Evolution of the electron distribution function in intense laser-plasma interactions

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We report a numerical investigation of the time evolution of the electron distribution function (EDF) in a laser-embedded, fully ionized plasma. A distinctive feature of the calculations is removal of the frequently adopted assumption of small anisotropy of the EDF in velocity space. This requires solving a two-dimensional partial differential equation for the EDF. Within the adopted range of parameters, the EDF undergoes significant changes. An initially isotropic EDF transforms rapidly into an anisotropic one characterized by a longitudinal velocity scale larger than the perpendicular one. This longitudinal stretching persists for several cycles of the radiation field, implying the establishment of a two-temperature EDF. We also investigate the time behavior of moments of the EDF for different initial conditions.

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I. INTRODUCTION

The laser-plasma interaction is one of the most important radiation-matter interactions, which continues to attract considerable attention. The permanent interest in laser-plasma interaction is a result of the fundamental character of the processes it makes possible and the prospects for important practical applications. Moreover, the availability of increasingly intense short pulse sources of laser radiation continually allows the laser-plasma interaction to acquire qualitative new features. Under the influence of intense laser radiation, plasma electrons may strongly depart from equilibrium conditions. Therefore only accurate knowledge of the electron distribution function (EDF) ensures an adequate description of the plasma properties and behavior. Unfortunately, the correct description of an EDF for a plasma embedded in an intense laser radiation field remains a difficult problem.

A number of recent theoretical papers [1–6] have addressed this problem generally by assuming a small anisotropy of the EDF in velocity space in order to apply the spherical harmonics expansion. This expansion does not take into account the angular dependence of the EDF. As a rule, the procedure based on spherical harmonics expansion is useful when the electron quiver velocity, $V_E=eE_0/(m\omega)$ is much smaller than the thermal velocity, $V_T=(T_e/m)^{1/2}$. Here, e and m are the electron charge and mass, E_0 and ω are the electric field amplitude and the frequency of the external high-frequency laser radiation, and T_e is the electron temperature. V_E may be much smaller than V_T, even in the case of very powerful laser sources, provided that the frequency ω is appropriately high. If the criterion $V_E/V_T \ll 1$ is fulfilled, all the information about the plasma properties is contained in the isotropic part of the EDF, $f(v)$, where v is the length of the velocity vector. Investigations in recent years [1–7], restricted to the case of a uniform plasma dominated by Coulomb collisions, have provided sufficient and reliable information on the evolution and characteristics of the small anisotropy EDF. For instance, applying a high-frequency electric field to a fully-ionized plasma, after a state of relaxation [6], an arbitrary initial distribution transforms into the self-similar distribution [1,2]

$$\ln f(v) \approx -v^5 \tag{1}$$

if, for the plasma parameters, we have

$$(V_E/V_T)^2 \gg 3/Z \tag{2}$$

with Z the ion charge.

A more general self-similar EDF than Eq. (1), corresponding to the case for which electron-electron (e-e) collisions are taken into account self-consistently, has been derived in Ref. 6. When terms that account for e-e collisions are retained, the equation for the isotropic part of the EDF is nonlinear, and further approximations are required for its solution. Nevertheless, comparison of the approximate, analytical, self-similar EDF and the numerical solution reported in Ref. 6 has shown that sufficiently accurate EDF can be obtained with e-e collisions included. Interesting contributions to the effort to construct an EDF in laser-embedded plasmas have also been reported recently in Ref. 4. Thus, on the whole, for cases in which laser radiation interacting with fully ionized plasmas produces EDFs with small anisotropy, these EDFs can be determined with good accuracy, and basic plasma properties and processes can be studied with confidence. For the case in which the laser radiation is expected to produce a strongly anisotropic EDF, the available information is sparse, and the study and solution of the related equations is still an open problem.

