# Ion wake fields in a plasma with negative ions

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Ion wake fields in a quasineutral plasma that contains negative ions are theoretically examined using two independent approaches. The two analyses described here are based on the initial assumption that a driving ion bunch may be injected into either a drifting plasma or a stationary plasma. Both methods predict that the ion wake fields may be oscillatory, constant, or decrease exponentially. A formula for the period of the oscillation frequency of the ion wake field is derived. Effects of driving ion bunches with different characteristics on the wake field excitation and modulation are presented.

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## I. INTRODUCTION

Since the concept of a plasma wake field accelerator was first suggested by Chen, Huff, and Dawson [1], the study of plasma wake fields has been an important area of investigation in plasma physics. Most of the initial attention was paid to wake fields excited by relativistic or nonrelativistic electron bunches injected into a cold plasma. The frequency of oscillation of the wake field which is in the wake of the electron bunch is of the order of the electron plasma frequency [2–4]. The possibility of using an ion wake field that is generated by an ion bunch instead of an electron bunch in a wake field accelerator was suggested by Jones and Keinigs [5]. It was found that it might be easier to tailor the characteristics of an ion bunch rather than an electron bunch since the time scale would be of the order of the ion plasma period instead of the much shorter electron plasma period. In addition, there may be an improvement in the value of energy gain using ion wake fields instead of electron wake fields. In their analysis, effects of a drifting plasma were also investigated.

The first experimental observation of large-amplitude ion wake fields in a plasma was made by Nishida *et al.* [6]. Using a standard double plasma machine in which an argon plasma was created, they demonstrated that an ion wake field could be excited. They found that the required velocity of the exciting ion bunch was approximately equal to the ion acoustic velocity and that the frequency of the ion wake field was of the order of the ion plasma frequency.

We recently extended the Nishida *et al.* [6] experiments in order to investigate ion wake fields in a quasineutral three component plasma consisting of positively charged ions, negatively charged ions, and electrons [7]. In the experiments performed also in a double plasma machine, a gas with a large electron attachment cross section, in our experiment sulfur hexafluoride ($SF_6$), was introduced into a previously quasineutral argon plasma. Hence, negative $SF_6$ or F ions of varying densities became a part of a quasineutral three component plasma. We found that the frequency of the ion wake field depended on several parameters: the ratio of positive ion mass to negative ion mass, the negative ion concentration, the velocity of the ion bunch, the ion acoustic velocity, and the positive ion plasma frequency. In addition, we were able to predict the period of oscillation of the ion wake field assuming that the plasma was stationary. The analytical description that we used at that time was an extension of a two component cold plasma model analyzed originally by Ruth *et al.* [8]. This extension allowed us to describe a three component stationary plasma.

In the present article, we investigate an ion wake field in a nonstationary three component plasma that has a finite electron temperature and cold positive and negative ions. The result that we will obtain reduces in the limit to the stationary three component plasma result that was found before [7]. The stationary plasma model of Ruth *et al.* [8] as applied to a plasma that has three components is also presented and extended here. Using the Jones and Keinigs procedure that is directed toward the drifting plasma model, we more accurately emulate the double plasma experiment and find that the characteristics of the ion wake fields have additional features than were obtained previously. In particular, we are able to analyze effects due to the wake field excitation with different applied voltage signals.

In Sec. II, the derivation of the ion wake field in a drifting plasma is presented following the procedure of Jones and Keinigs [5]. In Sec. III, the derivation of the ion wake field in a stationary plasma is presented following the procedure of Ruth *et al.* [8]. The spatial period of the ion wake field under the conditions of oscillation is also given in both cases. In Sec. IV, effects of changing the shape of the driving ion bunch on the oscillating ion wake fields are discussed. Section V is the conclusion.

