_i^2}\left[1+\frac{i\beta\delta}{\omega+i\eta}\right]-\frac{\omega_{pd}^2}{\omega^2}=0\ , \tag{16}$$

where the dust plasma frequency $\omega_{pd}^2 = 4\pi Q_{d0}^2 n_{d0}/m_d$, electron attachment frequency $\beta = |I_{e0}| n_{d0}/e n_{e0}$, $\lambda_{Da} = (T/4\pi e^2 n_{a0})^{1/2}$, and $\delta = n_{e0}/n_{i0}$. This can easily be written in the form

$$\frac{\omega^2}{k^2}=\frac{\omega_{pd}^2\lambda_{\text{eff}}^2}{1+k^2\lambda_{\text{eff}}^2+\dfrac{\beta}{\eta-i\beta}}\ , \tag{17}$$

where λ_{eff} is the effective Debye length. Equation (16) is the same as the dust-acoustic dispersion relation of Melandso, Aslakson, and Havens [11], if one identifies another charging frequency introduced by the authors in [11] with our electron attachment frequency β. Charge dynamics can cause damping of the dust-acoustic mode. The results obtained by us for the dust-acoustic mode is similar to one obtained by previous workers [9,13]. The reason for this similarity is that the electron and ion continuity equations become redundant when one ignores the inertia terms in the momentum equations of electrons and ions.

Next we consider the streaming instability of the ion-acoustic mode, with $kV_d, kV_i \ll \omega \ll kV_e$. For this we assume that the ions move relative to the dust with a uniform speed V_0. Morever, we take the dust inertia to be large so as to neglect the dust motion ($\omega_{pd} \to 0$). It is known that in an electron-ion plasma, with the electron streaming in the background of fixed ions, there is no instability when the ion inertia is ignored [14]. However, in a dusty plasma, when the ions are streaming in a fixed dust background, streaming ions may develop an instability [13].

We have obtained the dispersion relation using self-consistent charge dynamics, which can be written as follows, by treating the charging frequency η to be small compared to ω:

$$[(\varpi+i\beta\delta)^2-k^2C_{ds}^2+i\beta\delta kV_0](\omega+i\beta+i\eta) = -i\beta(1-\delta)k^2C_{ds}^2\ , \tag{18}$$

where $\varpi = \omega - kV_0$ and $C_{ds}^2 = \omega_{pi}^2\lambda_e^2$, where ω_{pi} is the ion-plasma frequency and λ_e the electron Debye length. Due to the presence of attachment frequency terms in the continuity and momentum balance equations of ions, i.e., Eqs. (3) and (13), the frequency ϖ term appears with an additional term $i\beta\delta$. In addition, there appears one more term which arises due to the coupling of the ion attachment frequency with the stream velocity V_0. If we drop both these terms with $\beta\delta$ on the left-hand side of Eq. (18), the dispersion relation would reduce to that of Ref. [13].

The physical regime where the charging and collision frequencies satisfy the condition $\eta, \beta < \omega$ is of special importance. In this limit the charge dynamics can have appreciable effects, and the damping effect on wave propagation is not very severe. The roots of Eq. (18) are then obtained perturbatively in this regime . The root which may exhibit the instability can be written as

$$\omega \simeq kV_0 - kC_{ds} + i\frac{\beta}{2}\frac{C_{ds}}{V_0 - C_{ds}} - i\beta\delta\left[\frac{1}{2}\frac{C_{ds}}{V_0-C_{ds}}+1+\frac{V_0}{2C_{ds}}\right]\ . \tag{19}$$

If one assumes that $\delta = 0$, then one has purely a growing negative energy mode when $V_0/C_{ds} > 1$, as reported in Ref. [13]. On the other hand, if $\delta \neq 0$, the parameters of the system have to satisfy yet another criterion for the development of the instability, i.e., $(V_0/C_{ds})^2 + (V_0/C_{ds}) < 1 + 1/\delta$. Now the parameter space of the system has to satisfy two conflicting criteria for the development of the instability, and thus it becomes restricted. For example, as δ increases, the parameter range over which the instability develops decreases. In the case of planetary atmosphere [4,9], generally $\delta \sim 1$, and thus the parameter range which allows the instability to develop shrinks to zero.

In Fig. 1 we have plotted the imaginary part of Eq. (19) as a function of V_0/C_{ds} and compared various $\delta \neq 0$ cases with $\delta = 0$ (solid curve) case. Figure 1 depicts the case when $\delta = 0.1$ (dotted curve), where soon after $V_0/C_{ds} \simeq 2.8$ the unstable mode becomes a purely damp-

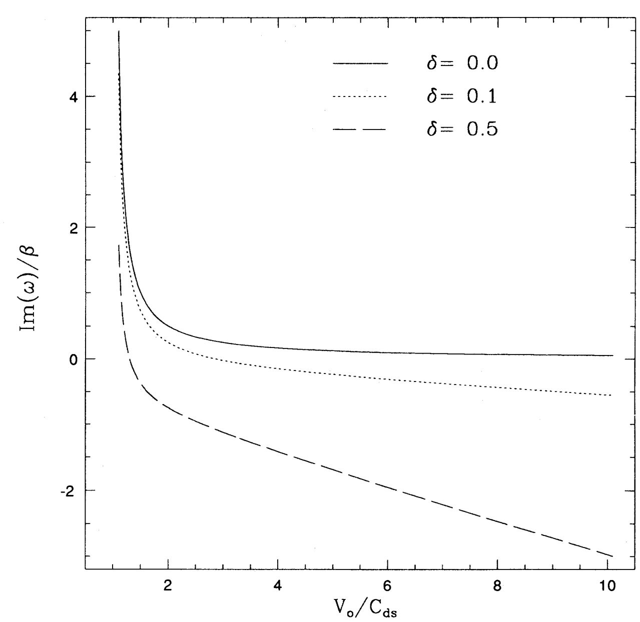

FIG. 1. Figure 1 depicts the imaginary part of the unstable root as a function of V_0/C_{ds}, for different values of the ratio of background densities of electrons to ions δ, $\delta = 0.1$, and $\delta = 0.5$. A comparison is made with the $\delta = 0$ case.

ing one and increases linearly in magnitude with V_0/C_{ds}. The dashed curve in the figure represents the $\delta=0.5$ case, where one can see that $V_0/C_{ds}\simeq 1.3$, and the unstable mode becomes a purely damping one. Thus Fig. 1 clearly indicates the following.

(1) $V_0/C_{ds}>1$ is not the general criterion for the instability. This is because as V_0/C_{ds} increases beyond 1 the frictional term in Eq. (19) dominates over the rest of the terms.

(2) With an increase in δ the parameter space available for the instability to develop decreases significantly.

We would like to add that when $V_0=0$ in Eq. (19), one obtains a purely damped ion-acoustic mode.

IV. CONCLUSIONS

By incorporating charge dynamics of the dust grains self-consistently, we have shown that the charging frequency of the dust grains is related to the attachment frequencies of ions and electrons. Also, we have shown that the parameter space of the negative energy mode is a sensitive function of the ratio of electron to ion densities. As this ratio increases, streaming ions lose more energy due to attachment with the dust, and thus restrict the range of V_0/C_{ds} from above.

High frequency phenomena, e.g., electron plasma oscillations, are not studied in the present work because we have used an expression for the electron current falling on the dust grains [Eq. (10)]. This is written in the Maxwellian limit of the electron distribution function. However, Eqs. (12)–(14) are written in a general form, and can be used to study the electron plasma oscillations if one obtains attachment frequency β_{e1} for the non-Maxwellian component of the electron distribution function. These studies will be carried out in the future.

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