Very intense laser sources are becoming increasingly available in laboratory practice. It is therefore worthwhile to extend investigation of laser-embedded

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EDFs to the case of an arbitrary degree of anisotropy in order to study plasma properties and processes under more general conditions. The present work is a contribution to the analysis of the EDF behavior and evolution without the limitation of the small-anisotropy assumption. Here we address only the case of a uniform, fully ionized plasma with a value of the ion charge Z that allows us to disregard e-e collisions. This requires solving a two-dimensional partial differential equation using a modified version [8] of the alternating direction method [9].

In Sec. II we concisely outline the theoretical framework used and define the quantities which are the object of our calculations. Section III is devoted to selected numerical calculations and the related discussion of the results. Concluding remarks are given in Sec. IV.

II. MASTER EQUATION

The evolution of the velocity EDF in a uniform, fully-ionized plasma dominated by Coulomb collisions is described by the well known equation [10]

$$\frac{\partial f(\mathbf{v})}{\partial t}+\frac{eE_0}{m}\cos(\omega t)\frac{\partial f(\mathbf{v})}{\partial \mathbf{v}} =\frac{1}{2}\frac{\partial}{\partial v_j}\{\nu_{ei}(v)[v^2\delta_{jl}-v_jv_l]\}\frac{\partial f(\mathbf{v})}{\partial v_l}\ , \tag{3}$$

where v_j is the jth component of the electron velocity, $\nu_{ei}=4\pi Ze^4N_e\ln\Lambda/(m^2v^3)$ is the electron-ion collision frequency, N_e is the electron density, and ln Λ is the Coulomb logarithm. The laser field is assumed to be monochromatic and linearly polarized along the z direction. Equation (3) does not contain terms to account for the e-e collisions. Equation (3) is valid provided that condition (2) is fulfilled. Silin [10] showed that Eq. (3) may be used to describe fast time-varying processes, such as the interaction of a plasma with a high-frequency field. For a time-varying process, we modify the definition of the Coulomb logarithm [10], which at fixed plasma density and field frequency may be considered a constant. From Eq. (3), using the spherical harmonic expansion and averaging over the field period, the well known equation for the isotropic part of the EDF results [1–6,9]. We intend to study the plasma properties without any additional assumption about the angular dependence of the distribution function and, hence, we must solve Eq. (3) directly. We cannot average over the field period since we do not know in advance the "fast" time dependence of the distribution function. It is only under the small-anisotropy assumption that the anisotropic part of the EDF oscillates together with the field, thus allowing the isotropic part to be averaged over the field period and the "slow" time dependence to be extracted. In our case, averaging over the field period is possible only after solving Eq. (3). As suggested by the symmetry of the problem, the natural coordinate system for our case is cylindrical. Moreover, in view of the numerical modeling developed below, it is convenient to define the following equation to remove the velocity gradient of the distribution function:

$$\mathbf{v}'=\mathbf{v}-\mathbf{z}V_E\sin(\omega t)\ , \tag{4}$$

with $\mathbf{v}'$ and $\mathbf{v}$, respectively, the electron velocity in the oscillating frame and in the rest frame, and $\mathbf{z}$ a unit vector directed along the z axis.

Then Eq. (3) may be written as

$$r\frac{\partial F}{\partial t}=\frac{\partial}{\partial r}\left[\nu_{ei}\frac{r}{2}\left[z_t^2\frac{\partial F}{\partial r}-z_tr\frac{\partial F}{\partial z}\right]\right] +\frac{\partial}{\partial z}\left[\nu_{ei}\frac{r}{2}\left[r^2\frac{\partial F}{\partial z}-z_tr\frac{\partial F}{\partial r}\right]\right]\ , \tag{5}$$

where $F(\mathbf{v}',t)=f(\mathbf{v},t), \nu_{ei}=4\pi Ze^4N_e\ln\Lambda/[m^2(r^2+z_t^2)^{3/2}]$, r and z are, respectively, the component of the electron velocity in the oscillating frame perpendicular and parallel to the oscillating electric field, and z_t is the z component of the electron velocity in the rest frame. As pointed out above, choosing the oscillating coordinate system removes the velocity gradient of the EDF but, however, the coefficients of Eq. (5) are now functions of time. The boundary conditions pertinent to Eq. (5) are that the distribution function is equal to zero for $v\to\infty$ and the particle flow vanishes for $r\to 0$. To calculate the moments of the EDF, the EDF is normalized to the condition