## II. DERIVATION OF ION WAKE FIELDS IN THE MANNER OF JONES AND KEINIGS

In order to derive the ion wake field in a three component plasma, we follow the procedure presented in Jones and Keinigs [5]. Wake fields in a one-dimensional, homogeneous, and field-free plasma can be written as

 

$$E(y)=\int_{-\infty}^{+\infty}G(y'-y)\rho_b(y')dy' , \tag{1}$$

where

$$G(y'-y)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{ik(y'-y)}G(k)dk \tag{2}$$

and

$$G(k)=\frac{-4\pi i}{k[1+4\pi\chi_p(k,\omega=kv_b)]} . \tag{3}$$

In (1)–(3), $\rho_b$ is the charge density of the driving ion bunch, $\chi_p$ is the plasma susceptibility, $v_b$ is the velocity of the bunch, and $y=v_bt-z$. Since the wake field trails behind the ion bunch, we hereafter assume that $y\geq 0$.

We consider that a quasineutral plasma consists of three species: cold positive ions, cold negative ions, and Boltzmann electrons. In addition, the plasma drifts with a constant velocity in the $z$ direction. The plasma response due to the driving ion bunch is computed using the linearized equations of continuity and momentum. They are written for the three species of the plasma as follows.

For *electrons,*

$$\frac{\partial n_{e1}}{\partial t}+n_{e0}\frac{\partial v_{e1}}{\partial z}+v_{e0}\frac{\partial n_{e1}}{\partial z}=0 , \tag{4}$$

$$\kappa_B T_e\frac{\partial n_{e1}}{\partial z}=-en_{e0}E . \tag{5}$$

In (5), the electron mass has been neglected in comparison with the ion mass.

For *positive ions,*

$$\frac{\partial n_{+1}}{\partial t}+n_{+0}\frac{\partial v_{+1}}{\partial z}+v_{+0}\frac{\partial n_{+1}}{\partial z}=0 , \tag{6}$$

$$m_+\left[\frac{\partial v_{+1}}{\partial t}+v_{+0}\frac{\partial v_{+1}}{\partial z}\right]=eE . \tag{7}$$

For *negative ions,*

$$\frac{\partial n_{-1}}{\partial t}+n_{-0}\frac{\partial v_{-1}}{\partial z}+v_{-0}\frac{\partial n_{-1}}{\partial z}=0 , \tag{8}$$

$$m_-\left[\frac{\partial v_{-1}}{\partial t}+v_{-0}\frac{\partial v_{-1}}{\partial z}\right]=-eE , \tag{9}$$

where $\kappa_B$ is Boltzmann's constant, $T_e$ is the electron temperature, $m_+$ and $m_-$ are the positive ion and negative ion masses, and $v_{j0}$ and $n_{j0}$ are the drifting velocity and the background density of the $j$ species respectively [$j=e,+,$ or $-$]. A time-harmonic dependence for the perturbations $v_{j1}$, $n_{j1}$, and $E$ is assumed. They are written as

$$v_j=v_{j0}+v_{j1}e^{i(kz-\omega t)},\quad n_j=n_{j0}+n_{j1}e^{i(kz-\omega t)},$$
$$E=E_1e^{i(kz-\omega t)} . \tag{10}$$

The perturbations of density and velocity can then be expressed in terms of the electric field perturbation as

$$n_{e1}=i\frac{en_{e0}}{\kappa_B T_e k}E_1 , \tag{11}$$

$$v_{e1}=i\frac{e(\omega-v_{e0}k)}{\kappa_B T_e k^2}E_1 , \tag{12}$$

$$v_{\pm 1}=i\frac{\pm e}{m_\pm(\omega-v_{\pm 0}k)}E_1 , \tag{13}$$

$$n_{\pm 1}=i\frac{\pm en_{\pm 0}k}{m_\pm(\omega-v_{\pm 0}k)^2}E_1 . \tag{14}$$

We write Maxwell's equation

$$\nabla\times\mathbf{B}_1=-\frac{i\omega}{c}\mathbf{E}_1+\frac{4\pi}{c}\mathbf{J}_1=-\frac{i\omega}{c}\eta\mathbf{E}_1 , \tag{15}$$

where the dielectric constant $\eta$ is a scalar since the plasma was assumed to be field-free. Since $\eta=1+4\pi\chi_p$, we can write that