$$2\pi\int F(r,z,t)r\,dr\,dz=1\ . \tag{6}$$

We define the following first moments of the EDF: (a) the average component of the electron velocity along the direction of the electric field

$$V_\parallel=2\pi\int_0^\infty r\,dr\int_{-\infty}^\infty dz\,F(r,z,t)[z+V_E\sin(\omega t)] =\langle z\rangle(t)+V_E\sin(\omega t)\ , \tag{7}$$

(b) the average parallel kinetic energy

$$E_\parallel=m\pi\int_0^\infty r\,dr\int_{-\infty}^\infty dz\,F(r,z,t)[z+V_E\sin(\omega t)]^2 =\frac{m}{2}[\langle z^2\rangle(t)+2\langle z\rangle(t)V_E\sin(\omega t)+V_E\sin^2(\omega t)]\ , \tag{8}$$

(c) the average perpendicular velocity

$$V_\perp=2\pi\int_0^\infty r\,dr\,r\int_{-\infty}^\infty dz\,F(r,z,t)\ , \tag{9}$$

(d) the average perpendicular kinetic energy

$$E_\perp=m\pi\int_0^\infty r\,dr\int_{-\infty}^\infty dz\,r^2F(r,z,t)\ . \tag{10}$$

III. NUMERICAL CALCULATIONS AND RESULTS

Equation (5) is now solved numerically, and Eqs. (7)–(9) are evaluated. To solve the two-dimensional partial differential Eq. (5), we make use of the alternating direction method [8]. However, because of the presence of mixed derivatives in Eq. (5), the usual version is not convergent. Therefore we use the modification of the method proposed in Ref. 7, to which the interested reader is referred for details. For the sake of brevity, we omit cumbersome and poorly instructive expressions and report the numerical results only.

Numerical calculations have been carried out in three different regimes characterized, respectively, by $V_E/V_T<1$; $V_E/V_T\approx 1$; $V_E/V_T>1$. The electron plasma is assumed to be described initially by a Maxwellian distribution function, and its parameters have been chosen in such a way that the electron-ion collision frequency is much less than the frequency of the external electromagnetic field. As it is well known, a collisionless plasma driven by an external electromagnetic field can be described, in the oscillating frame defined by Eq. (4), by a stationary EDF. Therefore in our case the EDF is expected not to change sensibly during a field period T, as $\nu_{ei}T\ll 1$.

We now illustrate in detail the behavior of the EDF in the above mentioned regimes.

A. $V_E/V_T<1$.

In this regime, our numerical results are in agreement with those of Refs. 1 and 6, in which an analytical expression for the EDF has been obtained. In particular, we find that for values of V_E smaller than $0.5V_T$, the shape of the EDF does not change appreciably during time intervals as large as twenty cycles of the oscillating field. By integrating the distribution function over the angular coordinates that define the electron velocity, we obtain a one-dimensional distribution function that behaves as the well known EDF reported in Ref. 1. In Fig. 1 this one-dimensional distribution function is shown as a function of $(V/V_T)^2$. In Fig. 2 the distribution function, evaluated at such a time that the velocity of the oscillating frame is zero, is shown as a function of z and r, and in Fig. 3 the momenta defined by Eqs. (7)–(10) are plotted as functions of time.

We observe that the ratio $E_\perp/E_\parallel$, evaluated at times

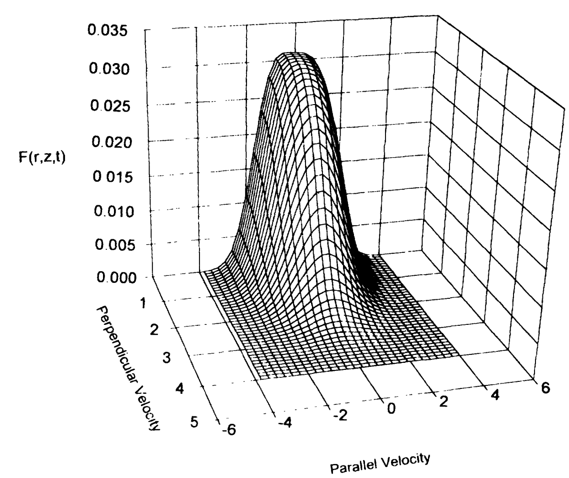

FIG. 2. The electron distribution function evaluated in the middle of the 20th period of the laser field. At this instant the velocity of the oscillating frame is zero. Laser and plasma parameters are as in Fig. 1. Both the parallel and the perpendicular velocities are expressed in units of V_T.

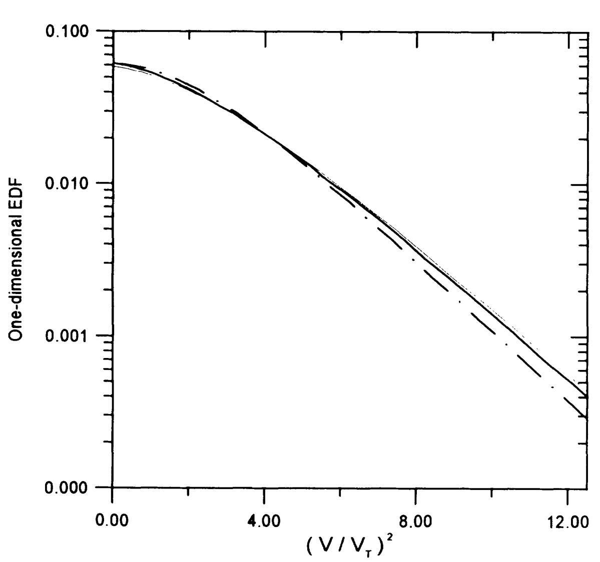

FIG. 1. The one-dimensional EDF as a function of the squared electron velocity evaluated at three different instants t. The laser and plasma parameters are: $E_0=2\pi\times 10^6$ V/cm, $\hbar\omega=0.4$ eV, $Z=5$, $\nu_{ei}T=0.1$, $V_E/V_T=0.5$. ν_{ei} and T are, respectively, the electron-ion frequency and the laser period. V_E and V_T are, respectively, the electron quiver velocity and the initial thermal velocity. Thin line: $t=19T+\pi/2$. Dash-dotted line: $t=19T+\pi$. Thick line: $t=19T+3\pi/2$.

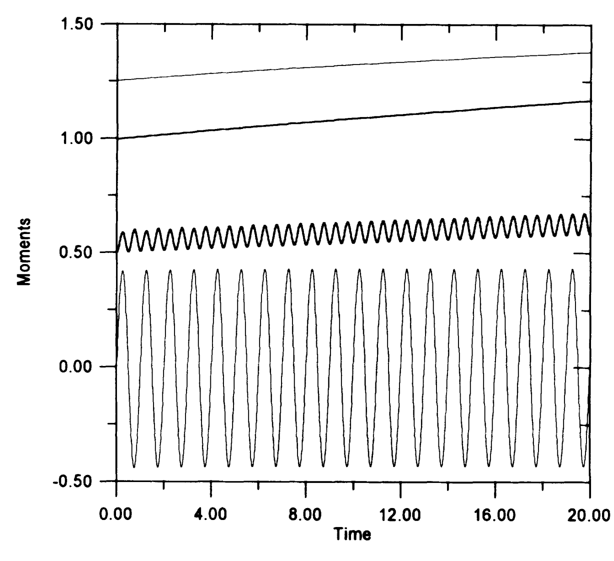

FIG. 3. The first moments Eqs. (7)–(10) as a function of time. The velocities and the energies are, respectively, expressed in units of the initial thermal velocity and the kinetic energy of the electron plasma. Laser and plasma parameters are as in Fig. 1. The time is expressed in units of the laser period T. Thin monotonic curve: the average perpendicular velocity. Thin, large-amplitude, oscillating curve: the average parallel velocity. Thick monotonic curve: the average perpendicular kinetic energy. Thick oscillating curve: the average parallel kinetic energy.