$$\begin{aligned}4\pi\chi_p&=i4\pi J_1/(\omega E_1)\\&=i(4\pi/\omega E_1)[(-n_{e0}ev_{e1}-n_{e1}ev_{e0})\\&\qquad+(n_{+0}ev_{+1}+n_{+1}ev_{+0})\\&\qquad+(-n_{-0}ev_{-1}-n_{-1}ev_{-0})] ,\end{aligned}$$

or finally

$$4\pi\chi_p=\Omega_{p+}^2\left[\frac{1-\varepsilon}{k^2C_{s+}^2}-\frac{1}{(\omega-v_{+0}k)^2}-\frac{\varepsilon M}{(\omega-v_{-0}k)^2}\right] , \tag{16}$$

where the parameter of negative ion concentration $\varepsilon=n_{-0}/n_{+0}$, the electron density $n_{e0}=(1-\varepsilon)n_{+0}$, the positive ion plasma frequency $\Omega_{p+}=(4\pi n_{+0}e^2/m_+)^{1/2}$, the ion acoustic velocity $C_{s+}=(\kappa_B T_e/m_+)^{1/2}$, and the ion mass ratio $M=m_+/m_-$. Thus the function $G(k)$ in (3) is given as

$$G(k)=-\frac{4\pi ik}{k^2+\Omega_{p+}^2\left[\frac{1-\varepsilon}{C_{s+}^2}-\frac{1}{(v_b-v_{+0})^2}-\frac{\varepsilon M}{(v_b-v_{-0})^2}\right]} . \tag{17}$$

In order to evaluate the wave field, we substitute (17) into (2) and perform the integral of (2) in the complex plane. This will be done under three plasma conditions.

(a) The first plasma condition assumes that

$$\frac{1-\varepsilon}{C_{s+}^2} < \frac{1}{(v_0-v_{+0})^2} + \frac{\varepsilon M}{(v_b-v_{-0})^2} .$$

In this case, Eq. (1) becomes

$$E(y)=2\pi\int_{-\infty}^{+\infty}\cos[k_p(y'-y)]\rho_b(y')dy' , \tag{18}$$

where

$$k_p=\Omega_{p+}\left[\frac{1}{(v_b-v_{+0})^2}+\frac{\varepsilon M}{(v_b-v_{-0})^2}-\frac{1-\varepsilon}{C_{s+}^2}\right]^{1/2} . \tag{19}$$

The integral in (18) can be easily evaluated if the driving ion bunch can be expressed as the Dirac δ function, $\rho_b(y)=\sigma e\delta(y)$, where $\sigma$ is a uniform surface number density. Other choices for this charge distribution will be analyzed later in Sec. IV. The resulting ion wake fields then have an oscillating form

$$E(y)=2\pi e\sigma\cos(k_py) . \tag{20}$$

Within a trivial constant, this reduces to our previous result [7] [Eq. (32)] that was obtained using an initial stationary plasma assumption, i.e., $v_{+0}=v_{-0}=0$. The period $T_{\rm wake}$ of the ion wake field is expressed as

$$T_{\rm wake}=\frac{2\pi}{\Omega_{p+}v_b\left[\frac{1}{(v_b-v_{+0})^2}-\frac{1}{C_{s+}^2}+\varepsilon\left\{\frac{M}{(v_b-v_{-0})^2}+\frac{1}{C_{s+}^2}\right\}\right]^{1/2}} . \tag{21}$$

If $\varepsilon=0$, Eq. (21) reduces to the wake field period in a two component plasma when $|v_b-v_{+0}|<C_{s+}$. Note that the difference of velocity between the driving ion bunch and the drifting positive ions has to be less than the ion acoustic velocity in order to generate an oscillating ion wake field. Furthermore, if the velocity direction of the driving ion bunch is the same as that of the positive ion, the direction of the negative ion velocity will be opposite to that of the driving ion bunch velocity, and *vice versa*. Assuming the former case, the term $(v_b-v_{-0})$ in (21) has to be replaced with the term $(v_0+v_{-0})$.