such that the velocity of the oscillating frame is zero, is nearly equal to two for all the times considered. The EDF, in the oscillating frame, therefore remains essentially isotropic [with the limitation $F(r,z,t)\approx F(r,-z,t)$]. For example, see Fig. 3, where Eqs. (7)–(10) have been plotted as functions of time. Hence, although the electron-ion collisions are responsible for the asymmetry in the distribution function, the energy absorption from the laser pump proceeds in such a way that, in the oscillating frame, it is equally shared among each degree of freedom of the plasma electron. Ultimately, all of the above properties of the EDF, in the regime $V_E/V_T<1$, corroborate the validity of the known procedure for solving the Boltzmann equation for a plasma driven by an electromagnetic field by expansion of the EDF in terms of spherical harmonics.

B. $V_E \approx V_T$

The shape of the EDF is similar to that for the previous case with, however, the degree of anisotropy now more emphasized (see Fig. 4). Note that for all the cases we use the same initially isotropic function. In the first case (case A), such a function in practice does not change, although the typical scale of the longitudinal motion is larger than the perpendicular one. The maximum value of $F(r,z,t)$ is reduced by approximately a factor of two. Further, more significant modifications appear in the tail of the one-dimensional EDF (see Fig. 5). In fact, in the course of a field period the tail of the EDF is considerably affected, while the bulk suffers only small modifications.

Another distinct feature is the modulation of the relative maxima of the curve for $E_{\parallel}$ as a function of time in Fig. 6. From inspection of this figure, it seems reasonable to assume that $\langle z^2\rangle(t)$ is a slowly increasing function of time. We can then linearize $\langle z^2\rangle(t)$ during half a cycle of the oscillating electromagnetic field. In this way, we can assume that the increase of $\langle z^2\rangle(t)$ between two consecutive maxima is roughly the same as that occurring between the two associated consecutive minima. This is, in fact, easily confirmed by scrutinizing the numerical values of $\langle z\rangle(t)$ and $E_{\parallel}$. Moreover, for $\langle z\rangle(t)=0$, the upper envelope of the curve for $E_{\parallel}$ as a function of time should behave in the same way as the lower one. The oscillations occurring in the maxima are, in fact, caused by the lack of symmetry suffered by the distribution function because of the combined action of the electromagnetic external field and the electron-ion collisions. Such an asymmetry [$F(r,z,t)\neq F(r,-z,t)$] gives rise, in the oscillating frame, to a drift velocity $\langle z\rangle(t)$ from which the

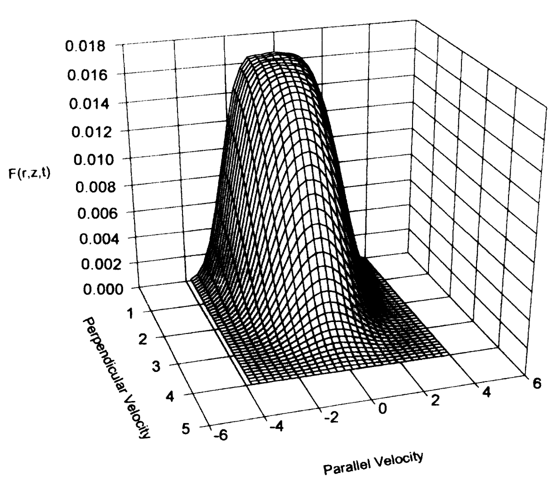

FIG. 4. The electron distribution function as in Fig. 2, with $E_0=4\pi\times10^6$ V/cm and $V_E/V_T\approx1$.

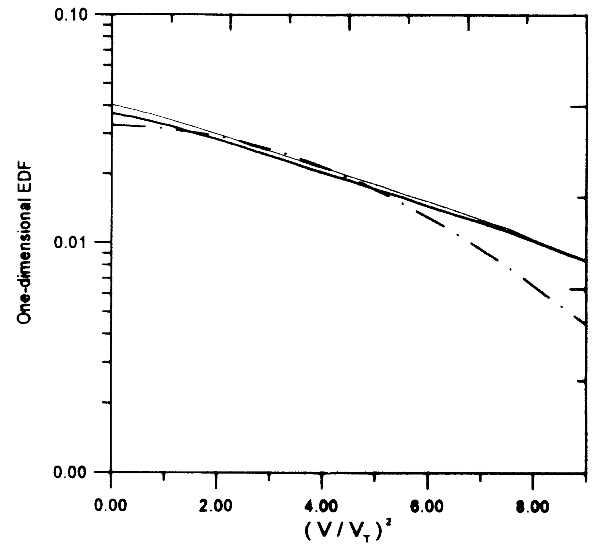

FIG. 5. The electron distribution function as in Fig. 1, with E_0 and V_E/V_T the same as in Fig. 4.

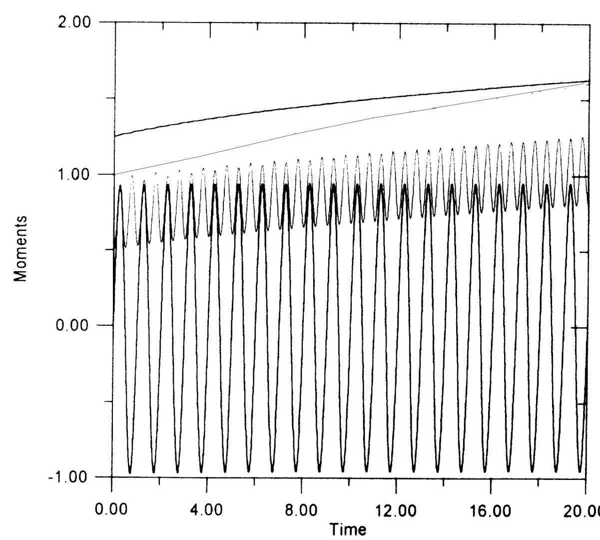

FIG. 6. The first moments of the EDF as in Fig. 3. Laser and plasma parameters are as in Fig. 4. Thick monotonic curve: the average perpendicular velocity. Thick, large-amplitude, oscillating curve: the average parallel velocity. Thin monotonic curve: the average perpendicular kinetic energy. Thin oscillating curve: the average parallel kinetic energy.

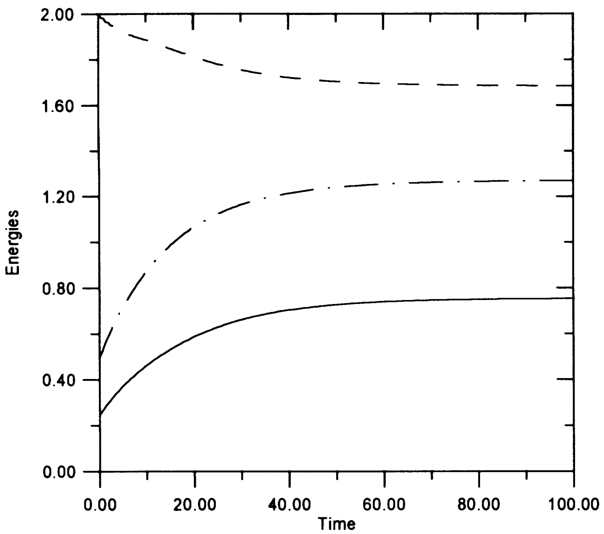

FIG. 7. The ratio $E_{\perp}/E_{\parallel}$ as a function of time evaluated at $\omega t = n\pi$ (n integer) for $V_E/V_T \approx 2$ (dashed curve). Dash-dotted curve: perpendicular energy. Full curve: parallel energy.