(b) The second plasma condition assumes that

$$\frac{1-\varepsilon}{C_{s+}^2} = \frac{1}{(v_b-v_{+0})^2} + \frac{\varepsilon M}{(v_b-v_{-0})^2} ,$$

yielding $G(y'-y)=2\pi$. Hence the wake field is computed from

$$E(y)=2\pi\int_{-\infty}^{+\infty}\rho_b(y')dy' . \tag{22}$$

Again using the Dirac δ function as the shape of the driving ion bunch, we find $E(y)=2\pi e\sigma$. This means that the wake fields are not oscillatory but are constant.

(c) The third plasma condition assumes that

$$\frac{1-\varepsilon}{C_{s+}^2} > \frac{1}{(v_b-v_{+0})^2} + \frac{\varepsilon M}{(v_b-v_{-0})^2} ,$$

yielding $G(y'-y)=-2\pi e^{k_p(y'-y)}$. The wake field is computed to be

$$E(y)=-2\pi\int_{-\infty}^{+\infty}e^{k_p(y'-y)}\rho_b(y')dy' . \tag{23}$$

With the Dirac δ function assumption for the driving ion charge distribution, we compute the electric field to be $E(y)=-2\pi e\sigma e^{-k_py}$. In this case, the wake fields decrease exponentially and are rapidly suppressed behind the driving ion bunch.

### III. DERIVATION OF ION WAKE FIELDS IN THE MANNER OF RUTH *et al.*

The procedure due to Ruth *et al.* [8] assumes that the plasma is stationary. We used this method previously in interpreting the experiment [7]. In this section, the derivation of the period $T_{\rm wake}$ of the ion wake field is reviewed and extended. Using (5) and (6)–(9) under the assumptions $v_{+0}=v_{-0}=0$, we obtain the following equations that describe the perturbed densities:

$$\frac{d^2n_{+1}}{dy^2}-\frac{C_{s+}^2}{(1-\varepsilon)v_b^2}\frac{d^2n_{e1}}{dy^2}=0 , \tag{24}$$

$$\frac{d^2n_{-1}}{dy^2}+\frac{\varepsilon MC_{s+}^2}{(1-\varepsilon)v_b^2}\frac{d^2n_{e1}}{dy^2}=0 . \tag{25}$$

Poisson's equation is written as

$$\frac{d^2n_{e1}}{dy^2}=-\frac{\Omega_{p+}^2}{C_{s+}^2}(n_{+1}-n_{-1}-n_{e1}+n_b) , \tag{26}$$

where $n_b$ is the number density of the driving ion bunch and the other notations are defined with the same meaning as before. Assuming that the driving ion bunch can be expressed as the Dirac δ function $n_b=\sigma\delta(y)$ where $\sigma$ is a uniform surface number density and integrating (26) over the ion bunch at $y=0$, we find that

$$\left[\frac{dn_{e1}}{dy}\right]_{-0}^{+0}=-\frac{\Omega_{p+}^2}{C_{s+}^2}\sigma . \tag{27}$$

Considering the nonexistence of any density perturbation for $y<0$ and assuming that the electron density perturbation has the form

$$n_{e1}=A_e\sin(k_p y) , \tag{28}$$

Eq. (27) becomes

$$n_{e1}=-(1-\varepsilon)\frac{\Omega_{p+}^2\sigma}{C_{s+}^2k_p}\sin(k_p y) . \tag{29}$$

Substituting (29) into (24) and (25) allows us to compute the positive and negative ion perturbations. We find

$$n_{+1}=-\frac{\Omega_{p+}^2\sigma}{v_b^2k_p}\sin(k_p y) \tag{30}$$

$$n_{-1}=\frac{\varepsilon M\Omega_{p+}^2\sigma}{v_b^2k_p}\sin(k_p y) . \tag{31}$$

From (5) and (29), the wake fields can be expressed as

$$E=-4\pi e\sigma\cos(k_p y) , \tag{32}$$

which agrees with the result computed earlier using the Jones and Keinigs [5] method. The two results differ only by a constant. The substitution of (29)–(31) into (26) yields for the region of the wake field $y>0$

$$k_p=\Omega_{p+}\left[\frac{1}{v_b^2}+\frac{\varepsilon M}{v_b^2}-\frac{1-\varepsilon}{C_{s+}^2}\right]^{1/2} . \tag{33}$$