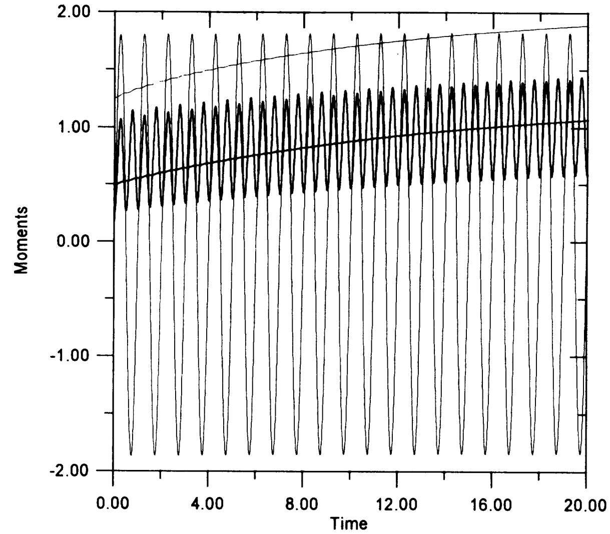

FIG. 9. The first moments of the EDF as in Fig. 3, with E_0 and V_E/V_T the same as in Fig. 7.

modulations result. Of course such modulations, though less pronounced, are also present in the regime $V_E/V_T < 1$.

C. $V_E > V_T$

The significant feature that distinguishes this regime is that the EDF, in the oscillating frame, experiences enhanced stretching in the direction of the oscillating electric field and is broadened and lowered further because of heating. As shown in Fig. 7, the ratio $E_{\perp}/E_{\parallel}$, calculated for up to 100 cycles of the radiation field, appears to increase its departure from the isotropic value (2). A slowly evolving anisotropic EDF has therefore been established. In Fig. 8 the EDF is shown as a function of the components of the electron velocity perpendicular and parallel to the oscillating electric field. The rate of energy absorption decreases with the time, as can be seen by the quite regular behavior followed by $E_{\perp}(t)$ and the dumping of the oscillations of the upper envelope of the curve for $E_{\parallel}(t)$ (see Fig. 9).

In addition, numerical data not reported here show that the drift velocity $\langle z \rangle(t)$, after increasing during the first few cycles of the oscillating electric field, starts decreasing. This is reflected in the dumping of the oscillations of the upper envelope of $E_{\perp}(t)$ and, correspondingly, in the reduction of the heating rate with increasing time.

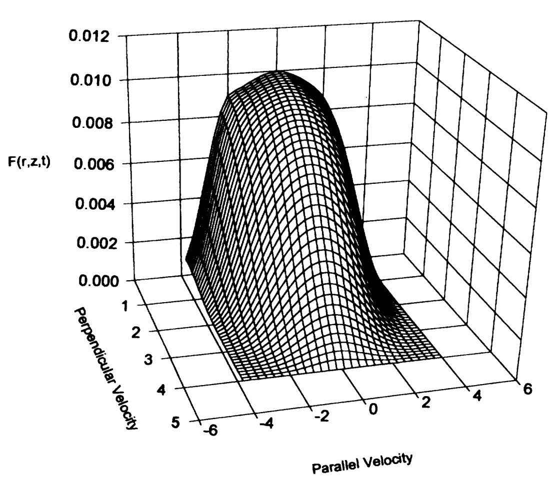

FIG. 8. The electron distribution function as in Fig. 2, with $E_0 = 6\pi \times 10^6$ V/cm and V_E/V_T the same as in Fig. 7.