We find that the period of the wake field $T_{\rm wake}$ has the same period that is given in (21) if there is no plasma drift, i.e., $v_{+0}=v_{-0}=0$. Since $k_p$ is real, the velocity of the ion bunch has to satisfy the relation

$$v_b<C_{s+}\sqrt{(1+\varepsilon M)/(1-\varepsilon)}\equiv v_f . \tag{34}$$

In (34), $v_f$ is identified to be the fast mode ion acoustic velocity in a three component plasma. Therefore, in order to generate an oscillating ion wake field, the velocity of the ion bunch has to be less than the fast mode velocity in a stationary plasma with negative ions. This assumes that the ions are cold so there is no slow mode in the long wavelength approximation [9–11]. In a two component plasma ($\varepsilon=0$), (34) reduces to $v_b<C_{s+}$. This means that the ion acoustic velocity replaces the fast mode ion acoustic velocity in (34) as the requirement in order to produce oscillating ion wake fields.

If $v_b=v_f$, the wake fields in (32) become constant with a value $E=-4\pi e\sigma$. If $v_b>v_f$, the form of the electron perturbation in (28) has to be replaced with the exponential form

$$n_{e1}=A_e e^{-k_p y} \tag{35}$$

in order to keep $k_p$ real. Then the electron perturbation is expressed as

$$n_{e1}=(1-\varepsilon)\frac{\Omega_{p+}^2\sigma}{C_{s+}^2k_p}e^{-k_p y} \tag{36}$$

and the wake fields are given by

$$E=-4\pi e\sigma e^{-k_p y} , \tag{37}$$

where $k_p$ can still remain real as

$$k_p=\Omega_{p+}\left[\frac{1-\varepsilon}{C_{s+}^2}-\frac{1}{v_b^2}-\frac{\varepsilon M}{v_b^2}\right]^{1/2} . \tag{38}$$

These results which are obtained using the Ruth *et al.* [8] procedure are similar to those obtained previously using the Jones and Keinigs [5] method if we take the limit of zero plasma drift.

## IV. EFFECT OF THE BUNCH SHAPE ON OSCILLATING WAKE FIELDS

In this section the effect of the shape of the driving ion bunch on the oscillating ion wake fields is investigated. Wake fields inside and outside of the bunches are calculated under the rubric of the Jones and Keinigs [5] procedure. We previously had assumed that the charge was localized and could be represented with the Dirac δ function. Three different bunches that have a trapezoidal shape, a triangular shape, and a half sine wave shape are examined. These are shown in Fig. 1.

First, we consider a trapezoidal shaped ion bunch in Fig. 1(a). The distribution of charge density can be written as

$$\rho_b(y)=\begin{cases}\rho_0\dfrac{y}{\alpha}, & 0\le y<\alpha\\ \rho_0, & \alpha\le y<\beta\\ \rho_0\dfrac{\gamma-y}{\gamma-\beta}, & \beta\le y\le\gamma .\end{cases} \tag{39}$$

Substituting this charge distribution into Eq. (18) and integrating the resulting integrals, we find that the wake fields $E_{\rm in}$ within the bunch are expressed with three terms to represent the three regions of charge distribution

$$E_{\rm in}=\begin{cases}\left[\dfrac{2\pi\rho_0}{k_p^2\alpha}\right][1-\cos(k_p y)], & 0\le y<\alpha\\ \left[\dfrac{4\pi\rho_0}{k_p^2\alpha}\right]\sin\left[\dfrac{k_p\alpha}{2}\right]\sin\left[k_p\left[y-\dfrac{\alpha}{2}\right]\right], & \alpha\le y<\beta\\ \left[\dfrac{2\pi\rho_0}{k_p^2\alpha}\right]\left[2\sin\left[\dfrac{k_p\alpha}{2}\right]\sin\left[k_p\left[y-\dfrac{\alpha}{2}\right]\right]+\dfrac{\alpha}{\gamma-\beta}\{\cos[k_p(\beta-y)]-1\}\right], & \beta\le y\le\gamma .\end{cases} \tag{40}$$

For $y > \gamma$, the wake field $E_{\text{out}}$ outside of the bunch has one term that is written as

$$E_{\text{out}} = \left[\frac{2\pi\rho_0}{k_p^2\alpha}\right]\left[2\sin\left[\frac{k_p\alpha}{2}\right]\sin\left[k_p\left[y-\frac{\alpha}{2}\right]\right]+\frac{\alpha}{\gamma-\beta}\{\cos[k_p(\beta-y)]-\cos[k_p(\gamma-y)]\}\right] . \tag{41}$$

There are several asymptotic limits of the charge distribution that have a trapezoidal shape that can be analyzed and are of technological interest. The first case assumes that the charge distribution rapidly returns to zero. The initial rise time may remain long. In this limit of $\gamma \Longrightarrow \beta$, the wake field given in (41) becomes

$$E_{\text{out}} = \left[\frac{2\pi\rho_0}{k_p^2\alpha}\right]\left\{2\sin\left[\frac{k_p\alpha}{2}\right]\sin\left[k_p\left[y-\frac{\alpha}{2}\right]\right]+k_p\alpha\sin[k_p(\beta-y)]\right\} . \tag{42}$$

The second case assumes that the charge distribution rapidly rises from zero to a constant value. Its return to zero may be long. In this limit of $\alpha \Longrightarrow 0$, the wake field given in (41) becomes

$$E_{\text{out}} = \left[\frac{2\pi\rho_0}{k_p^2}\right]\left[k_p\sin(k_py)+\frac{\cos[k_p(\beta-y)]-\cos[k_p(\gamma-y)]}{\gamma-\beta}\right] . \tag{43}$$

The third case assumes that the shape of ion bunch becomes rectangular with both a fast rise time and a fast decay time. This can be effected by taking the limit of $\gamma \Longrightarrow \beta$ in (43). In this case, the wake field becomes

$$E_{\text{out}} = \left[\frac{4\pi\rho_0}{k_p}\right]\sin\left[\frac{k_p\beta}{2}\right]\cos\left[k_p\left[y-\frac{\beta}{2}\right]\right] . \tag{44}$$

Thus, using the trapezoidal shaped ion bunch, a modulation of the oscillating wake field is found to occur for the wake field that is external to the ion bunch in the first two cases. This modulation disappears for the case of a rectangular shaped ion bunch.

If an asymmetrical triangular ion bunch shown in Fig. 1(b) is used, the charge distribution is given by

$$\rho_b(y) = \begin{cases} \rho_0\dfrac{y}{\alpha}, & 0 \le y < \alpha \\ \rho_0\dfrac{\beta-y}{\beta-\alpha}, & \alpha \le y \le \beta . \end{cases} \tag{45}$$

The wake fields $E_{\text{in}}$ inside of the ion bunch are found using this distribution for the ion bunch in (45). After integration, we find

$$E_{\text{in}} = \begin{cases} \left[\dfrac{2\pi\rho_0}{k_p^2\alpha}\right][1-\cos(k_py)], & 0 \le y < \alpha \\ \left[\dfrac{2\pi\rho_0}{k_p^2\alpha(\beta-\alpha)}\right]\{\beta\cos[k_p(\alpha-y)]-(\beta-\alpha)\cos(k_py)-\alpha\}, & \alpha \le y \le \beta . \end{cases} \tag{46}$$

External to the ion bunch $y > \beta$, the wake field $E_{\text{out}}$ is

$$E_{\text{out}} = \left[\frac{2\pi\rho_0}{k_p^2\alpha(\beta-\alpha)}\right]\{\beta\cos[k_p(\alpha-y)]-(\beta-\alpha)\cos(k_py)-\alpha\cos[k_p(\beta-y)]\} . \tag{47}$$

For the case that assumes that the charge distribution rapidly returns to zero and the initial rise time may remain long , we let $\beta \Longrightarrow \alpha$. The wake field given in (47) becomes

$$E_{\text{out}} = \left[\frac{2\pi\rho_0}{k_p^2\alpha}\right]\{k_p\alpha\sin[k_p(\alpha-y)]-\cos(k_py) +\cos[k_p(\alpha-y)]\} . \tag{48}$$

For the case that assumes that the charge distribution rapidly rises from zero and its return to zero may be long, we let $\alpha \Longrightarrow 0$. The wake field given in (47) becomes

$$E_{\text{out}} = \left[\frac{2\pi\rho_0}{k_p^2\beta}\right]\{k_p\beta\sin(k_py)+\cos(k_py) -\cos[k_p(\beta-y)]\} . \tag{49}$$

Therefore the modulation of wake fields always exists outside of an asymmetrical triangular bunch.