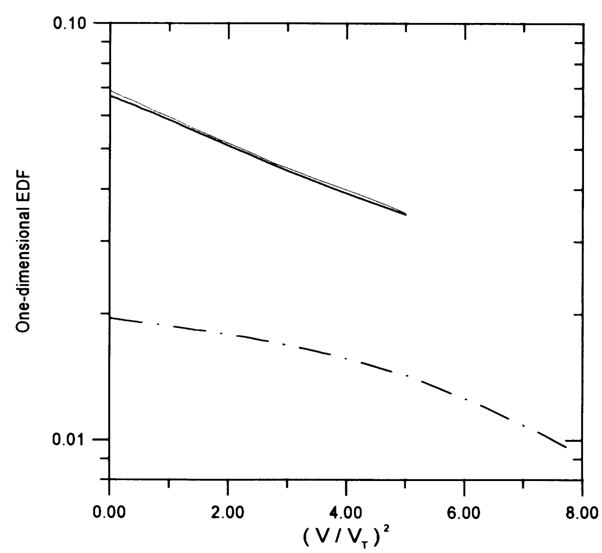

FIG. 10. The electron distribution function as in Fig. 1 with E_0 and V_E/V_T the same as in Fig. 7. Thick line: $t = 19T + \pi/2$. Thin line: $t = 19T + 3\pi/2$. The dash-dotted line corresponds to $t = 19T + \pi$ and must be multiplied by 5.

Finally, in Fig. 10 the instantaneous one-dimensional EDF exhibits a behavior rather different as compared with that in Ref. [1] for the case of small anisotropy.

IV. CONCLUSION

The reported numerical results allow us to conclude that the time evolution of the EDF, for the case under consideration, may be divided into two stages distinguished by different energy absorption rates.

During the first stage, the initially isotropic EDF, under the joint action of an intense electromagnetic field and the electron ion-collisions, transforms rapidly into an anisotropic EDF characterized by a longitudinal velocity scale larger than the perpendicular one. The value of this relative stretching depends on the ratio V_E/V_T and becomes significant when this ratio is close to or larger than one. In other words, the initial electron temperature and the radiation field parameters control the shape of the "first-stage" anisotropic EDF. In the case of small anisotropy ($V_E/V_T \ll 1$), the EDF shape remains similar to the initial one for practically all times considered, changing only in scale because of the heating. For values $V_E/V_T > 1$, the relative longitudinal stretching persists for many cycles of the radiation field, implying establishment of two-temperature EDF. Because these calculations are very time consuming, we presently are not in the position to make definite statements about the evolution of the EDF for times larger than 100 cycles.

The reported results are based on a numerical solution of the Boltzmann equation. These results complement similar results [1,2,4,6,7] obtained using approximate analytical treatments and serve as a stimulus for further work. As a matter of fact, the approximate analytical treatments, in principle, may have an heuristic power not shared by the numerical treatments. The latter may provide rigorous results and, ultimately, a check on the limiting, approximate treatments of the same problem. In this context, it is hoped that rigorous numerical results can be used as "input data" for developing useful and predictive analytical treatments of the important but difficult topic of nonstationary EDFs in plasmas subject to strong laser field radiation.

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[1] A. B. Langdon, Phys. Rev. Lett. **44**, 575 (1980).
[2] R. Balescu, J. Plasma Phys. **27**, 553 (1982).
[3] R. D. Jones and K. Lee, Phys. Fluids **25**, 2307 (1982).
[4] B. N. Chichkov, S. A. Shumsky, and S. A. Uryupin, Phys. Rev. A **45**, 7475 (1992).
[5] D. Deck, Laser and Part. Beam **5**, 49 (1987).
[6] P. Porshnev, G. Ferrante, and M. Zarcone, Phys. Rev. E **48**, 2081 (1993).
[7] S. McKee and A. R. Mitchell, Comput. J. **13**, 81 (1970).
[8] M. B. Allen III, I. Herrera, and G. F. Pinder, *Numerical Modeling in Science and Engineering* (Wiley, New York, 1988).
[9] I. P. Shkarofsky, T. W. Johnston, and M. P. Bachinski, *The Particle Kinetics of Plasma* (Addison-Wesley, Reading, MA, 1966).
[10] V. P. Silin, Zh. Eksp. Teor. Fiz. **38**, 1771 (1960) [Sov. Phys. JETP **11**, 1277 (1960)].