Finally, we consider a half-cycle sinusoidal ion bunch shown in Fig. 1(c). The charge distribution is

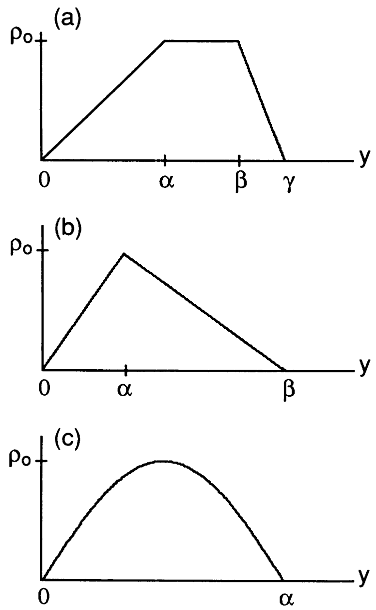


FIG. 1. Shapes of the driving ion bunches. (a) Trapezoidal shape. (b) Triangular shape. (c) Half sine wave shape.

$$\rho_b(y)=\rho_0 \sin\left[\frac{\pi y}{\alpha}\right], \quad 0\le y\le\alpha . \tag{50}$$

For $0\le y\le\alpha$, the wake fields computed from (18) are

$$E_{\rm in}=\left[\frac{2\pi^2\alpha\rho_0}{k_p^2\alpha^2-\pi^2}\right]\left[\cos\left[\frac{\pi y}{\alpha}\right]-\cos(k_p y)\right] \tag{51}$$

and for $y>\alpha$

$$E_{\rm out}=-\left[\frac{4\pi^2\alpha\rho_0}{k_p^2\alpha^2-\pi^2}\right]\cos\left[\frac{k_p\alpha}{2}\right]\cos\left[k_p\left[y-\frac{\alpha}{2}\right]\right] . \tag{52}$$

Therefore no modulation of wake fields is found outside of a half-cycle sinusoidal bunch.

## V. CONCLUSION

In one-dimensional, homogeneous, field-free, drifting plasmas with negative ions where cold ions and the finite electron temperature are assumed, a formula for the period $T_{\rm wake}$ of an oscillating ion wake field is derived using the procedure of Jones and Keinigs [5]. This formula is valid if the ion bunch can be approximated as the Dirac δ function and the following criteria

$$\frac{1-\varepsilon}{C_{s+}^2}<\frac{1}{(v_b-v_{+0})^2}+\frac{\varepsilon M}{(v_b-v_{-0})^2}$$

is satisfied. Otherwise the wake fields are constant or decrease exponentially. The period of the oscillating ion wake field depends on several plasma parameters: the ratio of positive ion mass to negative ion mass, the negative ion concentration, the ion acoustic velocity, the ion plasma frequency, the velocity of the ion bunch, and the difference of velocity between the driving ion bunch and the drifting positive or negative ions.

In a two component plasma, an oscillating wake field exists only if $|v_b-v_{+0}|<C_{s+}$. Furthermore, in a stationary plasma, an oscillating wake field is expected when $v_b<C_{s+}$. Thus the ion acoustic velocity is a critical parameter to determine the oscillation of the ion wake fields. If a stationary plasma is assumed, the same formula for the wake field period can also be derived using the scheme of Ruth *et al.* [8], where an ion bunch is assumed to be represented as the Dirac δ function.

The effect of the shapes of three different ion driving bunches on the oscillating wake fields is calculated. Bunches with a symmetrical shape do not lead to a modulation in the wake field. The amplitude of the wake field depends on the shape of the ion bunch and is directly proportional to the charge density of the bunch. The phase shift of the wake field depends on the parameter $k_p$ given in (19).

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